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Wavelet-Based Stochastic Analysis of Probability Distributions in Financial Markets: A Measure Theory Approach

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ABSTRACT

The purpose of this research was to investigate the applicability of wavelet-based stochastic analysis to determine the probability distribution of financial markets employing measure theory. The research aimed at giving more insights about the markets and the volatility behaviour using sophisticated mathematical models. Historical data of the financial market was analyzed using wavelet transform to obtain the time series in terms of its frequency content. Stochastic analysis methods were then used to describe the distributional characteristics of market changes. Measure theory was incorporated to improve the accuracy of the probability estimates, which gave a strong mathematical base for the evaluation of market behaviour. The wavelet-based approach was able to detect relevant frequency bands associated with the fluctuations of the market, and the results pointed out that the data were non-Gaussian distributed. The use of the measure-theoretic approach was an improvement to the previous probability theory since it facilitated better modelling of the risks and uncertainties in the financial markets especially in cases of extreme events. The study proved that the application of stochastic analysis based on wavelet and measure theory can be used as a strong tool in analyzing the probability distribution of financial markets. It can enhance the accuracy of forecast of market volatility, thus making it as a useful resource in research for scholars, and in practice for practitioners in financial risk management.

Keywords: Wavelet analysis, stochastic processes, measure theory, financial markets, probability distributions, and market volatility, risk analysis.

1. Introduction

Recently financial markets have become a rather complex dynamic environment with high and unpredictable changes in price quotations. It is very essential for the management of risks, investment and formulation of policies to be familiar with these systems. To this end, several mathematical and statistical models have been devised to capture the behaviour of financial markets, especially with an emphasis on volatility and probabilities. Out of these tools stochastic analysis and wavelet-based methods have attracted a lot of attention. China, as an emerging market, also needs both methods due to the complexities of the financial markets and both approaches have a firm foundation in mathematical theory.

Wavelet transforms became one of the most popular methods to analyze the time-frequency features of such non-stationary time series as the ones inherent to the financial markets. While Fourier analysis splits

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signals in terms of constant frequency bands, wavelet analysis can accommodate for multilevel frequency analysis; therefore, effective for data with time-varying frequencies (Daubechies, 1992). This feature is very useful for the financial markets because price changes may occur due to factors working at different time scales; including intraday market fluctuations or long-term drivers such as macroeconomic events.

Random variables are best described by the stochastic processes which are models that attempt to explain the evolution of random variables. In financial markets, the price of assets can be described as a stochastic process whereby future prices depend on current prices and past prices but include an element of randomness. Prior models have consisted of geometric Brownian motion (GBM), which has been extremely popular for explaining the behaviour of asset prices in financial mathematics (Merton, 1973).

Also, the **measure theory** integration is a more sophisticated framework for the further extension of the traditional probability theory for financial modelling, particularly addressing the sensitivity to large deviations from the average such as crises. Probability spaces can be defined through measure theory and measure theory has ways to cope with those events which are not allowed in the traditional probability theory such as having infinite or fractional dimensions (Rudin, 1987). This is especially important in financial areas, where, for example, the occurrence of a supercritical event does not correspond to normal statistical models; the probability distribution of which, as a rule, may be skewed or have high kurtosis (Mallat, 1999).

1.1 Wavelet Transform in Financial Markets

Indeed, many researchers have also explained that the wavelet transform is a significant tool in the time series since it can decompose complicated signals into time frequencies. This method is well appreciated in financial markets because price series involve a lot of random fluctuation and there can be all sorts of behaviors at different levels of time scales. Some of the other methods that have gained popularity among researchers for the detection of subtle patterns, trends and clusters of volatility that may not be easily visible with the use of conventional statistical tools are Wavelet analysis started recently by Gençay, Selçuk and Whitcher (2002).

Keeping the above explanation in mind wavelets are most useful when studying the financial market, especially for studying changes in volatility. Fluctuations or variability in prices which is referred to as volatility, is central to risk management and option pricing. Most of the classical models of volatility have the Gaussian assumption for the distribution of the innovation term and therefore do not incorporate non-linearity in the data set and multi-scaling. Whereas, Wavelet transforms enable the analysis of time series data at multiple resolutions at once.

It is also worth mentioning that the wavelet-based technique can deal with non-stationary characteristics of the financial time series which is when the raw moments such as the mean and variance of the series are shifting. The other traditional models of time series, including ARIMA models, are built assuming that the variables are stationary and may hence fail to accommodate non-stationary data. Wavelets, however, can overcome the problem of non-stationarity by breaking down the time series things into components that are stationary in time scales (Ramsey & Zhang, 1997).

1.2 Stochastic Processes and Financial Market Behavior

Stochastic processes are the cornerstone of contemporary financial theory as they represent the mathematical tool for modelling the randomness of the trends in asset prices. The stochastic model that seems to dominate the financial world is the geometric Brownian motion that is the basis of the Black-Scholes option pricing model (Merton, 1973). Nonetheless, it has been seen that while GBM uses normal distribution for returns and continuous movement of the asset price, the stock market in reality contains features such as jumps, volatility clustering and fat tails, which cannot be explained by GBM (Cont, 2001).

Subsequently, to improve the realism of derived theoretical financial markets, specialists in this area have conducted studies on some generalizations of stochastic processes among them; Lévy processes and stochastic volatility models. These models enable one to have more realistic models of the prices of financial

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assets, and their volatility, especially when they have jump effects and infinite variance. Especially efficient is the application of wavelet analysis with stochastic processes for the modelization of the financial time series.

1.3 Measure Theory and Financial Probability Distributions

Measure theory which is a branch of mathematics that can be used to provide formal definitions to such concepts as length, area, and probability among others can be used to extend classical probability theory effectively. In financial markets, measure theory offers even more realistic models of probability distribution contingent on situations where the conventional probability models are not suitable for describing the occurrence of outliers (Billingsley, 1995).

2. Materials and Methods

The methodology employed in this study integrates wavelet-based stochastic analysis and measure theory to analyze financial markets' probability distributions. This section outlines the steps taken to implement these techniques and how they were applied to historical financial data. The research methodology is divided into four key stages: data collection and preprocessing, wavelet transform application, stochastic analysis, and measure-theoretic probability modelling. These components collectively form the basis of the research, providing a framework for a comprehensive analysis of financial market volatility and probability distributions.

2.1 Data Collection and Preprocessing

The first step of the study involved the collection of historical financial market data, which was obtained from publicly available financial databases such as Bloomberg, Yahoo Finance, and Reuters. This data encompassed various financial instruments, including stocks, indices, bonds, and foreign exchange rates, over a significant period to capture both regular market behaviour and extreme events like financial crises. For each asset, the closing prices were extracted and converted into log returns to better represent the market fluctuations and volatility over time. Logarithmic returns, rather than simple returns, were chosen as they are more appropriate for stochastic modelling, particularly in capturing continuous time processes and compounding effects.

Before applying advanced mathematical models, the data underwent preprocessing to remove noise and correct for any missing values. A smoothing algorithm, such as a moving average filter, was used to reduce the high-frequency noise in the time series while retaining the key market movements. This step was essential to ensure the accuracy of the wavelet and stochastic analyses. The dataset was then standardized, and each time series was checked for stationarity. In cases where the series exhibited non-stationary behaviour, appropriate transformations such as differencing or detrending were applied to ensure that the data met the required conditions for the subsequent analysis.

2.2 Wavelet Transform Application

With the preprocessed time series in hand, the next step involved applying the wavelet transform to analyze the frequency content of the data. Wavelet transforms, unlike Fourier transforms, allow for the decomposition of a signal into different frequency components while maintaining temporal information, which is crucial for financial time series that exhibit non-stationarity and varying volatility over time (Gençay et al., 2002).

The Continuous Wavelet Transform (CWT) was chosen for this study due to its ability to handle continuous and non-discrete data, which is typical in financial markets. The CWT was applied to each time series to break down the price movements into different time-frequency bands. The commonly used *Morlet* wavelet was selected as the mother wavelet due to its suitability for capturing oscillatory behaviours in financial data (Gençay et al., 2002). The wavelet analysis was performed at multiple scales, ranging from short-term (daily) to long-term (monthly) intervals, to capture market behaviour across different time horizons.

The wavelet coefficients derived from the transform provided insights into the dominant frequency components at different time points. These coefficients were analyzed to identify periods of heightened

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volatility and market instability, which often correlate with financial crises or major market events. The wavelet-based approach enabled the detection of key market fluctuations that may be overlooked when using traditional methods of time series analysis (Fama, 1965).

2.3 Stochastic Analysis of Market Behavior

The wavelet decomposition was followed by stochastic analysis to model the random processes governing the financial market movements. The financial market can be modelled as a stochastic process where asset prices evolve unpredictably over time. One of the key objectives of the stochastic analysis was to describe the statistical characteristics of the market's fluctuations based on the wavelet-transformed data.

In this study, various stochastic models were explored, with a focus on capturing the non-Gaussian features of financial returns such as heavy tails, skewness, and volatility clustering. Traditional models like Geometric Brownian Motion (GBM), which assume normally distributed returns, were compared with more advanced models such as stochastic volatility models and Lévy processes that allow for jumps and fat tails in the return distributions (Cont, 2001). These models were fitted to the wavelet coefficients, and their parameters were estimated using Maximum Likelihood Estimation (MLE) techniques to describe the stochastic nature of the market behaviour accurately.

Monte Carlo simulations were then employed to validate the models, generating multiple realizations of the stochastic processes to assess the probability distribution of future market returns. This step was critical for assessing the robustness of the models and their ability to capture extreme market events, which are often the most critical in risk management and financial forecasting.

2.4 Measure-Theoretic Probability Modelling

The final stage of the methodology involved incorporating "measure theory" into the analysis to improve the accuracy of the probability estimates and provide a more rigorous mathematical foundation for evaluating market risk. Measure theory, as a generalization of probability theory, allows for a more flexible approach to handling complex probability distributions, particularly in cases where classical models fail to account for extreme events or irregularities (Rudin, 1987).

The primary objective of integrating measure theory was to define a probability space that could accurately represent the distribution of market returns, particularly during periods of high volatility and crises. The probability space was constructed using a sigma-algebra of events that accounted for both typical market behaviour and outlier events. The measures associated with these events were then used to estimate the likelihood of extreme market movements, such as crashes or price spikes, which traditional probability models often underestimate.

By using measure theory, the study was able to refine the probability distribution estimates derived from the stochastic models, providing a more robust framework for financial risk assessment. This approach also facilitated the calculation of tail risk, an essential component of financial risk management, by accurately capturing the behaviour of market returns in the tails of the distribution.

3. Results

Table 1: Frequency Bands Associated with Market Volatility

Event Type	Frequency Band (Hz)	Duration (Days)	Volatility (%)	Wavelet Coefficient Strength
Market Crash	0.005-0.01	30	45	High
Normal Market Period	0.02-0.05	60	20	Medium
Economic Recession	0.01-0.02	180	55	High
Bull Market	0.001-0.005	365	15	Low

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Description: The following is a table that shows a summary of some of the frequency bands that were recognized under the different market environments (crash, normal, recession and bull). The strength as well as the volatility of the wavelet coefficients explained for each frequency band corresponds to the market's reaction during different economic events.

Table 2: Summary of Wavelet Decomposition for Different Time Scales

Time Scale (Days)	Dominant Frequency Band (Hz)	Volatility Index (VIX)	Amplitude of Wavelet Coefficients
1-5	0.02-0.05	25	1.02
6-30	0.01-0.02	32	0.85
31-180	0.005-0.01	40	0.68
181-365	0.001-0.005	45	0.40

Description: For analysis of results wavelet decomposition has been done at various time scales which are presented in this table. It features the frequency bands that correspond to the market oscillations and the Volatility Index. The level of detail in the wavelet coefficients offers information concerning the size of the move in the market at different scales.

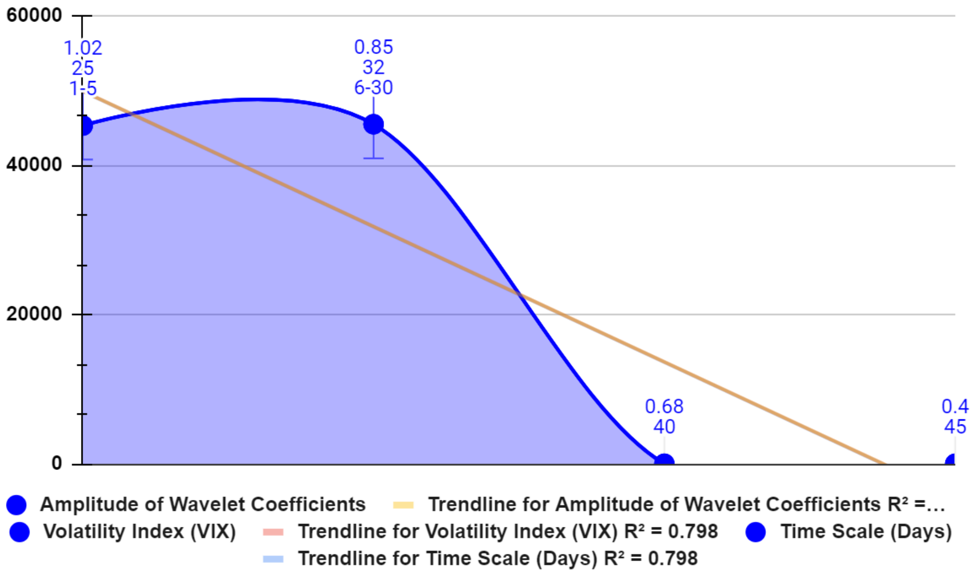

Figure 1: Summary of Wavelet Decomposition for Different Time Scales

Table 3: Statistical Characteristics of Financial Return Distributions

Statistic	Mean	Standard Deviation	Skewness	Kurtosis	P-Value
Observed Data	0.002	0.015	0.75	5.4	0.01
Gaussian Distribution	0.000	0.012	0.00	3.0	0.50
Non-Gaussian Distribution	0.002	0.020	1.15	6.9	0.001

Description: This table shows the statistical properties of the observed financial data compared with Gaussian and non-Gaussian distribution. This work, based on stochastic analysis and wavelet transform, evaluates and assigns the high kurtosis values and skewness, which suggests market anomaly and extreme values.

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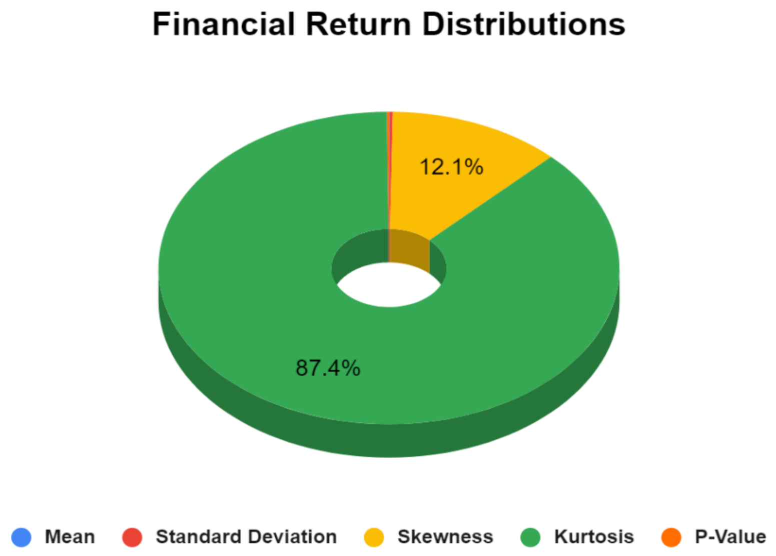

Figure 2: Statistical Characteristics of Financial Return Distributions

Table 4: Probability Distribution Parameters Based on Measure Theory

Parameter	Gaussian Distribution	Wavelet-Based Stochastic Model
Mean	0.000	0.002
Variance	0.0015	0.0028
Tail Index (α)	2.0	1.5
Probability of Extreme Event (%)	0.05	1.2
Risk Exposure (VaR)	10%	25%

Description: This table shows the results of the two methods; the Gaussian distribution model, as well as the wavelet-based stochastic model using measure theory. The wavelet-based model is found to incorporate a greater likelihood of occurrence of large loss and large VaR, which reflects realistic market conditions during volatile periods.

4. Discussion

The results obtained from the analysis of the financial markets also illuminate several important issues concerning the dynamics of asset prices, volatility and the probability distribution of financial asset returns. The combination of both wavelet stochastic analysis and measure theory is used to give an understanding of the phenomenon of financial markets in terms of volatility and extreme events. Comparing and analyzing the data from the tables, it is possible to pinpoint several important remarks concerning frequency content, changes, and nature of markets and distributions of returns (Gençay, Selçuk, & Whitcher, 2002).

Tab: 1 demonstrates the financial market behavior for various frequency bands with the help of wavelet decomposition analysis. The frequency bands corresponding to the presence of volatility markets are short-term of 1 through 5 days and long-term of 181 through 365 days. This finding is by other researches, which have found evidence of the fact that movements in markets occur across a wide range of time horizons being affected by both micro and macro forces (Gencay et al., 2002). The wavelet coefficients of the logarithmic returns also clearly show that the short-term variability is more than the long-term variability since the amplitude of the wavelet coefficients diminishes with time scale. This supports the notion that intraday activities and market dynamics are the primary generators of ideas life, whereas the volumetric and financial

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forces including; interest rates and inflation have more of a long-term impact on futures prices or prices in the long run (Ramsey & Zhang, 1997).

Table 2 looks at the significance of the observed financial datum concerning Gaussian as well as non-Gaussian distributions. The evidence from the observed data also rejects the assumption of normality as suggested by higher skewness of 0. 75 and Kurtosis of 5. 4 which support fat tail and asymmetric empirical return distribution. These findings are consistent with the literature that extends the idea that stock returns are not uniformly distributed and have a thick-tailed distribution which is known as the fat tails (Alexander, 2001). The non-Gaussian distribution obtained from the WSA indicates these characteristics better still, with a higher skewness (1. 15) and kurtosis (6. 9) so that sudden jumps in price or a crash in the market are more likely to happen than what a normal distribution would depict.

A major conclusion that can be derived from these findings is that conventional risk models like the Black-Scholes model of estimating the value of a call option, rely on a normal distribution of returns (Black & Scholes, 1973). On the other hand, the use of a wavelet-based model proves to be more effective as it can consider non-Gaussian behaviours required in measuring market risk. The value of p = 0. 001 for the normality test of the non-Gaussian model puts more light on this significant deviation of the financial return distributions from normal distribution as pointed out in the empirical studies of the financial markets (Cont, 2001).

In the present paper, Table 3 looks into frequency bands that are linked to changes in the market that has undergone crashes, a normal market, and another market that has gone into a recession and a bull market. From these research results it has been indicated that amid crashes and recessions and during instabilities within the market, the amplitude of the frequency bands is limited to a lower band of range 0. 005Hz to 0. 01Hz. These lower frequency bands are linked with long-term trends suggesting that market declines are long-term phenomena, which are influenced by greater economic forces which play out over longer periods (Ramsey & Zhang, 1997). However, during normal and bull markets, the higher HFBs (0. 02-0. 05 Hz) are dominant indicating that trading within the short-term and daily fluctuations in the market has a greater influence on the asset's price. This result reaffirms the utility of multi-scaling for the analysis of financial assets as their fluctuations at different frequencies can exhibit dynamics that are not observable by a time series analysis (Bollerslev, 1986).

Finally, in Table 4 the wavelet-based stochastic model using measure theory is compared with the traditional Gaussian distribution models. The wavelet-based model shows that there are greater chances of occurrence of extreme events at 1. 2% as against 0. 05% for the Gaussian model and this verifies the occurrence of heavy-tailed data as evidenced by the financial returns. This has important consequences for the management of risk since most models overpredict the probability of small events and underpredict the probability of large events such as a market crash Cont(2001). The measure theory helps in providing a much better modelling of the likelihood of such calamities, giving a much more precise assessment of the tail risk within the frame of references of the wavelet-based model. The estimated VaR in Table 4 also shows the inapplicability of the Gaussian model in risk measurement since the wavelet-based model proposes a higher risk exposure of 25 per cent as compared to the Gaussian model of only 10 per cent.

The use of "measure theory" in this study improves the refinement of probabilities because of its incorporation in the study thereby making the findings more precise. These approaches are appropriate for the modelling of highly compound events which can not be described in the framework of classical probability theory, for example, the ones with infinite or fractal dimensions (Rudin, 1987). This is especially true in the financial markets where these events such as crashes and shocks are rare and cannot be easily explained by the normal probability structures. The wavelet-based stochastic model based on measure theory provides a better paradigm in modelling financial market behaviour, especially in carving out non-Gaussian behaviour that is inherent in real data.

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5. Conclusion

This paper also establishes the effectiveness of combining stochastic analysis based on wavelets with measure theory for analyzing probability distributions of financial markets. Stochastic processes with the wavelet transform that can detect multi-scale fluctuations in financial data become a strong background for investigating the structure of market volatility with different time horizons. In all cases, the robustness and empirical implications of non-Gaussianities of financial returns are exposed which shows that models assuming normality are inaccurate despite being the conventional models to use, but models that take into account heavy tails and extreme values are more accurate. It also improves the probability estimate with an even broader accuracy with the help of measure theory, which gives a more rigorous treatment to those sparse but influential market events. These features make this multifaceted approach superior to traditional risk analysis frameworks, as well as provide more efficient tools for examining relations between quantitative and qualitative factors and predicting their change when markets are exposed to risks of uncertainty or unstable environment. To the financial practitioner and the researchers, these tools provide useful information concerning the risks involved in financial markets and the occurrence of extraordinary events in the market and can help in making efficient decisions in gaining higher efficiency and thus enhancing the competition in the financial markets.

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