

# Spectrum of Responding using the Integration of GREEN

Tahar Brahim<sup>1</sup>, Mustapha Amor<sup>2</sup>.

<sup>1</sup> Ecole Normale Supérieure de Laghouat, laboratoire des chimiques et physique appliquées- Laghouat, Algeria. [t.brahimi@ens-lagh.dz](mailto:t.brahimi@ens-lagh.dz)

<sup>2</sup> Civil Engineering and Sustainable Development Laboratory, Faculty of Sciences and Technology, Ziane Achour University of Djelfa, Algeria, [m.amor@univ-djelfa.dz](mailto:m.amor@univ-djelfa.dz)

ARTICLE INFO	ABSTRACT
Received: 25 Dec 2024	The creation of a spectrum of response to a seismic force for a region is one of the most important things that are focused on building facilities and all projects in any Region. The spectrum of response is the graphic curve of the maximum (displacement, speed or acceleration) in terms of the role, but the approved is acceleration We will draw the curve, the spectrum of responding to two regions in Algeria, relying on the results of my article [3], which we found the Hustriya energy in terms of the role, as well as the displacement.
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	As well as the use of mathematical relationships such as integration and the theory of Green, which will help us find a spectrum of response.
	<b>Keywords:</b> Spectrum of response, Hastre energy, integration of Green.

## 1. Introduction

In this study, we will determine the acceleration and displacement response spectrum for two earthquakes in Algeria, namely Blida on (01/03/2018) and Bejaia on (03/18/2021), where we rely on the article that gives us the linear relationship for both hysteretic energy and displacement in terms of the role and importance of the edge of this envelope.

## 2. Spectrum presence

It is a graph of the maximum response of a structure with one degree of freedom as a function of periodic time under the action of a given dynamic signal (such as an earthquake).

## 3. Create an Earthquake Intensity Response Spectrum

A large number of single degree of freedom (SDOF) structures with different properties and periodicity are brought from different so that they all have the same decay rate.

For each structure, the maximum response to displacement, velocity, or acceleration is found. A graph of the maximum response with periodic time is plotted

## 4. Theory Green [2]

Let  $D$  be continuous and compact  $\partial D$  whose edge is a closed directed path

$f : U \rightarrow \mathbb{R}^2 / (x, y) \rightarrow (f_1, f_2)$  radial follower of class  $C^1$  where  $U \subset \mathbb{R}^2$  is open  $D \subset U$

$$\int_{\partial D^+} (f_1(x, y)dx + f_2(x, y)dy) = \iint_D \left( \frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y} \right) dx dy \quad (1)$$

## 5. Response Spectra

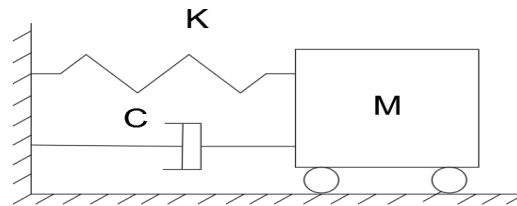
Structural response spectra provide an excellent means to describe a ground acceleration time series. To obtain response spectra, a single degree of freedom system with viscous damping is used. Fig. 9. System stiffness can be bilinear, or stiffness degrading. The method used to obtain response spectra for a particular acceleration time series

is step by step numerical integration of the general equation assuming linear acceleration in each time step, [12]. The equation is:

$$MU''(t) + cU(t) + R(U, t) = -MU''_g(t) \quad (2)$$

Where  $U(t)$  is the relative displacement of the mass relative to the ground,  $M$  is the mass,  $c$  is the damping coefficient,  $U''_g(t)$  is the ground acceleration relative to the fixed reference axis, and  $R(U, t)$ .

**Figure.1** Mass Spring System



## 6. Damage Measures

Several researchers have proposed several measures. These measures are expressed as functions of the structural response parameters to summarize the effect of a time series of acceleration on linear and nonlinear systems. A comparative study of the measures was conducted by [10], and these measures were also applied by [11].

### Maximum displacement

For design purposes, it is generally important to know the maximum absolute value of the response subject to a time series of acceleration

$$U_{max} = \max|U(t)| \quad (3)$$

A plot of the maximum value  $U_{max}$  as a function of the natural vibration frequency or period with systems with the same damping value and a range of periods, provides a conventional spectrum. Maximum Displacement Ductility The maximum displacement ductility is defined as the normalized quantity given by dividing the maximum absolute of the displacement response during its complete excitation by the yield displacement of the system.

A maximum displacement ductility  $\mu$  less than one indicates an elastic response. Normalized Hysteretic Energy. The normalized hysteretic Energy is defined by the amount of energy dissipated by the system for its complete excitation divided by twice the energy absorbed at first yield plus one. The energy dissipated in a structure with hysteretic load deformation relationship is given by

$$E_H = \int R(U, t)dt - E_s \quad (4)$$

Where  $E_s$  is the elastic strain energy given by

$$E_s = \frac{1}{2} K_y U_y^2 \quad (5)$$

He Normalized Hysteretic

Energy will be then

$$E_{NH} = 1.0 + \frac{E_H}{R_y U_y} \quad (6)$$

## 7. Hysteretic Energy and Distance in Terms of Period

We found earlier in the article that the logarithm of the distance is linearly proportional to the logarithm of the period. The logarithm of the hysteretic energy is also linearly proportional to the logarithm of the period.

$$\log(\mu) = C_1 + C_2 \log(T) \quad (7)$$

$$\log(EH) = C_3 + C_4 \log(T) \quad (8)$$

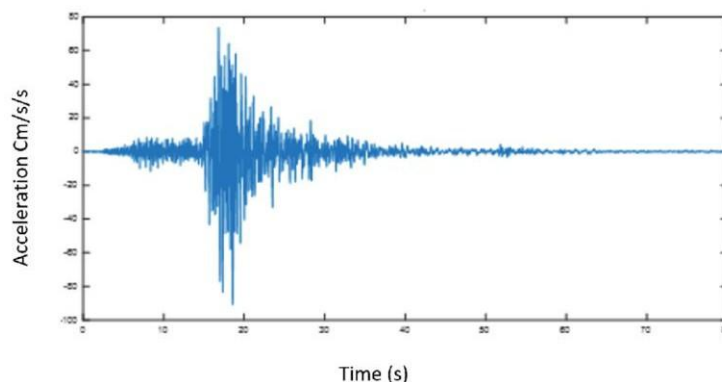
where  $C_1$ ,  $C_2$ ,  $C_3$ ,  $C_4$  are constants, and  $T$  is period

## 8. Applications

### Acceleration Time Series Modeled

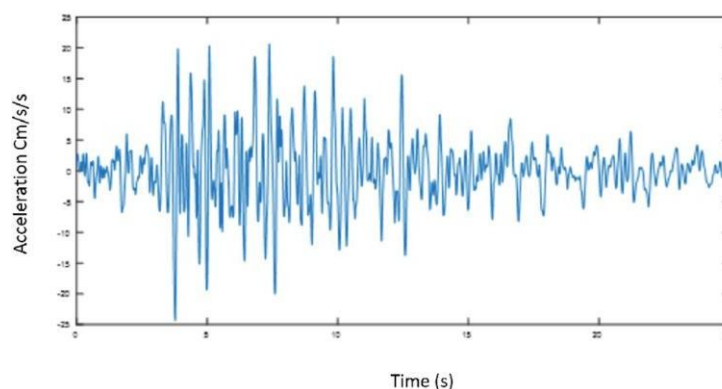
It is worth noting that data for two acceleration time series were consistently used in this study. These countries included Bejaia with 15900 data points (a digitization increase of 0.005 seconds), and Blida with 5498 data points (a digitization increase of 0.005 seconds). Thus, the crucial difference between the series was the number of points to be estimated.

Time series plots of the measured acceleration are shown in Figure.2 and Figure.3.



**Figure.2** Bejaia Acceleration Time Series

Illustrates acceleration in ( $cm/s^2$ ) and time  $t$  in( $Sec$ ), it has zero mean and variable standard deviation. it is a non-stationary time series.

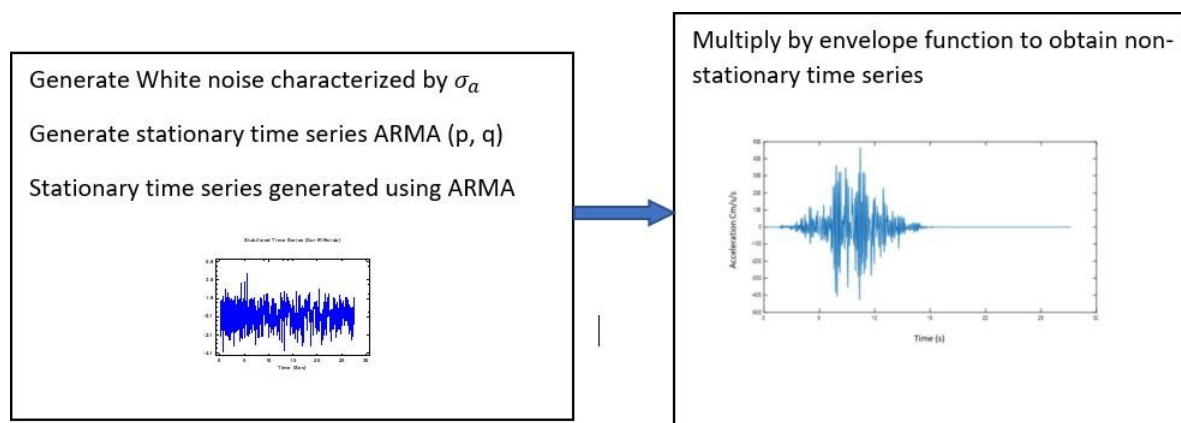


**Figure. 3** Blida Acceleration Time Series

Illustrates acceleration in ( $cm/s^2$ ) and time  $t$  in( $Sec$ ), it has zero mean and variable standard deviation. it is a non-stationary time series.

## Envelope function

In this study of an acceleration time series, the choice of model depends on the nature of the intended application. For design purposes, it is necessary to use as few parameters as possible with a parametric envelope function. It is understood from the previous sections that an acceleration time series event is a non-stationary time series. The event is divided by the adjustment function to obtain the stationary value of the series. The adjustment function obtained is used to fit the smooth parametric envelope function. A simple model for a single peak event is given by [7]

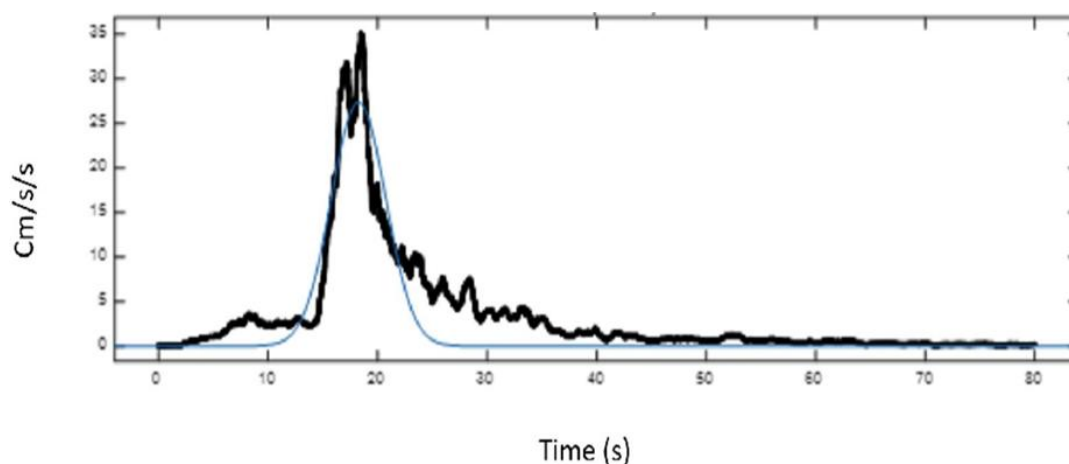


**Figure. 4** Generate stationary time series ARMA

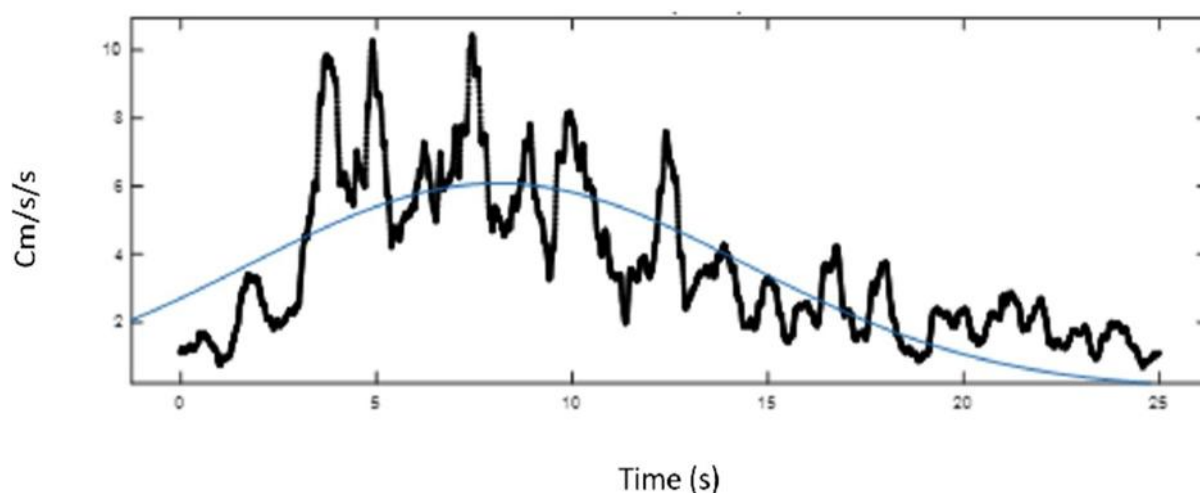
They are  $s(t) = \alpha e^{-\left(\frac{t-\beta}{\gamma}\right)^2}$  constants found by fitting the function to the estimated variance using the least squares method. This function  $\alpha, \beta, \gamma$  where is effective in fitting adjustment functions with narrow peaks (9)

**Table 1** : Envelope Function Parameters

Parameters	Blida	Bejaia
$\alpha$	27,37	126,19
$\beta$	18,27	8,13
$\gamma$	3,30	3,29



**Figure. 5** Blida envelope fuction



**Figure. 6** Bejaia envelope fuction

### 9. Results:

We calculate the area of the envelope representing the hysteretic energy using Green's integral. Through the relationship (1) and also the relationship that gives us hysterical energy (8) The relationship of period and movement

$$\omega^2 = 2\pi T \quad (10)$$

equation of movement

(11)

From all the  $x = x_0 \cos(\omega t + \varphi_0)$

equations we mentioned in the result (1), (8), (10) and (11) We get the differ-ential equation

$$x'' = 2\pi T x \quad (12)$$

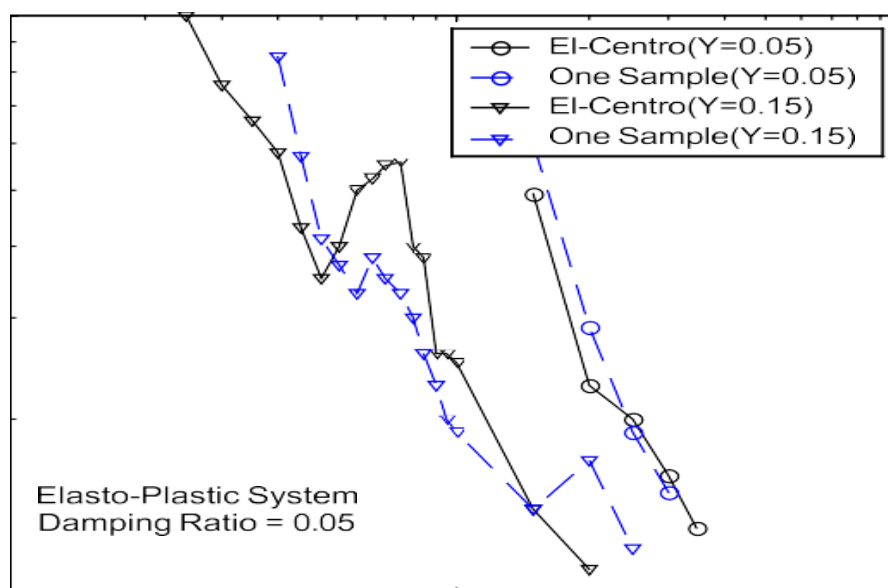
This differential

equation gives us the acceleration results in terms of the period, through which we draw the response spectrum curve, which is relied upon in construction facilities in any region.

### 10.Results of Applications:

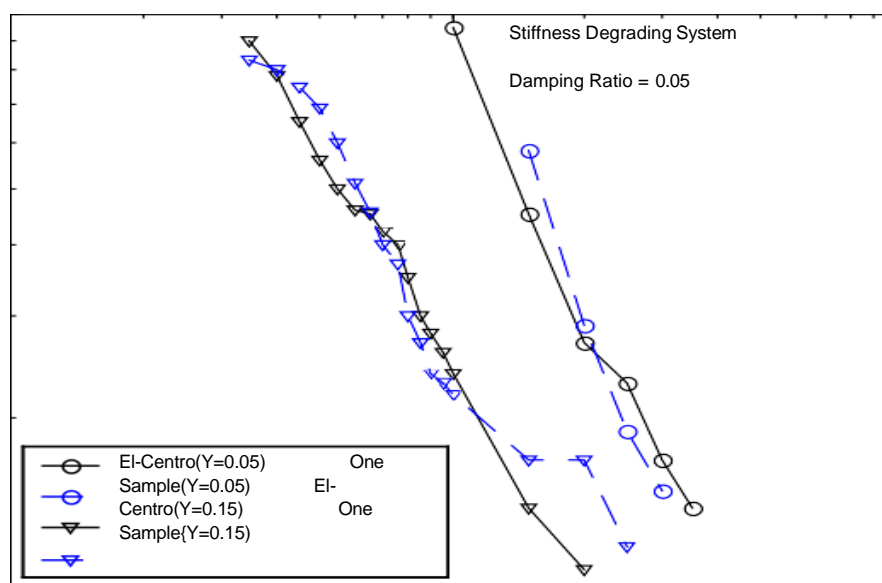
Figure (7),(8),(9)and(10) Confirmation of the results of the previous article equation 7 and 8

While Figure ( 11) Confirmation of the results of the equation (12)



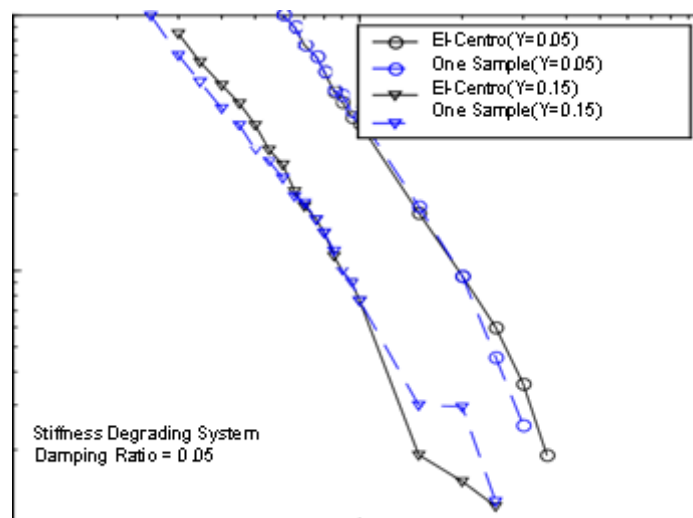
**Figure. 7** Response spectra – original and one sample

**Figure 7** is the displacement ductility as an output from a Stiffness Degrading, single degree of freedom system. (Damping 5% and Yield Ratio 5%). The input is Blida acceleration time series, and ten simulated acceleration time series from the same ARMA (2, 1) and the envelope function that represent Blida. The output mean displacement ductility and the original acceleration time series displacement are within the (mean  $\pm$  1 Standard Deviation).



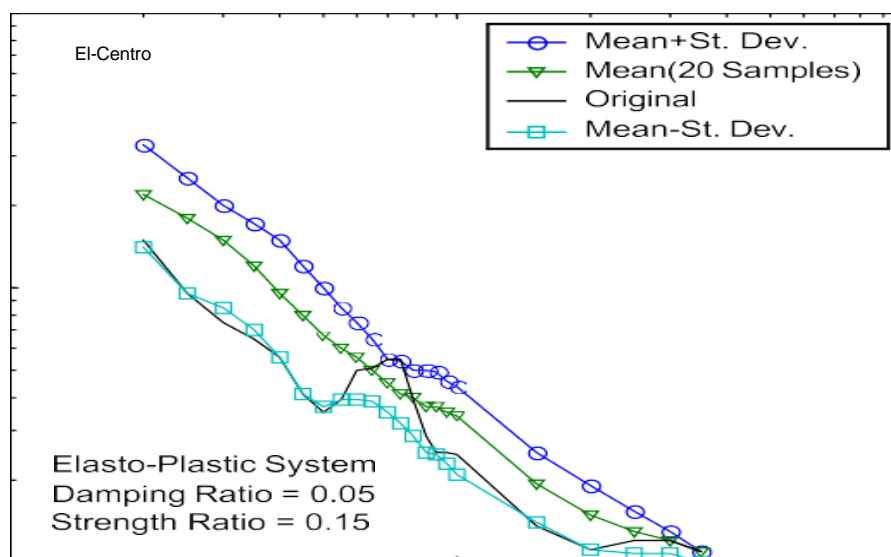
**Figure. 8** Is the displacement ductility as an output from a Stiffness

Degrading, single degree of freedom system. (Damping 5% and Yield Ratio 5%). The input is Bejaia acceleration time series, and ten simulated acceleration time series from the same ARMA (2, 1) and the envelope function that represent Bejaia. The output mean displacement ductility and the original acceleration time series displacement are within the (mean  $\pm$  1 Standard Deviation).



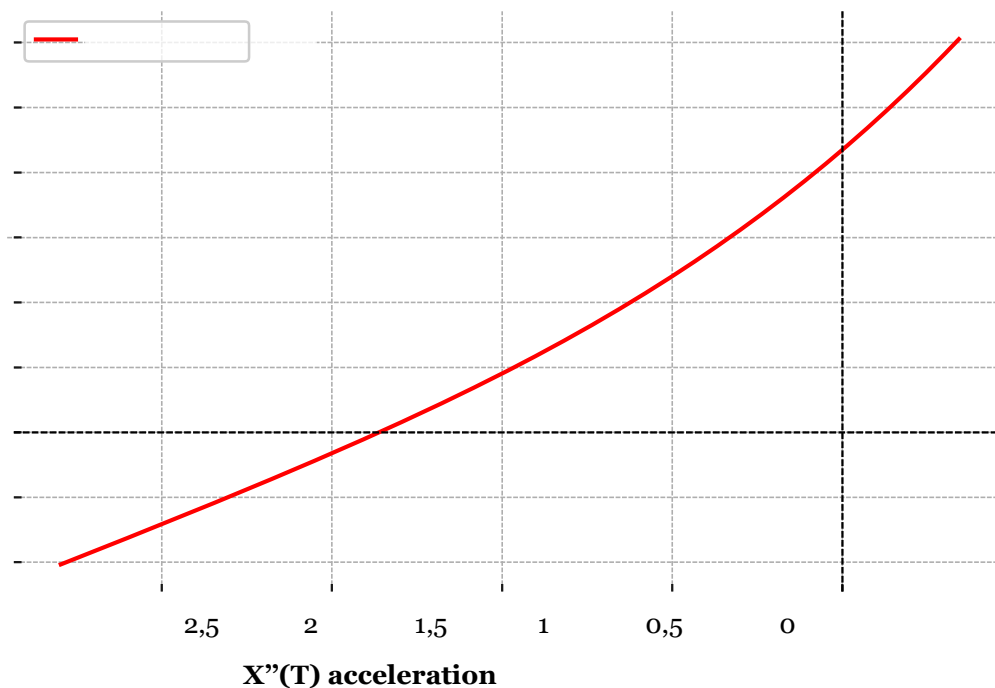
**Figure. 9** Shows the effect of yield ratio on hysteretic energy using a bilinear system and Blida time series as input

For 5% damping the hysteretic energy decrease when the yield ration increases.



**Figure. 10** Shows the effect of yield ratio on hysteretic energy using a bilinear system and Bejaia time series as input

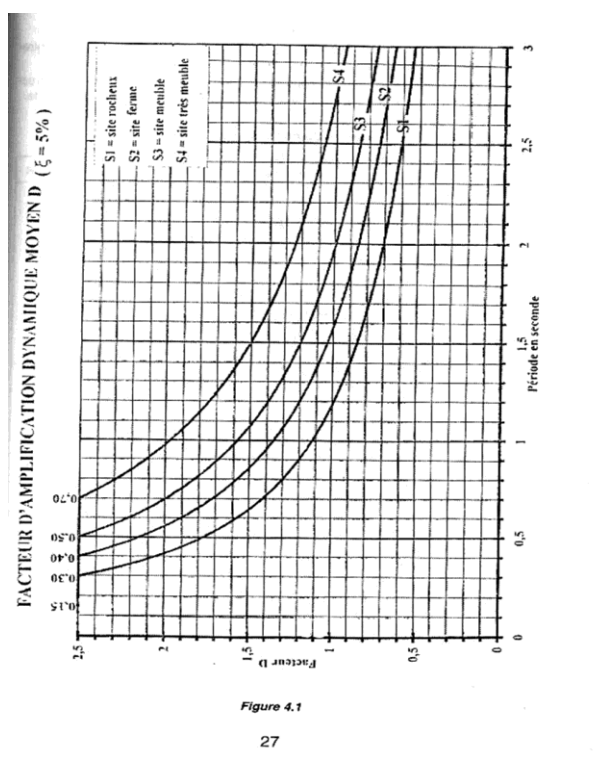
For 5% damping the hysteretic energy decrease when the yield ration increases.



**Figure. 11** This curve is the result of the equation and may match

What is practiced according to the National Authority for the Control of Facilities in Algeria, as shown in the following figure.(12).

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**Figure. 12.** Image of the approved reference page [21]



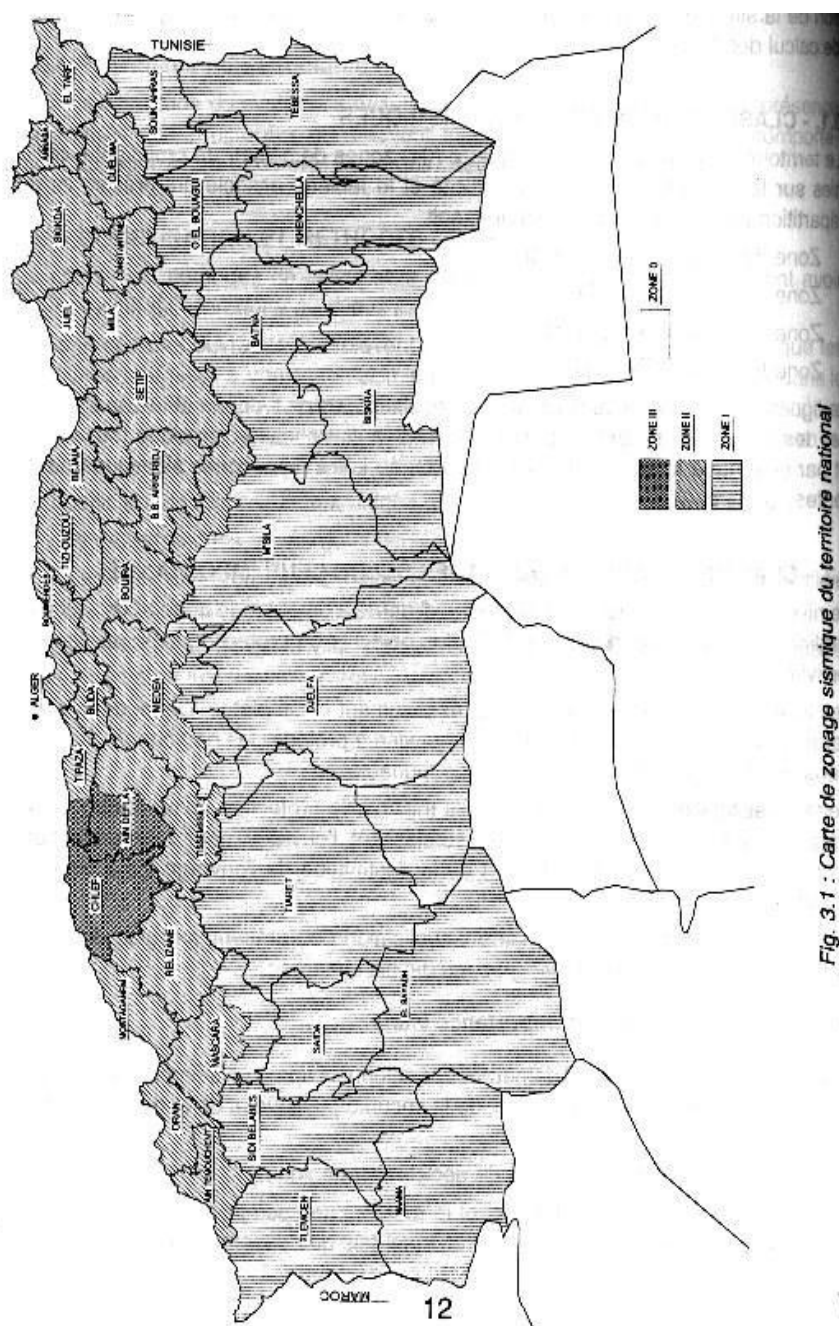


Fig. 3.1 : Carte de zonage sismique du territoire national

Figure. 13. Image of the approved reference page[21]

### 11.CONCLUSIONS :

This approach provides a time domain using a limited number of parameters. The coupled relationship is widely used in practice. Assuming that the acceleration time series (earthquake) corresponds to the main event, this allows for a good description of the response spectra. The response spectra (acceleration) are approximately related to the envelope parameters of the system period, making the result more reliable in determining the building space of a structure. In the future, Stokes' theorem can be used because it is more general than Green's theorem.

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