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#### **Research Article**

# **Soft b Connectedness in Soft Tritopological Spaces**

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ARTICLE INFO	ABSTRACT
Received: 28 Dec 2024	This paper deals Soft tritopological spaces which is based on soft bitopological spaces. The characteristic of trio separated soft b sets and trio connected soft b sets were discussed. The
Revised: 18 Feb 2025	final observation in this paper will extended to future study on soft tritopological spaces.
Accepted: 26 Feb 2025	<b>Keywords:</b> trio separated soft b sets, trio connected soft b sets

#### 1.INTRODUCTION

In 1999, Molodtsov.D [9] instituted soft sets theory as a new mathematical device for dealing with undetermine objects and to solve complex problems in engineering, economics, medicines, computer science, social sciences and environment. In 2012, H.Hazra, P.Majumadav, S.K.Samanta[7] instigated the notions of topology on soft subsets and softtopology.In 2017, Asmhan Flieh Hassan [1]set forth some new definitions of soft open sets in soft tritopological spaces. In 2014, Sabir Hussain[12] developed the properties and description of soft connected spaces in soft topological spaces. In 2017,N.Revathi and K.Bageerathi [11] introduced the concept of (1,2)\* soft b separated sets and (1,2)\* soft b connected spaces. In 2015, Sabir Hussain[14] introduced properties of soft semi open and soft semi closed sets. This paper discusses the concept and properties of trio soft b separated sets and trio soft b connected sets in the soft tritopological spaces.

#### 2. PREMILARIES

# **Definition 2.1[3,5,6]**

A set X along with three other topologies is called soft tritopological space.

Shown as  $(X, \tau_1, \tau_2, \tau_3)$ .

## **Definition 2.2[3,5,6]**

Soft tritopological space is  $(X, \tau_1, \tau_2, \tau_3, E)$  and  $(A, E) \subseteq X$  and then (A, E) be called  $(1, 2, 3)^*$  open soft b-set  $(A, E) \subseteq \tau_{1,2,3} - \operatorname{int}(\tau_{1,2,3} - \operatorname{cl}(A, E)) \cup \tau_{1,2,3} - \operatorname{cl}(\tau_{1,2,3} - \operatorname{int}(A, E))$ 

# **Definition 2.3[3,5,6]**

Soft tritopological space is  $(X, \tau_1, \tau_2, \tau_3, E)$  and  $(A, E) \subseteq \overline{X}$ . Hence (A, E) be called  $(1,2,3)^*$  closed soft b set if  $\tau_{1,2,3} - int(\tau_{1,2,3} - cl(A, E)) \cup \tau_{1,2,3} - cl(\tau_{1,2,3} - int(A, E)) \subseteq (A, E)$ 

#### Definition 2.4[11]

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Soft bitopological space is  $(X, \tau_1, \tau_2, E)$ . Two non-empty disjoint soft subsets  $F_{E_1}$  and  $F_{E_2}$  of  $\overline{X}$  are called  $(1,2)^*$ soft separated sets over X if  $\left(\tau_{1,2} - cl(F_{E_1})\right) \cap F_{E_2} = F_{E_1} \cap \left(\tau_{1,2} - cl(F_{E_2})\right) = \emptyset$ .

#### Definition 2.5[11]

Soft bitopological space is  $(X, \tau_1, \tau_2, E)$ . Hence X is called  $(1,2)^*$  soft b connected space if  $\bar{X}$  cannot be indicated as the union of soft two  $(1,2)^*$  soft b separated sets.

#### 3. TRIO SEPARATED SOFT B SETS

#### **Definition 3.1**

Two not-empty soft subsets  $A_{E_1}$  and  $A_{E_2}$  of  $\bar{X}$  are mentioned to be disjoint if  $A_{E_1} \cap A_{E_2} = \emptyset$ .

# **Definition 3.2**

. Two not-empty soft disjoint subsets  $A_{E_1}$  and  $A_{E_2}$  of  $\bar{X}$  are called trio separated soft sets over X if  $\left(\tau_{1,2,3} - cl(A_{E_1})\right) \cap A_{E_2} = A_{E_1} \cap \left(\tau_{1,2,3} - cl(A_{E_2})\right) = \emptyset$ .

# **Definition 3.3**

Soft tritopological space is  $(X, \tau_1, \tau_2, \tau_3, E)$ . Two non-empty soft disjoint subsets  $A_{E_1}$  and  $A_{E_2}$  of  $\bar{X}$  are called trio separated soft b set over X if  $(\tau_{1,2,3} - sbcl(A_{E_1})) \cap A_{E_2} = A_{E_1} \cap (\tau_{1,2,3} - sbcl(A_{E_2})) = \emptyset$ .

## **Definition 3.4**

A trio separation soft b set of a soft tritopological space  $(X, \tau_1, \tau_2, \tau_3, E)$  is a pair of trio separated soft b sets  $A_{E_1}$  and  $A_{E_2}$  whose soft union is  $\bar{X}$ .

# Example 3.5

Let 
$$X = \{l, m\}$$
,  $E = M = \{e_1, e_2\}$  and  $\overline{X} = \{(e_1, \{l, m\}), (e_2, \{l, m\})\}$ . The soft subsets are 
$$A_{E_1} = \{(e_1, \{l\}), (e_2, \{l\})\} \quad A_{E_9} = \{(e_1, \{X\}), (e_2, \{l\})\}$$
 
$$A_{E_2} = \{(e_1, \{l\}), (e_2, \{m\})\} \quad A_{E_{10}} = \{(e_1, \{X\}), (e_2, \{m\})\}$$
 
$$A_{E_3} = \{(e_1, \{l\}), (e_2, \{X\})\} \quad A_{E_{11}} = \{(e_1, \{X\}), (e_2, \{X\})\}$$
 
$$A_{E_4} = \{(e_1, \{l\}), (e_2, \{\emptyset\})\} \quad A_{E_{12}} = \{(e_1, \{X\}), (e_2, \{\emptyset\})\}$$
 
$$A_{E_5} = \{(e_1, \{m\}), (e_2, \{l\})\} A_{E_{13}} = \{(e_1, \{\emptyset\}), (e_2, \{l\})\}$$
 
$$A_{E_6} = \{(e_1, \{m\}), (e_2, \{M\})\} \quad A_{E_{14}} = \{(e_1, \{\emptyset\}), (e_2, \{M\})\}$$
 
$$A_{E_7} = \{(e_1, \{m\}), (e_2, \{\emptyset\})\} \quad A_{E_{15}} = \{(e_1, \{\emptyset\}), (e_2, \{X\})\}$$
 
$$A_{E_8} = \{(e_1, \{m\}), (e_2, \{\emptyset\})\} \quad A_{E_{16}} = \{(e_1, \{\emptyset\}), (e_2, \{\emptyset\})\}$$

Consider the soft tritopological spaces  $(X, \tau_1, \tau_2, \tau_3, E)$  where  $\tau_1 = \{\bar{X}, \emptyset, A_{E_1}, A_{E_4}\}$ ,  $\tau_2 = \{\bar{X}, \emptyset, A_{E_5}, A_{E_{13}}\}$  and  $\tau_3 = \{\bar{X}, \emptyset, A_{E_9}\}$ . Then  $\tau_{1,2,3}$  open sets  $= \{\bar{X}, \emptyset, A_{E_1}, A_{E_4}, A_{E_5}, A_{E_{13}}\}$ ,

$$\tau_{1,2,3} \text{ Closed sets=} \big\{ \overline{X}, \emptyset, A_{E_2}, A_{E_6}, A_{E_7}, A_{E_{10}}, A_{E_{14}} \big\},$$

trio SbO(X)= 
$$\{\bar{X},\emptyset,A_{E_1},A_{E_2},A_{E_3},A_{E_5},A_{E_6},A_{E_7},A_{E_{13}},A_{E_9},A_{E_{15}}\},$$

trio SbC(X)=
$$\{\bar{X}, \emptyset, A_{E_2}, A_{E_4}, A_{E_6}, A_{E_7}, A_{E_8}, A_{E_5}, A_{E_{10}}, A_{E_{12}}, A_{E_{14}}\}$$
. Take $\bar{X} = A_{E_4} \cup A_{E_7}$ , then trio  $sbcl(A_{E_4}) = \{(e_1, \{l\}), (e_2, \{\emptyset\})\} = A_{E_4}$ , trio  $sbcl(A_{E_7}) = \{(e_1, \{m\}), (e_2, \{X\})\} = A_{E_7}$ . We have trio  $sbcl(A_{E_4}) \cap A_{E_7} = \emptyset$  and

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trio  $sbcl(A_{E_7}) \cap A_{E_4} = \emptyset$ . Therefore  $A_{E_4}$  and  $A_{E_7}$  are trio separated soft b sets. Hence  $A_{E_4}$  and  $A_{E_7}$  are trio separation soft b sets of X.

## Remark 3.6

From the fact that trio  $sbcl(A_E) \subset \tau_{1,2,3} - cl(A_E)$ , for every soft subset  $A_E$  of X, every trio separated soft set is trio separated soft b set. But the reverse will not be true in the given example.

## Example 3.7

Consider the example 3.5. Take  $\bar{X} = A_{E_1} \cup A_{E_{12}}$ , then  $\tau_{1,2,3} - cl(A_{E_4}) = \{(e_1, \{l\}), (e_2, \{m\})\} = A_{E_2}, \tau_{1,2,3} - cl(A_{E_7}) = \{(e_1, \{l\}), (e_2, \{X\})\} = A_{E_7} \}$ . We have  $\tau_{1,2,3} - cl(A_{E_4}) \cap A_{E_7} = \{(e_1, \{l\}), (e_2, \{\emptyset\})\} = A_{E_4} \neq \emptyset$  and  $A_{E_7} \cap \tau_{1,2,3} - cl(A_{E_4}) = \emptyset$ . Therefore  $A_{E_4}$  and  $A_{E_7}$  are not trio separated soft sets. Hence  $A_{E_4}$  and  $A_{E_7}$  are not trio separation soft sets of X.

#### Theorem 3.8

Soft tritopological space is  $(X, \tau_1, \tau_2, \tau_3, E)$ . Two trio soft b closed sets  $A_E$  and  $G_E$  are trio separated soft b sets if and only if they are soft disjoint.

Proof: Let  $(X, \tau_1, \tau_2, \tau_3, E)$  be soft tritopological space and  $A_E$ ,  $G_E$  be two  $(X, \tau_1, \tau_2, \tau_3, E)$  soft b closed sets in X which are  $(X, \tau_1, \tau_2, \tau_3, E)$  separated soft b sets. Then by definition 3.3  $A_E$  and  $G_E$  are soft disjoint.

Converse: Let  $A_E$  and  $G_E$  are both soft disjoint and  $(X, \tau_1, \tau_2, \tau_3, E)$  soft b closed sets. Then are  $(\text{trio } sbcl(A_E)) \cap G_E = A_E \cap (\text{trio } sbcl(G_E)) = \emptyset$ . Therefore  $A_E$  and  $G_E$  are trio separated soft b sets.

## Theorem 3.9

Consider  $G_E$  and  $H_E$  are not empty soft sets of a soft tritopological space  $(X, \tau_1, \tau_2, \tau_3, E)$ . Consider: If  $G_E$  and  $H_E$  are trio separated soft b sets,  $G_{E_1} \subseteq G_E$  and  $G_{E_2} = G_E$  and  $G_{E_3} = G_E$  and  $G_{E_4} = G_E$  and

Proof: Let  $G_E$  and  $H_E$  are trio separated soft b sets. Then trio  $sbcl(G_E) \cap H_E = G_E \cap trio \, sbcl(H_E) = \emptyset$ . Since  $G_{E_1} \subseteq G_E$ , trio  $sbcl(G_{E_1}) \subseteq (1,2,3)^* \, sbcl(G_E)$ . Hence  $H_{E_1} \cap trio \, sbcl(G_{E_1}) \subseteq H_E \cap trio \, sbcl(G_E) = \emptyset$ . Similarly  $G_{E_1} \cap trio \, sbcl(H_{E_1}) = \emptyset$ . Thus  $G_{E_1}$  and  $G_{E_1} \cap trio \, sbcl(H_{E_1}) = \emptyset$ . Thus  $G_{E_1} \cap trio \, sbcl(H_{E_1}) = \emptyset$ . Thus  $G_{E_1} \cap trio \, sbcl(H_{E_1}) = \emptyset$ .

#### 4.Trio CONNECTED SOFT B SETS

#### **Definition 4.1**

Consider  $(X, \tau_1, \tau_2, \tau_3, E)$  as a soft tritopological space in which X is called trio connected soft b space then  $\bar{X}$  will not be shown as the soft union of two trio separated soft b sets.

#### Remark 4.2

- i) Let soft tritopological space, soft empty set is trivially trio connected soft b set.
- ii) Let soft tritopological space, each soft singleton set is trio connected soft b set because it will not be exhibited as a union of two soft not empty trio separated soft b sets.

## Theorem 4.3

Soft tritopological space is  $(X, \tau_1, \tau_2, \tau_3, E)$ . Then the succeeding utterance are same

- i)  $\bar{X}$  is a trio connected soft b space.
- ii) The trio soft b clopen set in X are  $\bar{X}$  and  $\emptyset$ .
- iii) $\bar{X}$  will not be indicated as the combination of two disassociate not empty trio soft b open sets.
- iv)  $\overline{X}$  will not be indicated as the combination of two disassociate not empty trio soft b closed sets.

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Proof: i)  $\to$  ii) Let  $\bar{X}$  is trio connected soft b space. Let  $A_E$  be not null actual subset of X.That is trio soft b clopen. Then  $\bar{X} \setminus A_E$  is a not empty trio soft b clopen set and  $\bar{X} = A_E \cup (\bar{X} \setminus A_E)$ . It is a disagreement to  $\bar{X}$  is a trio connected soft b space. Hence  $\bar{X}$  and  $\emptyset$  are the only trio soft b clopen set in X.

- ii)  $\rightarrow$  iii) Assume that  $\bar{X}$  and  $\emptyset$  are the only trio soft b clopen set in X . Suppose (iii) is false. Then  $\bar{X} = A_{E_1} \cup A_{E_2}$  where  $A_{E_1}$  and  $A_{E_2}$  are disjoint non empty trio soft b open sets. Then  $A_{E_2} = \bar{X} \setminus A_{E_1}$  is trio soft b closed set and not empty. Thus  $A_{E_2}$  is a not empty actual trio soft b clopen sets in X, which is contradicts (ii)
- iii)  $\to$  iv) Assume  $\bar{X}$  will not be indicated as the combination of two disjoint not empty trio soft b open sets. Suppose (iv) false. Then  $\bar{X} = A_{E_1} \cup A_{E_2}$  where  $A_{E_1}$  and  $A_{E_2}$  are disassociate not null trio soft b closed sets. Then  $A_{E_1} = \bar{X} \setminus A_{E_2}$  and  $A_{E_2} = \bar{X} \setminus A_{E_1}$  are disassociate not null trio soft b open sets in X. Thus  $\bar{X}$  is the soft combination of two disassociate not null trio soft b open sets. This is contradicts (iii).
- iv)  $\to$  i) Suppose  $\bar{X}$  is non trio connected soft b space. Then  $\bar{X} = A_{E_1} \cup A_{E_2}$  where  $A_{E_1}$  and  $A_{E_2}$  are disassociate not null trio soft b open sets. Then  $A_{E_1} = \bar{X} \setminus A_{E_2}$  and  $A_{E_2} = \bar{X} \setminus A_{E_1}$  are disassociate not null trio soft b closed sets in X. This is contradicts (iv).

#### **Proposition 4.4**

Every trio connected soft b space is trio connected soft set.

Proof: Let  $A_E$  be a trio connected soft b set in the tritopological space  $(X, \tau_1, \tau_2, \tau_3, E)$ . Then it does not occur a trio separation soft b set of  $A_E$ . Since every  $\tau_{1,2,3}$  open set is a trio soft b open set, there does non exist a trio separation soft set of  $A_E$ . Hence  $A_E$  is a trio connected soft set in the soft tritopological space.

#### Remark 4.5

In the given example, the discourse may not be accurate.

#### Example 4.6

Trio connected soft set does not imply trio connected soft b set . Let  $(X, \tau_1, \tau_2, \tau_3, E)$  be a soft tritopological space, Where X and its soft subsets are considered as in Example 3.5.

$$\tau_1 = \{\bar{X}, \emptyset, A_{E_1}, A_{E_4}\} \ , \tau_2 = \{\bar{X}, \emptyset, A_{E_5}, A_{E_{13}}\} \ \text{and} \ \tau_3 = \{\bar{X}, \emptyset, A_{E_9}\}. \\ \text{Then } \tau_{1,2,3} \ \text{open sets} = \{\bar{X}, \emptyset, A_{E_1}, A_{E_4}, A_{E_5}, A_{E_9}, A_{E_{13}}\}, \\ \tau_{1,2,3} \ \text{closed sets} = \{\bar{X}, \emptyset, A_{E_2}, A_{E_6}, A_{E_7}, A_{E_{10}}, A_{E_{14}}\}, \\ \tau_{1,2,3} \ \text{closed sets} = \{\bar{X}, \emptyset, A_{E_2}, A_{E_6}, A_{E_7}, A_{E_{10}}, A_{E_{14}}\}, \\ \tau_{1,2,3} \ \text{closed sets} = \{\bar{X}, \emptyset, A_{E_2}, A_{E_6}, A_{E_7}, A_{E_{10}}, A_{E_{14}}\}, \\ \tau_{1,2,3} \ \text{closed sets} = \{\bar{X}, \emptyset, A_{E_1}, A_{E_2}, A_{E_6}, A_{E_7}, A_{E_{10}}, A_{E_{14}}\}, \\ \tau_{1,2,3} \ \text{closed sets} = \{\bar{X}, \emptyset, A_{E_2}, A_{E_6}, A_{E_7}, A_{E_{10}}, A_{E_{14}}\}, \\ \tau_{1,2,3} \ \text{closed sets} = \{\bar{X}, \emptyset, A_{E_1}, A_{E_2}, A_{E_6}, A_{E_7}, A_{E_{10}}, A_{E_{14}}\}, \\ \tau_{1,2,3} \ \text{closed sets} = \{\bar{X}, \emptyset, A_{E_2}, A_{E_6}, A_{E_7}, A_{E_{10}}, A_{E_{14}}\}, \\ \tau_{1,2,3} \ \text{closed sets} = \{\bar{X}, \emptyset, A_{E_1}, A_{E_2}, A_{E_6}, A_{E_7}, A_{E_{10}}, A_{E_{14}}\}, \\ \tau_{1,2,3} \ \text{closed sets} = \{\bar{X}, \emptyset, A_{E_1}, A_{E_1$$

Since the only trio soft clopen sets are  $\emptyset$  and  $\bar{X}$ ,  $\bar{X}$  is trio connected soft set. Also

trio SbO(X)=
$$\{\bar{X},\emptyset,A_{E_1},A_{E_2},A_{E_3},A_{E_5},A_{E_6},A_{E_7},A_{E_{13}},A_{E_9},A_{E_{15}}\},$$

trio SbC(X)= 
$$\{\bar{X},\emptyset,A_{E_2},A_{E_4},A_{E_6},A_{E_7},A_{E_8},A_{E_5},A_{E_{10}},A_{E_{12}},A_{E_{14}}\}$$
.

Take  $\overline{X} = A_{E_4} \cup A_{E_7}$ , then trio  $sbcl(A_{E_4}) = A_{E_4}$ , trio  $sbcl(A_{E_7}) = A_{E_7}$  and trio  $sbcl(A_{E_4}) \cap A_{E_7} = \emptyset$  and trio  $sbcl(A_{E_7}) \cap A_{E_4} = \emptyset$ . Hence  $\overline{X}$  will be demonstrated as a combination of two trio separate soft b sets  $A_{E_4}$  and  $A_{E_7}$ . Hence  $\overline{X}$  is non connected soft b set.

# Example 4.6

Trio connectivity soft b set is not a hereditary property.

Consider  $(X, \tau_1, \tau_2, \tau_3, E)$  be a soft tritopological space, Where X and its soft subsets are considered as in Example 3.5. Let  $\tau_1 = \{\bar{X}, \emptyset, A_{E_4}\}, \tau_2 = \{\bar{X}, \emptyset, A_{E_{12}}\}, \tau_3 = \{\bar{X}, \emptyset, A_{E_1}\}.$  Then  $\tau_{1,2,3}$  open sets= $\{\bar{X}, \emptyset, A_{E_1}, A_{E_2}, A_{E_3}, A_{E_4}, A_{E_3}, A_{E_4}, A_{E_1}, A_{E_2}, A_{E_{12}}\}$ , trio SbO(X)= $\{\bar{X}, \emptyset, A_{E_1}, A_{E_2}, A_{E_3}, A_{E_4}, A_{E_3}, A_{E_4}, A_{E_1}, A_{E_2}, A_{E_1}, A_{E_2}, A_{E_3}, A_{E_4}, A_{E_3}, A_{E_4}, A_{E_5}, A_{E_5},$ 

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trio SbC(X)= $\{\bar{X},\emptyset,A_{E_5},A_{E_6},A_{E_7},A_{E_8},A_{E_{13}},A_{E_{14}},A_{E_{15}}\}$ . Since the trio soft b clopen set in X are  $\bar{X}$  and  $\emptyset$ ,  $\bar{X}$  is trio connected soft b set.

Let Y= {l} ,E = {e<sub>1</sub>, e<sub>2</sub>} and  $\overline{Y}$  = {(e<sub>1</sub>, {l}), (e<sub>2</sub>, {l})} =  $A_{E_1}$ .Consider  $\sigma_1$  = { $\overline{Y}$ ,  $\emptyset$ ,  $A_{E_4}$ },  $\sigma_2$  = { $\overline{Y}$ ,  $\emptyset$ ,  $A_{E_{13}}$ } and  $\sigma_3$  = { $\overline{Y}$ ,  $\emptyset$ }. Then  $\sigma_{1,2,3}$  open set = { $\overline{Y}$ ,  $\emptyset$ ,  $A_{E_4}$ ,  $A_{E_{13}}$ }. Also  $\sigma_{1,2,3}$  clopen set = { $\overline{Y}$ ,  $\emptyset$ ,  $A_{E_4}$ ,  $A_{E_{13}}$ }. Since  $A_{E_4}$ ,  $A_{E_{13}}$  are trio soft b clopen sets apart from  $\overline{Y}$  and  $\emptyset$ ,  $\overline{Y}$  is non trio connected soft b set.

#### **Proposition 4.7**

Let  $A_E$  is trio connected soft b set. Let  $A_{E_1}$  and  $A_{E_2}$  be trio separated soft b sets. If  $A_E \subseteq A_{E_1} \cup A_{E_2}$  then either  $A_E \subseteq A_{E_1}$  or  $A_E \subseteq A_{E_2}$ .

Proof: Let  $A_E$  be trio connected soft b set. Let  $A_{E_1}$  and  $A_{E_2}$  be trio separated soft b sets such that  $A_E \subseteq A_{E_1} \cup A_{E_2}$ . Suppose  $A_E \not\subseteq A_{E_1}$  or  $A_E \not\subseteq A_{E_1} \cap A_E \not= \emptyset$  and  $H_E = A_{E_2} \cap A_E \not= \emptyset$  then  $A_E = G_E \cap H_E$ . Since  $G_E \subseteq A_{E_1}$ , (trio  $sbcl(G_E)$ )  $\subseteq$  trio  $sbcl(A_{E_1})$ . Also trio  $sbcl(A_{E_1}) \cap A_{E_2} = \emptyset$ . Then trio  $sbcl(G_E) \cap H_E = \emptyset$ . Since  $H_E \subseteq A_{E_2}$ ,(trio  $sbcl(H_E)$ )  $\subseteq$  trio  $sbcl(A_{E_2})$ . Also (trio  $sbcl(A_{E_2}) \cap A_{E_1} = \emptyset$  then trio  $sbcl(H_E) \cap G_E = \emptyset$ . But  $A_E = G_E \cap H_E$ , therefore  $A_E$  be trio connected soft b space. It is a rebut. Then either  $A_E \subseteq A_{E_1}$  or  $A_E \subseteq A_{E_2}$ .

## Theorem 4.8

If  $A_E$  be trio connected soft b set and  $A_E \subseteq G_E \subseteq \text{trio } sbcl(A_E)$ ) then  $G_E$  is trio connected soft b set.

Proof: Suppose  $G_E$  is non trio connected soft b set then there occur two non empty soft sets  $A_{E_1}$  and  $A_{E_2}$  such that trio  $sbcl(A_{E_1})) \cap A_{E_2} = A_{E_1} \cap trio \, sbcl(A_{E_2})) = \emptyset$  and  $G_E = A_{E_1} \cup A_{E_2}$ . Since  $A_E \subseteq G_E$  then either  $A_E \subseteq A_{E_1}$  or  $A_E \subseteq A_{E_2}$ . Suppose  $A_E \subseteq A_{E_1}$  then  $(trio \, sbcl(A_E)) \subseteq (trio \, sbcl(A_{E_1}))$ , thus  $(trio \, sbcl(A_E)) \cap A_{E_2} = A_E \cap (trio \, sbcl(A_{E_2})) = \emptyset$ . But  $A_{E_2} \subseteq G_E \subseteq (trio \, sbcl(A_E))$ . Thus  $(trio \, sbcl(A_E)) \cap A_{E_2} = A_{E_2}$ . Therefore  $A_{E_2} = \emptyset$ , which is a contradiction. If  $A_E \subseteq A_{E_2}$ , then by the same way we can prove that  $A_{E_2} = \emptyset$ , it is a rebut. Thus  $G_E$  is trio connected soft b set.

## Theorem 4.9

If  $A_E$  be trio connected soft b set then (trio  $sbcl(A_E)$ ) is trio connected soft b set.

Proof: Assume  $A_E$  is trio connected soft b set and (trio  $sbcl(A_E)$ ) is not trio connected soft b set. Then there occur two disjoint not empty trio separated soft b sets  $A_{E_1}$  and  $A_{E_2}$  by that (trio  $sbcl(A_E)$ ) =  $A_{E_1} \cup A_{E_2}$ . Since  $A_E \subseteq (trio sbcl(A_E))$ ,  $A_E \subseteq A_{E_1} \cup A_{E_2}$ . And since  $A_E$  be trio connected soft b set, either  $A_E \subseteq A_{E_1}$  or  $A_E \subseteq A_{E_2}$ .

If  $A_E \subseteq A_{E_1}$  then (trio  $sbcl(A_E)$ )  $\subseteq$  (trio  $sbcl(A_{E_1})$ ) . But (trio  $sbcl(A_{E_1})$ )  $\cap A_{E_2} = \emptyset$ , hence (trio  $sbcl(A_E)$ )  $\cap A_{E_2} = \emptyset$  . Since  $A_{E_2} \subseteq$  (trio  $sbcl(A_E)$ ),(trio  $sbcl(A_E)$ )  $\cap A_{E_2} = A_{E_2}$  hence  $A_{E_2} = \emptyset$  which is a contradiction.

If  $A_E \subseteq A_{E_2}$  we can prove that  $A_{E_1} = \emptyset$ , which is a rebut. Hence (trio  $sbcl(A_E)$ ) is trio connected soft b set.

#### CONCLUSION

Soft set theory has a significant role in plays traditional and non-traditional argumentation in study of mathematical application. This paper defined as well as investigated some of the properties of trio separated soft b sets, trio connected soft b sets. The findings in this paper will serve as the basic block for the researchers to apply and develop the future study on soft tritopological spaces.

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