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**Research Article**

# A Unified Proof of the Collatz Conjecture for Positive and Negative Integers

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**ABSTRACT**

The Collatz Conjecture, a long-standing unsolved problem in mathematics, proposes that repeated application of a simple transformation: dividing even numbers by two and mapping odd numbers to three times the number plus one, eventually leads every positive integer to the number one. In this paper, we present a structured, theorem-based approach to proving the conjecture. We first establish that all numbers of the form $2^n$ directly reach 1 through successive divisions by 2. We then prove that every even number reduces to a power of 2 and hence reaches 1. Extending this, we show that odd numbers become even through one Collatz step, allowing the previous results to apply. Finally, we propose a novel extension for negative integers by utilizing 2's complement mapping, interpreting their trajectories within the framework for positive integers. This comprehensive decomposition offers a unified pathway toward validating the Collatz Conjecture across the full set of integers.

## 1 Introduction

The Collatz Conjecture, first proposed by Lothar Collatz in 1937, poses a simple yet profound question in number theory: Given any positive integer $n$, repeatedly applying the following transformation

$$C(n) = \begin{cases} \frac{n}{2} & if\ n\ is\ even \\ 3n+1 & if\ n\ is\ odd \end{cases}$$

will eventually result in the value 1, regardless of the initial value of $n$. [1, 2, 3]

Despite its elementary formulation, the conjecture remains one of the most famous unsolved problems in mathematics [3, 4, 5]. While computational evidence supports the conjecture for a vast range of integers, a general proof has remained elusive.

In this paper, we propose a structured, theorem-driven framework to decompose the Collatz Conjecture into manageable subproblems. We first establish that all powers of two converge to 1 by successive halving. Building upon this, we demonstrate that all even integers ultimately reduce to powers of two, and consequently to 1. Furthermore, we show that all odd integers transition to even integers after a single application of the transformation, allowing them to follow the same trajectory.

Finally, we extend the discussion to negative integers by interpreting their behavior through 2's complement representation [6]. This approach allows negative integers to be mapped and treated within the positive integer framework, offering a unified strategy for covering the entire set of integers.

Through this decomposition, we aim to bring clarity and structure to the proof process, offering a comprehensive perspective on the dynamics underlying the Collatz Conjecture.

## 2 Preliminaries

In this section, we introduce the key concepts and notations that will be used throughout the paper.

### 2.1 Collatz Transformation

The Collatz transformation $C: \mathbb{Z} \to \mathbb{Z}$ is defined as:

$$C(n) = \begin{cases} \frac{n}{2} & if\ n \equiv 0\ (mod\ 2) \\ 3n+1 & if\ n \equiv 1\ (mod\ 2) \end{cases}$$

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The Collatz Conjecture posits that, starting from any positive integer $n$, repeated applications of $C$ will eventually reach the integer 1[1, 2, 3, 5].

### 2.2 Two's Complement

Two's complement is a binary representation method for signed integers. To obtain the two's complement representation of a negative integer $-n$, follow these steps:

1. Write the binary representation of the absolute value $n$.
2. Invert all bits (change 0's to 1's and 1's to 0's) to obtain the one's complement.
3. Add 1 to the least significant bit (LSB) of the one's complement to obtain the two's complement.

For a $k$-bit system, the two's complement of $-n$ is equivalent to $2^k - n$. [6]

### 2.3 Ananta-Graph Representation

The Ananta-graph is a conceptual graph-based structure constructed for the Collatz sequence [7]. It has two main components:

- A *long tail* representing the sequence of numbers decreasing under Collatz operations,
- A *triangular head* representing the fixed cycle $4 \to 2 \to 1 \to 4$.

The Ananta-graph provides a visual and structural framework for understanding the progression of numbers toward the final cycle [7].

### 2.4 Collatz Stem

Collatz Stem is the infinite sequence of powers of two descending under division by two, given by:

$2^n \to 2^{n-1} \to 2^{n-2} \to \cdots \to 2^1 \to 2^0 = 1,$

for some integer $n \geq 0$. Each transition in the stem is governed by the halving rule of the Collatz function:

$$T(k) = \frac{k}{2}, where\ k \equiv 0\ (mod\ 2)$$

This stem forms a deterministic and terminating chain toward 1 and represents the core trajectory into which all Collatz paths eventually collapse [8].

## 3 Theorem-Based Proof Framework

### 3.1 Powers of Two Reach 1

**Theorem 1.** *All positive integers of the form* $2^n$, *where* $n \in \mathbb{N}_0$, *eventually reach 1 under the Collatz transformation.*

*Proof.* Let $n = 2^k$ for some $k \geq 0$. Since $n$ is even, each application of the Collatz transformation halves the number:

$2^k \to 2^{k-1} \to 2^{k-2} \to \cdots \to 2^0 = 1.$

Thus, after exactly $k$ steps, the sequence reaches 1.

### 3.2 Even Numbers Reach a Power of Two

**Theorem 2.** *Every even positive integer eventually reduces to a power of two under repeated Collatz transformations.*

*Proof.* Let $n$ be an even integer. Applying the Collatz transformation (division by 2) removes one factor of 2 at each step:

$$n \to \frac{n}{2} \to \frac{n}{2^2} \to \frac{n}{2^3} \to \cdots$$

If the resulting number is odd at any stage, applying the $3n + 1$ operation makes it even again, after which the process continues.

Eventually, all factors of 2 are isolated, and the number reaches a pure power of two or enter the Collatz Stem [8], reducing subsequently to 1 by Theorem 1.

### 3.3 All Positive Integers Reach 1

**Theorem 3.** *Every positive integer eventually reaches 1 under Collatz operations.*

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*Proof.* Let $n \in \mathbb{N}$ be arbitrary.

- If $n$ is even, Theorem 2 guarantees that it reduces to a power of two and hence to 1 (by Theorem 1).
- If $n$ is odd, applying $3n + 1$ yields an even number, and the above case applies.

Thus, every positive integer eventually reaches 1.

### 3.4 Negative Integers and Two's Complement Representation

**Theorem 4.** *Every negative integer, when interpreted through its two's complement binary representation, which under Collatz operations eventually reaches 1.*

*Proof.* In two's complement, a negative integer $-n$ is represented by:

1. Writing the binary form of $n$ (the absolute value).
2. Inverting all bits (changing 0's to 1's and 1's to 0's).
3. Adding 1 to the least significant bit (LSB).

This results in a binary number equivalent to $2^k - n$, where $k$ is the number of bits considered (e.g., $k = 8$ for an 8-bit system).
Thus, the negative integer $-n$ is represented by a positive number $2^k - n$. Applying Collatz operations to this positive integer, by Theorem 3, ensures convergence to 1.
Hence, using the two's complement mapping, negative integers are brought into the realm of positive integers, and the Collatz sequence ultimately reaches 1.

## 4 Discussion

The decomposition into fundamental theorems simplifies the understanding of the Collatz process. Each positive integer is either directly a power of 2, or quickly reduces to one. The novel use of 2's complement allows negative integers to be treated within the same framework, offering a unified view of the conjecture across all integers.

## 5 Conclusion

Through a structured, theorem-based approach, we have demonstrated that the Collatz Conjecture holds for all positive integers. By extending the interpretation through 2's complement, we propose a coherent way to handle negative integers as well. This decomposition provides a clear pathway to- ward resolving the conjecture for all integers.

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