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Research Article

An Enhanced Probabilistic Graphical Model Framework, With Application to Community Identification In Dynamic Networks

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ABSTRACT

Real-world complex networks like social and biological networks generally exhibit inhomogeneity, leading to tightly coupled nodes, clusters or communities which serve a crucial functional purpose for the original system. Analyzing these communities in larger networks has become rapidly one among the hot areas in the complex networks. More general inorder to detect these communities in a variety of such complex networks, various strategies have been put forth. However probabilistic graphical model like Non negative matrix factorization (NMF) approach and its derivatives can efficiently address the issue of dynamic community detection in such complex networks, which is impossible often for other dynamic community detection approaches. Thus one may argue that NMF has a lot of flexibility when it comes to dynamic community detection. Unlike the existing approaches, we have formulated a framework comprising of 3 key steps: 1) information dynamics to quantify information propagation among involved nodes, thereby calculating the number of communities in the respective graph; 2) graph-regularization so as to attain precise representation of topology; 3) NMF soft-community-membership vectors so as to allot the nodes to the communities.

We believe strongly that this work can be a valuable resource for research workers who have interest in dynamic community detection, and encourage them to put in more effort so as to increase the versatility of NMF-based dynamic community detection.

Keywords: Probabilistic Graphical Model, Non-Negative Matrix Factorization, Community Detection, Information Dynamics.

INTRODUCTION

Complex networks possessing temporal features are referred to as dynamic networks. These networks are common in the real world and can be separated into several network slices. For example online social media networks are modelled as dynamic networks due to the reason of joining of new users or quitting of old users as times passes, and establishing or dismissing of relationships among the users. The ability of identifying the sets of users that interact with each other more frequently empowers us to comprehend how influence, cognition and even felicity flows through a social network. Identifying these sets or groups is known as community detection in social network analysis.

Detecting communities has of utmost importance in the fields of sociology, computer science and biology wherein systems are usually represented as the graphs. Graphs can be transformed into many different matrices to reveal their internal structure; D the degree matrix, A the adjacency matrix and L the laplacian matrix are most commonly used. As a result, matrix factorization can be used directly to these matrices so as to learn the hidden node representation. In an intuitive manner, if every dimension of representation is considered as the community; the vector nodes can also specify their membership in every community to resolve the community overlapping problem. This is what led to the popularity of non negative matrix factorization.

The matrix factorization approaches decomposes the matrix into smaller matrices thus improves the ability of detecting hidden patterns (features) and the ways nodes are linked together. To obtain more accurate results for community detection, one of these decomposed matrices may be utilized to identify one characteristic in the network while the others can be used to identify other features and then utilize them parallelly. Additionally, we don't need to worry because these decomposed matrices too are likewise capable of preserving as much info as possible.

Discovering communities enables us to alter the representation's resolution and create a comprehensible network map. The majority of social networks exhibit community structure, or node clusters with a significantly larger percentage of links inside the clusters and less connections between the clusters. Since the social networks are evolving, the community structure needs to be updated efficiently in response to the changing issues of network community structure and therefore the social network community detection problem needs the augmented solution. In this study an attempt has been made to address the issue.

LITERATURE SURVEY

Over the last ten years, numerous community detection methods have been put forth. However, many such algorithms neglect the temporal nature of networks and instead carry out community detection in static networks. Numerous networks originating from nature and society are, in fact, dynamic, meaning that their structure changes across different time intervals or conditions. [2] For instance, connections frequently alter in reaction to how a person's social partners behave [3]. It is therefore valuable to monitor the evolution of communities in dynamic networks (sometimes referred to as evolving or dynamic communities) [4].

NMF has drawn a lot of interest in community identification clustering because to its natural interpretability and strong practical performance. Matrix factorization (decomposition) is a collaborative filtering algorithm that works by decomposing the input matrix into two lower dimensional matrices making it easier to infer and calculate information from them. To enhance NMF's performance in community detection, numerous researchers are working on it; some of their notable efforts are as follows:

Prorakis et al. [5] suggested the Bayesian NMF approach to carry out community detection. The benefits of this approach include its automatic community count calculation and its non-resolution-limited nature. Unfortunately, the factorization could be misled and provide an inaccurate answer by its built-in estimate of the number of communities. Iteratively developing an optimisation method, Wang et al. [7] looked at the application of NMF to community detection for directed networks, undirected networks, and hybrid networks.

A constrained NMF triple decomposition approach called BNMTF was presented by Zhang et al. [8], that may detect communities in directed and undirected networks using the unified model. However, the number of communities must be predetermined, though, according to BNMTF. A popular technique for community analysis and their evolution while taking network dynamics into account is FacetNet [10]. Unlike typical techniques, which tackle two stage processes asynchronously, it merges communities and their evolutions in a unified way utilising NMF. However, the FacetNet method requires prior knowledge and the number of community divisions to be specified. Very often, it is challenging to estimate the number of community divisions ahead of time.

Moreover, a multi objective genetic optimisation approach built on evolutionary clustering called DYNMOGA [11] discovers the community structure within dynamic networks by maximising NMI and Modularity. However it needs the parameter setting which is a cumbersome task in itself. Despite significant efforts towards improving dynamic community detection, numerous issues are still unresolved.

To assess clustering drift, for example, the existing methods choose to use either the local information or the global information of the network over the previous time slice, thereby falling short of accurately capturing community dynamics. However, we address these issues by regularising both the network and community structure from the previous time slice via Graph-Regularized NMF. Furthermore deciding the no. of communities is yet an unresolved issue. Traditionally community detection algorithms based on NMF can obtain the no. of communities through optimized embedded target-function. But, it is challenging to pinpoint the exact no. of communities because these techniques are dependent on a variety of variables like predetermining the no. of communities and optimized

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target-function. However, our approach addresses this issue by employing information dynamics to quantify information propagation among involved nodes, thereby calculating the number of communities automatically in the respective graph based on the information flow.

RESEARCH GAP

Because of the dynamic characteristic, it is more difficult to detect communities in dynamic networks as that of static networks, therefore detecting communities in real world networks is however a challenge. The challenge is not restricted to evaluation quality of the detected communities or the algorithm scalability but the other few challenges which are important to get approached are as under:

- Identifying communities within dynamic networks, wherein new nodes may connect the network, existing nodes may leave or new edges may be produced or existent edges may terminate.
- Examining the stability of the communities detected by various methods, especially in dynamic networks.
- Combining non-structural and structural information to identify more realistic communities, if such knowledge is available.
- Most of the community detection approaches needs in advance the specification of the number of communities before implementing their primary procedures, so how to choose the community no is still a matter of debate.
- Interpreting what the detected communities reveal about the system functioning and how the results of a community identification algorithm might be applied in various ways.

OUR CONTRIBUTION

Identifying communities in large scale heterogeneous dynamic networks is a challenging task, thus arises the need of devising a framework that is not only capable enough for dynamic community detection but posses scalability and good info fusion capability also. Since NMF has a lot of flexibility when it comes to dynamic community detection, the development of such an adaptable NMF-based community detection framework is therefore promising.

Assuming the time as discrete, we observe the social interactions as several subgraphs of persons in every time step (every individual need not be observed at every time step). The groupings thus observed will serve as our basis for identifying true communities with their development over a course of time so that the community structure inferred can explain the majority of the interactions observed.

To summarize we have formulated a framework comprising of 3 key steps: 1) information dynamics to quantify information propagation among involved nodes, thereby calculating the number of communities in the respective graph; 2) graph- regularization so as to attain precise representation of topology; 3) NMF soft- community-membership vectors so as to allot the nodes to the communities.

Our method's effectiveness in identifying and describing community dynamics within extremely dynamic networks is ofcourse a key benefit. Therefore, it is desirable to use our approach to identify and examine the community evolution within larger networks that experiences a lot of change over time.

OUR FRAMEWORK

This research introduces a method of nonnegative matrix factorization approach towards temporal networks that detect community structures and analyzes their evolution. Graph adjacency matrices are factorised into two non negative matrices by employing NMF based approach, wherein each column of the factor -matrix may be utilized to analyze the node's inclination to belong to various communities and the other may be used to recognize mappings in between the community membership and original network. Inorder to identify the community dynamics and to track the community evolution we have presented an enforced graph-regularized non-negative matrix factorization framework.

A given complex network can be represented as a non-negative feature matrix (for instance adjacency matrix). This feature matrix can be factorised using NMF to create the node-community indicant matrix. Because of the non-negative constraint each element of this matrix may be interpreted as the puissance of related nodes of the corresponding community, thus making the results of the community detection more interpretable. However, we can see that the adjacency-matrix A is typically chosen as the feature-matrix X by the majority of the NMF-based community discovery algorithms for factorization. Despite being easy and straightforward way to follow, as A is most readily available; it often produces results that are unstable and much slower to converge because of being very sparse often and representing local structural features. Thus the effectiveness of community detection undoubtedly will suffer if the feature matrix is not containing enough information. To resolve this issue so as to adopt an advanced initialization mode to construct more informative feature-matrix became one of our main focuses.

Following explicit assumptions regarding individual behaviour are made by us so as to derive an optimisation formulation for dynamic community identification:

- Every group represents a different community at every time step. If the two groups happen to be present simultaneously, they are separate for a reason and so signify two different communities.
- At any given time, a person belongs to one community only. Although the person's affiliation with a community may change throughout the course of time, it is only ever affiliated with one and only one community at a time.
- A person is most often present in that group which represents the community to which it is associated with. It hardly ever misses its presence with the group in its own community and hardly ever attends the groups in other communities. In other words, people in one community interact with one another more often than people in other groups.

Our framework is capable of uncovering the connected components in a graph without explicitly specifying the number and size of communities as was required in previous studies [10], [12]. Moreover the approach is scalable enough to accommodate very large networks as often observed in real data.

Dynamic graph, also named as temporal graph in this paper, refers to a particular type of graphs which can be dynamically changed. This graph characteristic will lead to node community evolution, which usually happens in social media or other real-world scenarios where the user profiles and interactions are updated frequently. There may be relatively persistent members (core nodes) in a community. However, it also contains a lot of "fluid" members, or those that join communities periodically but leave after a few time steps, besides having new members which influences the community dynamics.

In general, dynamic communities are the communities that have the tendency to change dynamically, such changes affect the structure of the built in communities synchronously. We learnt that the community during its life time undergoes through following stages:

- Growth: An existing community can grow by gaining new nodes.
- Contraction: An existent community rejects some member-nodes.
- Merging: Two or more than two existent nodes merge to form the new community.
- Splitting: An existent community bursts into two or more communities.
- Birth: A new community comes, compiled of any no. of nodes.
- Death: An existing community may disappear at any moment of time by losing all member-nodes.
- Revival: An existing community disappears for some time and comes back after some time as if it has never disappeared.

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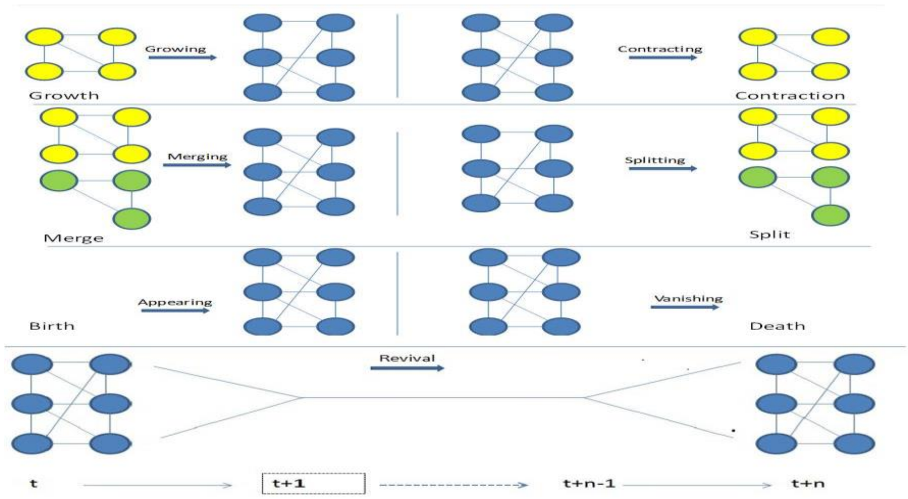

Fig.1. Classic view of Events occur in a Community

Over the period of time the dynamic communities undergoes different mutations that modifies the structure of the community. As the community emerges, it mutates state. The state of a community is determined in particular by the current activity of that community. Each community may be in one of the following states at a given time as described in Fig. below:

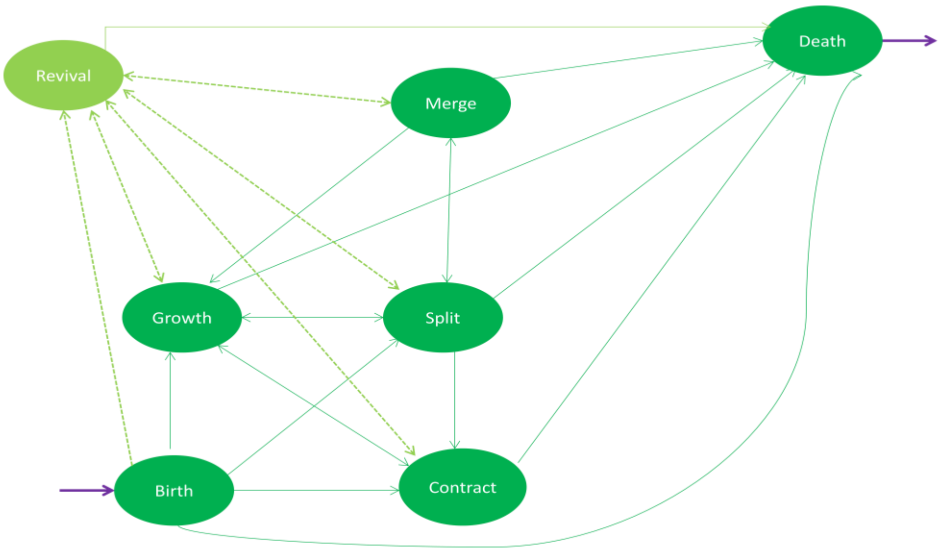

Fig.2. State diagram showing the life cycle of a Community

Inorder to analyze these network events in a much better way that mutates the community state dynamically, we took a small network consisting of 15 (1 to 15) nodes initially and over a period of time different operations performed on it were categorized numerically as:

i) Nodes Adding: When we talk about "adding nodes," we mean the new nodes that have been added to current time slice G_t network since G_{t-1}. Let NA stand for the set of newly added nodes wherein NA is determined as:

$$NA=\{V \mid v \notin V_{t-1}, v \in V_t\}$$

wherein V_{t-1} represent the node set in network G_{t-1} and V_t represents the node set in network G_t.

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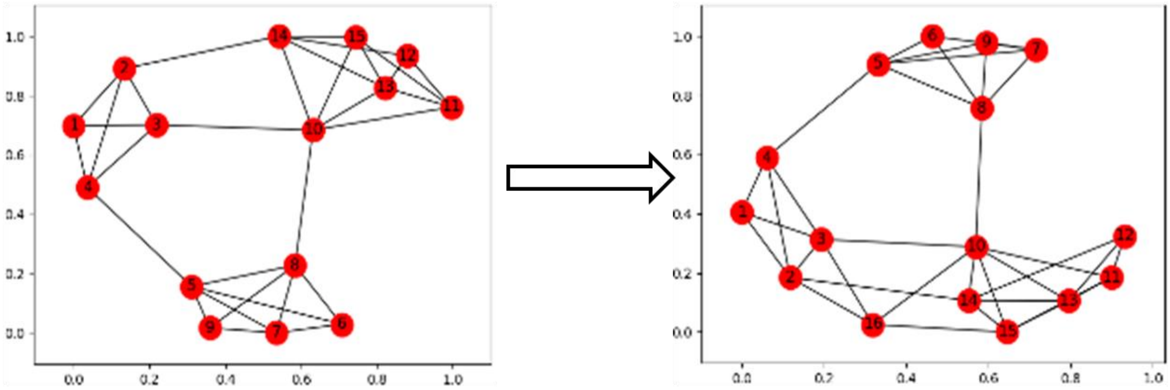

Fig.3. 1) Original graph *2) After node addition*

Fig.3. Showing how the original graph gets changed when new nodes came into existence.
ii) Nodes Deleting: When a node is destroyed, it means that it has been eliminated from the current time slice network Gt in comparison to the preceding time slice network Gt-1.

Here we took the resultant graph of fig.3.1 and performed node deletion event on it. The results were shown in fig.4.2. Let ND stand for the deleted node set, wherein ND is determined as:

$ND=\{V \mid v \in V_{t-1}, v \notin V_t\}$

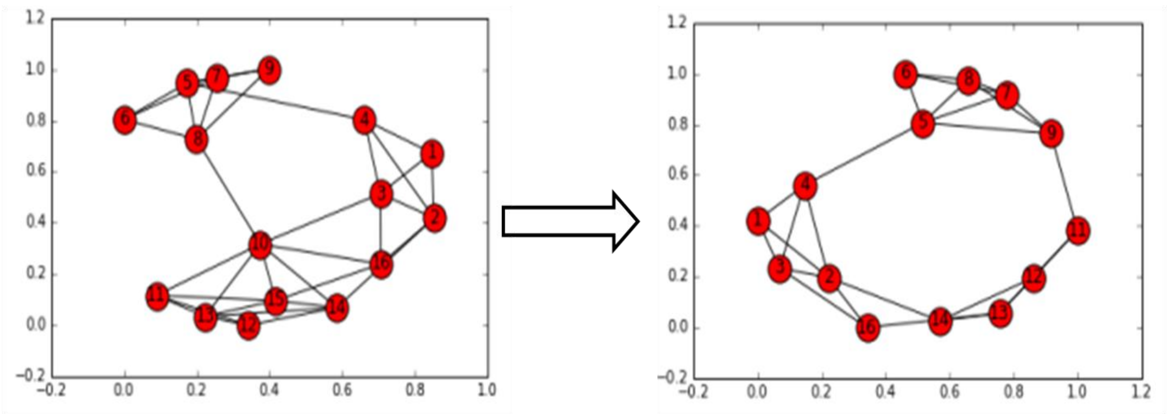

Fig.4. 1) Original graph *2) After node deletion*

iii)Edges Adding: In a similar manner, the newly edges added corresponds to the newest edges into the current time slice network G_t in comparison to the previous time slice network G_{t-1}. Let EA stand for the added edge set, wherein EA is defined as:

$EA=\{E \mid e \notin E_{t-1}, e \in E_t\}$

wherein E_{t-1} represent the edge set in the network G_{t-1} and and E_t represent the edge set in the network G_t.

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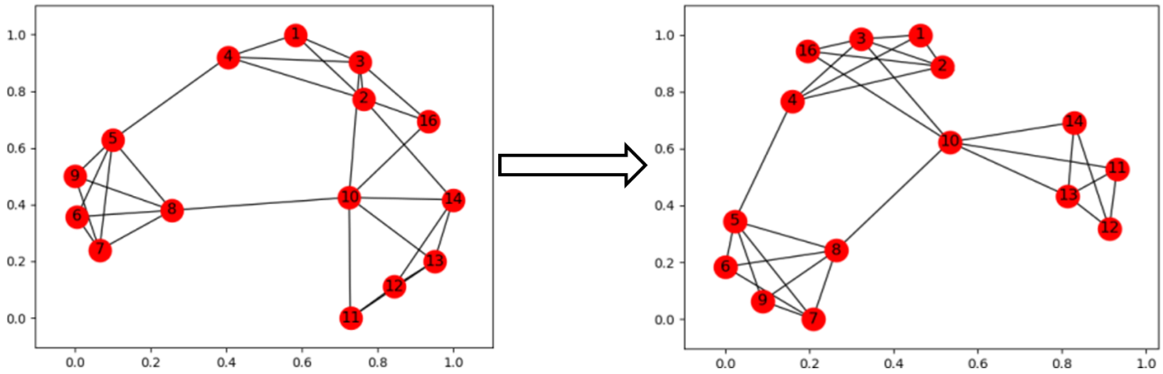

Fig.5. 1) Original graph *2) After Edges addition*

iv)Edges Deleting: The edge deleted is referring towards the edge that is removed in current time slice network G_t in comparison to the previous time-slice network G_{t-1}. Let ED stand for the deleted edge set, wherein ED is determined as:

$$ED = \{E |\ e \notin E_t ,\ e \in E_{t-1}\}$$

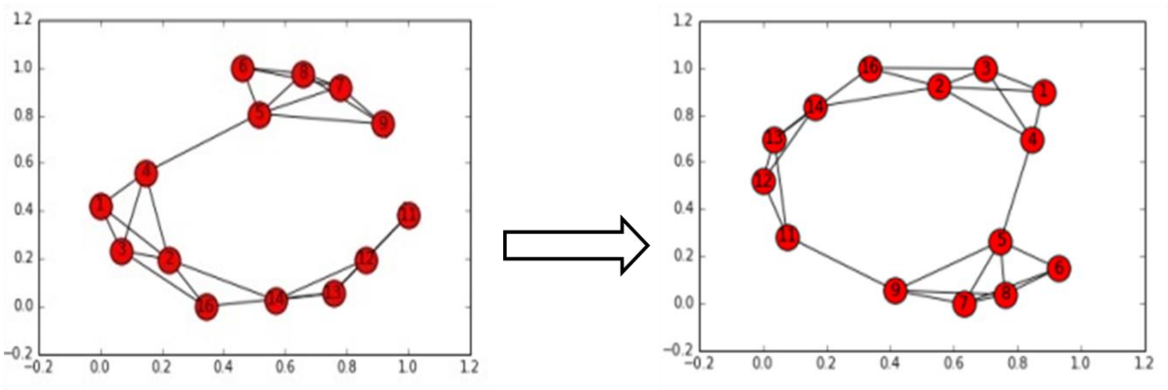

Fig.6. 1) Original graph *2) After edges deletion*

The majority of the currently used NMF-based techniques primarily concentrate on the first order topological info of a network described by the adjacency matrix regardless of the implicit relationships amongst the implicated nodes taking into account. However, this work attempts to provide efficient and effective community detection strategy through the NMF model by involving the second and third-order nodes as well besides incorporating such implicit associations amongst nodes quantized.

The steps of our framework are summarized as follows:

Algorithm I Graph-regularized NMF

Input: G = {Graph 0, Graph1, ..., Graph k}

Output: C = {Community 1, Community 2, ..., Community k}

// Detecting Communities Initially

1: Embed *G* using information flow score

and topological information

Return Y_{Gs} with no. of clusters as *cl*

2: *V, U*←NMF (Y_{Gs}, *cl*)

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3: U←Regularize(U)

4: U←Normalize(*Regularized* U)

5: **for** i in range [1 to cl] **do**

6: Ci ←those nodes in row u_i whose entries are 1 or greater than or equal to 0.75 along with their neighbour nodes

7: **end for**

8: Compute quality score

// **Community Dynamics**

9: **for** t = [1 to k] **do**

10: Calculate NA,ND,EA,ED using steps 11 to 14 below

11: ΔG_t ← *Nodes_adding*(NA,C_{t-1})

12: ΔG_t ← *Nodes_deleting* (ND,C_{t-1})

13: ΔG_t ← *Edges_adding*(EA, C_{t-1})

14: ΔG_t ← *Edges_deleting*(ED, C_{t-1})

15: $Y_{Gs}^{(t)}$ ←Dynamic Embedding of ΔG_t and return no. of clusters as cl

//**Recalculating Communities**

16: Repeat steps 2 to 8

17: **end for**

Algorithm II Nodes_adding

Input: NA, C

Output: Δg

1: **for** every node v belongs to NA **do**

2: **if** N(v) belongs to same community C_N(v) **then**

3: C_N(v) ←v

4: **else**

5: Δg ←v

6: **for** every node u belongs to N(v) **do**

7: Δg ←Cu

8: **end for**

9: **end if**

10: **end for**

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Algorithm III Nodes_deleting

Input: ND, C

Output: Δg

1: **for** every node v belongs to ND **do**

2: **for** every node u belongs to N(v) **do**

3: Δg ←Cu

4: **end for**

5: **end for**

Algorithm IV Edges_adding

Input: EA, C

Output: Δg

1: **for** every edge e belongs to EA **do**

2: **if** $C_u \neq C_v$ **then**

3: Δg ← C_u

4: Δg ←C_v

5: **end if**

6: **end for**

Algorithm V Edges_deleting

Input: ED, C

Output: Δg

1: **for** every edge e belongs to ED **do**

2: **if** $C_u = C_v$ **then**

3: Δg ← C_u

4: **end if**

5: **end for**

This research presented a novel hybrid approach that is more topologically motivated and intuitive for detecting dynamic communities. Information dynamics and topological information are employed in such away, so that these two parts are integrated into a unified framework.

The proposed framework mainly consists of two key steps: 1) information dynamics to quantify information propagation among involved nodes, thereby calculating the number of communities in the respective graph; and 2)

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graph regularized NMF soft-community-membership vectors so as to allot the nodes to the communities. On conducting experiments employing a combination of small and large 3 real-world networks, the framework showed favourable precision, performance and good conductance in comparison with other baseline approaches.

EXPERIMENTAL SETUP

For many diverse applications, accessing and analysing large scale real-world datasets is essential. It is not at all simple to gather and process real data. The challenges may be of both general as well as technical nature. Obtaining data access, ethical and privacy concerns, processing the dataset beforehand and conducting analysis of the same data are merely few challenges that must be resolved prior to the data utilization for the application. The processing of the enormous volume of data presents the biggest challenge, though. The method of collecting data must keep-up with how quickly data is formed or received. Typically, sampling data, processing summaries of data, or limiting analysis to snapshots of data collected within specific time intervals are unavoidable.

SNAP offers a large collection of network datasets containing more than fifty networks having millions of edges and nodes in each. They consist of a variety of networks used in real-world scenarios, such as social networks, citation networks, road networks, web graphs, etc. Most of the papers that we employed utilized dataset from this network. In addition to the SNAP datasets, other publicly accessible and commonly used datasets include the American Football dataset, Zachary Karate dataset etc.

The laptop used for all the studies has an i5 @ 2.40 GHz processor, 4 GB RAM, and 64-bit Windows operating system. Furthermore Python was used to implement all the framework.

DATA SETS DEPLOYED

We deployed the combination of different real-world and artificial social networks as discussed below in order to validate the effectiveness of our model.

W. Zachary investigated a karate club's social network for three years, from 1970 to 1972 [14]. One way to summarise the data is as a list of integer pairs. Every integer denotes a single karate club member, and a pair denotes the interaction between two members.

Fall 2000 saw the network of division IA American football games taking place. Teams are represented as vertices(115), while the regular season games between the two teams they connect to are represented by edges(616) [15].

We used Twitter API and selected the tweets related to Covid-19 issues and data was collected online at COVID-19_Sentiment Analysis & Social Networks _ Kaggle.html. We explore which Twitter users share similar content on Covid-19 issues. The collected tweets were then used in constructing the social network inorder to identify communities.

EVALUATION METRICS USED

We used normalised mutual information (NMI) and adjusted random index (ARI to gauge how well our model performed by comparing the discovered community structures to the ground truth and the findings are displayed. Information theory is where the NMI and ARI similarity metrics has its roots and have shown to be trustworthy. It asserts that minimal further information is required to infer one from the other if two partitions are comparable. Given two network partitions A and B, NMI $(X; Y)$ is computed as:

$$\text{NMI}\ (X;Y) = \frac{2I(X;Y)}{H(X)+H(Y)}$$

Wherein $I\ (X;\ Y)$ represents mutual information of X with Y and $H\ (Y)$ denotes entropy from Y. This NMI value ranges between 0 to 1 wherein NMI = 0 occurs if the obtained partition fully differs from the true partition. On the other hand, NMI = 1 occurs when the obtained division fully matches the actual partition.

$$\text{ARI}= \frac{RI-Expected\ RI}{max\ RI-Expected\ RI}$$

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Partitions include every pair of samples, while RI stands for the similarity in between two networks. Next, it determines how many pairs are assigned to distinct or identical network partitions in thee expected and real network partitions.

RESULTS

This research aims to generate strong community detection outcomes in dynamic networks. Initially, our methodology was used on these two crucial and basic networks (Fig. 7.) Thankfully, when compared to the baseline methods, we were able to get satisfactory outcomes.

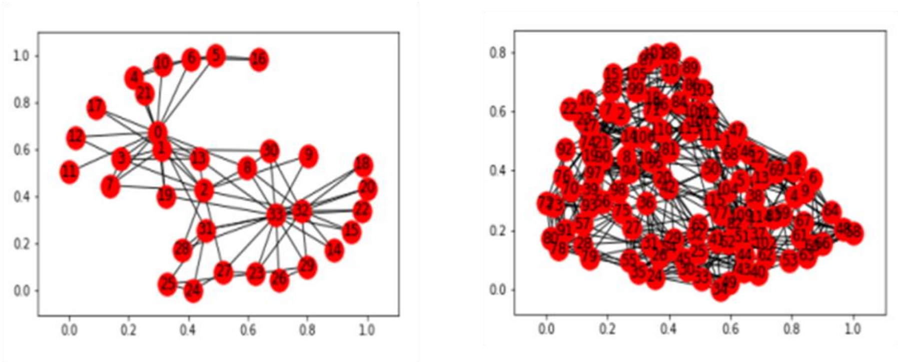

Fig.7.1. Graphs generated from Karate dataset and American football dataset

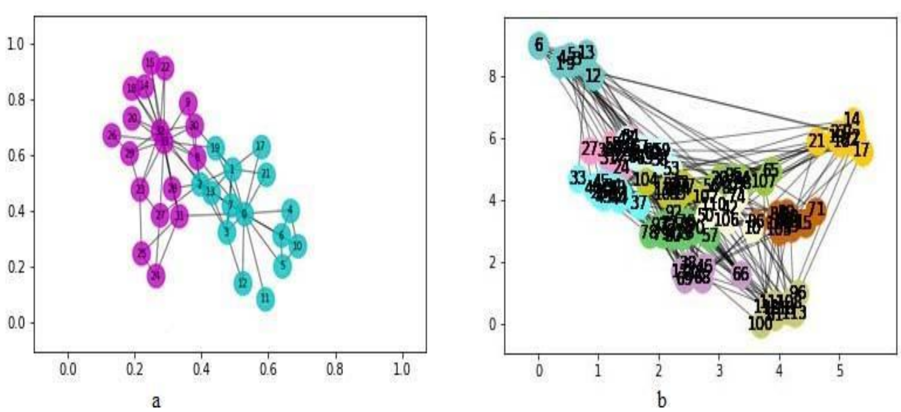

Fig.7.2. Our model identified the community structures of the a) American football network and b) karate network, with each colour denoting a different community.

Fig. below shows the graph generated from Twitter dataset, whose results are shown beneath.

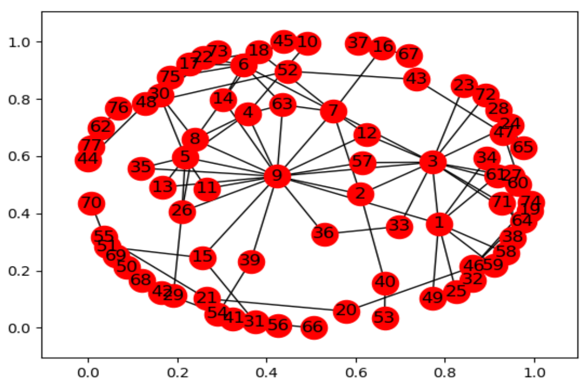

Fig.8.1. Twitter Network

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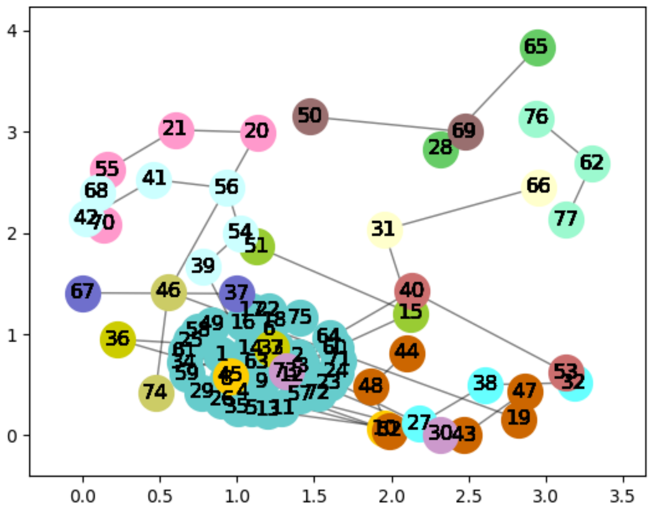

Fig.8.2 Community structure of the Twitter data set.

To assess the performance of our framework further, we compared our approach with other state of the approaches like FacetNet [10] and DYNMOGA [11]. Consider a sample network of fig.3.1 within which communities undergoes through different mutations like its communities gets expanded and contracted dynamically upon addition of nodes and edges or deletion of nodes and edges (as shown in figures 3 to 6). Moreover, to visualise the mutational communities, we used this very sample network as the case study for the purpose of expositional clarity as shown below:

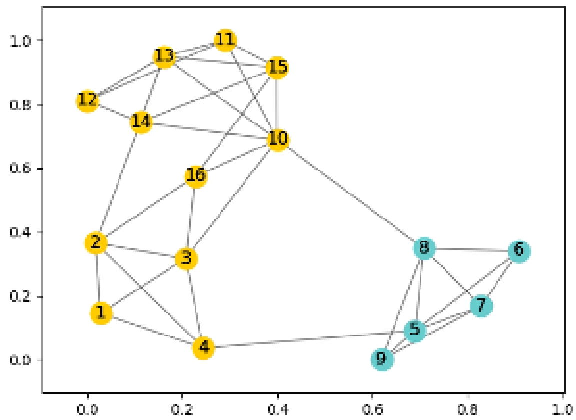

Fig.9. Visualization of community detection

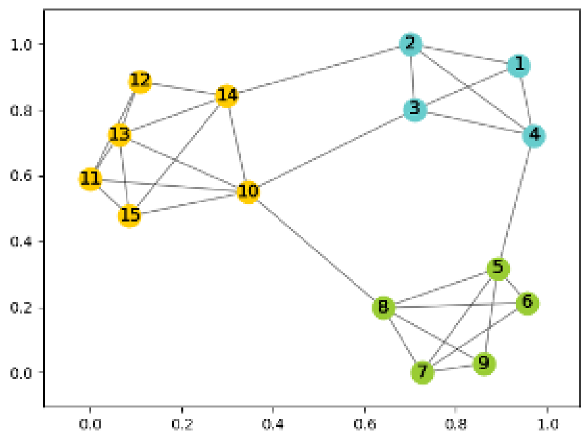

Fig.10. shows the mutational community evolution

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Furthermore, based on such mutations, we compared our approach possessing such mutated events against the two above mentioned approaches by employing two metrics NMI and ARI and fortunately we got satisfactory results as is clearly visible from the graph below:

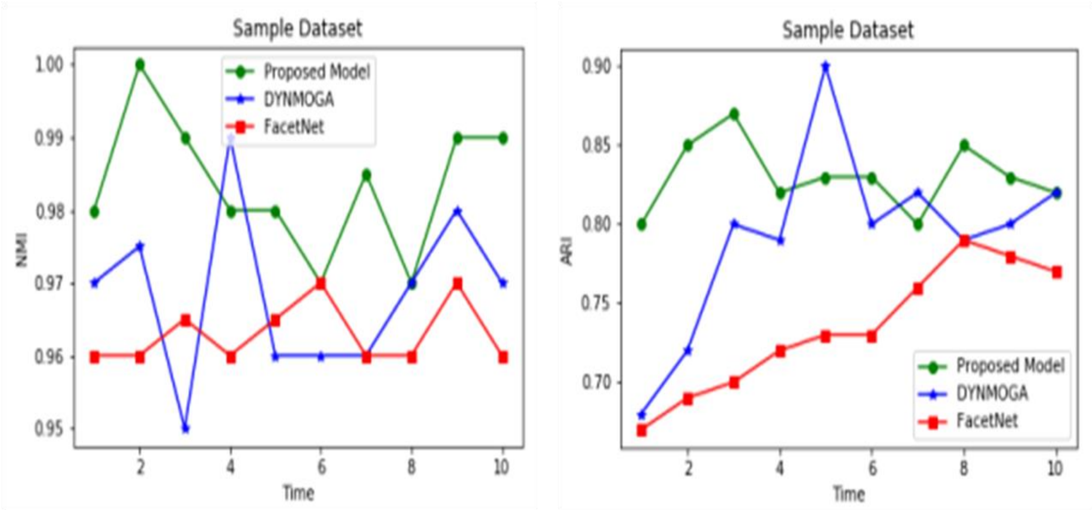

Fig.11. Performance comparison in term of NMI and ARI on a Sample network

CONCLUSION

This framework is the first in a series of global investigation of dynamic networks. The continuous data generation leads to an evolving network dynamics. Currently majority of the networks possess dynamic nature which implies that they don't exhibit fixed topology. Usually, a graph represents a network having nodes and the links that connect these nodes. However, this concept of a network needs to be adjusted to include the other crucial dimension, time. This temporal dimension eases improvised insight of the network on embedding valuable knowledge to it and such improved network definition is the subject of the study reported here.

Furthermore the evaluation revealed that the quality of the model's factorization has a significant impact on the community detection results. All of the target communities' nodes are generated by a good factorization, and there are either none or very few extraneous nodes that can be removed during community detection.

FUTURE SCOPE

Though our framework yielded promising results, yet there is room for further improvements with reference to performance. Moreover, the possible avenue towards future-work involves extending our framework so as to handle enhancing our framework to handle networks of exceptionally greater sizes besides considering the incorporation of supervised techniques of community detection. Furthermore the provision of benchmark graphs is still an unresolved issue in the domain of dynamic community detection.

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CONFLICT OF INTEREST

We hereby declare that we have no conflict of interest.

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