

Generalized Intuitionistic Trapezoidal Fuzzy Numbers in Transportation Problems: An Optimizational Method

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ABSTRACT

In this study, we propose an optimization method for transportation problems utilizing Generalized Intuitionistic Trapezoidal Fuzzy Numbers (GITFNs). This method effectively handles uncertainty and imprecision in transportation systems, enabling more realistic and reliable decision-making. A GITFN-based transportation model is developed, incorporating fuzzy costs, capacities, and demands. An optimization algorithm is designed to minimize transportation costs while satisfying supply and demand constraints. The method is applied to a numerical example, demonstrating its efficiency in solving complex transportation problems. The results show significant improvements in transportation effectiveness and cost reduction.

Keywords: Generalized Intuitionistic Trapezoidal Fuzzy Numbers, Transportation Problems, Optimization, Linear Programming, fuzzy ranking

1. INTRODUCTION

In operations research, transportation problems involve studying the transportation distribution of resources to minimize costs. Real-world scenarios require transporting products from various sources to different destinations efficiently. The transportation problem is a specialized linear programming problem that optimizes goods distribution from sources to destinations, considering factors like capacities, demands, and transportation costs. The general transportation problem was first developed by Hitchcock [1], who addressed parameters like transportation cost, resource availability, and demand uncertainty. Modern science and technology involve complex processes with incomplete information, making precise transportation cost determination challenging. Uncertainty is often represented using fuzzy numbers. Fuzzy set theory, established by Zadeh [2] in 1965, provides methods for ranking fuzzy numbers to process uncertainty. Many researchers [3-5] have applied fuzzy set theory to fuzzy transportation problems. Zimmermann [3] developed Zimmermann's fuzzy linear programming, which evolved into various fuzzy optimization methods. Kaur and Kumar [4] proposed a new method based on ranking function for solving fuzzy transportation problems using generalized trapezoidal fuzzy numbers. Intuitionistic fuzzy sets, introduced by Atanassov [7], offer a more precise approach by incorporating degrees of membership and non-membership.

Researchers [8-10] and [15-19] have explored intuitionistic fuzzy sets, with Aggarwal and Gupta [9], Antony et al. [10] and Singh and Yadav [12] proposing methods for type-2 intuitionistic fuzzy transportation problems. Ranking intuitionistic fuzzy numbers is crucial, with methods including membership ranking, non-membership ranking, composite, and distance-based ranking. Recent studies [6,11] have introduced novel ranking methods. Babu et al. [6] proposed ranking generalized fuzzy numbers using centroids of centroids, while Chhibber et al. [11] solved type-1 and type-2 fuzzy transportation problems using incentre of centroids.

Gupta and Kumar [5] developed an efficient method for solving intuitionistic fuzzy transportation problems of type-2. Researchers [20-24] have implemented arithmetic operations and ranking functions for intuitionistic fuzzy numbers and applied to transportation problems. The arithmetic operations and the ranking function of IFNs have been implemented by many researchers. the PSK method was applied to solve intuitionistic fuzzy solid assignment problems [13]. A transportation system was developed for urban districts, accompanied by a case study [14].

This paper is organized as follows : Section 2 provides preliminaries on fuzzy concepts, including generalized intuitionistic trapezoidal fuzzy numbers and transportation problems. Section 3 explains the existing methodology for ranking generalized trapezoidal fuzzy numbers. A new methodology for solving transportation problems with generalized trapezoidal intuitionistic fuzzy numbers is presented in section 4. Section 5 illustrates a numerical example applying this methodology to transportation problems with generalized intuitionistic trapezoidal fuzzy numbers. Finally, Section 6 discusses comparative analysis and Section 7 deliberates the results and future research directions.

2. PRELIMINARIES

This section contains several simple descriptions as defined in reference [11].

Definition 2.1: Fuzzy Set

Let U be a universal set. A fuzzy set A of U is defined by a membership function $f_A: U \rightarrow [0,1]$, where $f_A(x)$ represents the degree of membership of x in A . The fuzzy set A is represented as: $A = \{(x, f_A(x))/x \in U\}$.

Definition 2.2 : Intuitionistic Fuzzy Set

An Intuitionistic Fuzzy Set A in U is defined by $A = \{(x, f_A(x), g_A(x))/x \in U\}$ where f_A , and g_A are functions from U to $[0, 1]$ representing the degree of membership and non-membership of x in U , respectively, such that : $0 \leq f_A(x) + g_A(x) \leq 1$, for all $x \in U$.

Definition 2.3 : Intuitionistic Fuzzy Number

An intuitionistic fuzzy set $A = \{(x, f_A(x), g_A(x))/x \in U\}$ is called an intuitionistic fuzzy number on real line \mathbf{R} if it satisfies

(i) Intuitionistic fuzzy normality ($\exists z \in \mathbf{R}, f_A(z) = 1$ and $g_A(z) = 0$), (ii) Intuitionistic fuzzy convexity ($f_A(\lambda x + (1 - \lambda)y) \geq \min(f_A(x), f_A(y))$ and $g_A(\lambda x + (1 - \lambda)y) \leq \max(g_A(x), g_A(y))$), where $x, y \in U, \lambda \in [0,1]$, (iii) $f_A(x)$ and $g_A(x)$ are piecewise continuous real-valued functions, and (iv) Support of A is bounded.

Definition 2.4 : Intuitionistic Trapezoidal fuzzy number (Figure 1)

An intuitionistic fuzzy number A is said to be Intuitionistic Trapezoidal Fuzzy Number (ITFN) and denoted by $A = (a_1, a_2, a_3, a_4); (a'_1, a_2, a_3, a'_4)$ with membership function f_A and non-membership function g_A defined by

$$f_A = \begin{cases} 0, & x < a_1 \\ \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ 1, & a_2 \leq x \leq a_3 \\ \frac{x-a_4}{a_3-a_4}, & a_3 \leq x \leq a_4 \\ 0, & a_4 < x \end{cases} \quad \text{and} \quad g_A = \begin{cases} 0, & x < a'_1 \\ \frac{x-a'_1}{a'_1-a'_2}, & a'_1 \leq x \leq a'_2 \\ 1, & a_2 \leq x \leq a_3 \\ \frac{x-a'_3}{a'_4-a'_3}, & a_3 \leq x \leq a'_4 \\ 0, & a'_4 < x \end{cases}$$

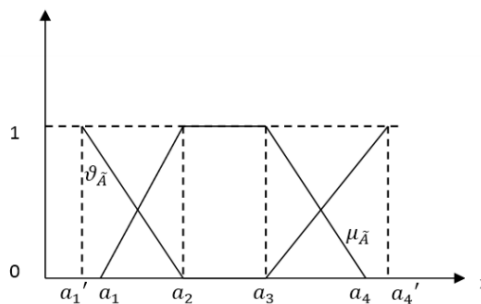


Figure 1: Intuitionistic Trapezoidal Fuzzy Number (ITFN)

Definition 2.5: Generalized ITFN (Figure 2)

Generalized Intuitionistic Trapezoidal Fuzzy Number is represented by

$A = (a_1, a_2, a_3, a_4; \omega_a)(a'_1, a_2, a_3, a'_4; \sigma_a)$ with membership function f_A and non-membership function g_A defined by

$$f_A = \begin{cases} 0, & x < a_1 \\ \frac{x-a_1}{a_2-a_1} \omega_a, & a_1 \leq x \leq a_2 \\ \omega_a, & a_2 \leq x \leq a_3 \\ \frac{x-a_4}{a_3-a_4} \omega_a, & a_3 \leq x \leq a_4 \\ 0, & a_4 < x \end{cases} \quad \text{and} \quad g_A = \begin{cases} 0, & x < a'_1 \\ \frac{x-a'_1}{a'_1-a'_2} \sigma_a, & a'_1 \leq x \leq a'_2 \\ \sigma_a, & a_2 \leq x \leq a_3 \\ \frac{x-a_3}{a'_4-a_3} \sigma_a, & a_3 \leq x \leq a'_4 \\ 0, & a'_4 < x \end{cases}$$

where ω_a and σ_a correspond to a high level of contribution and a low level of non-contribution and $0 \leq \omega_a \leq 1, 0 \leq \sigma_a \leq 1, 0 \leq \omega_a + \sigma_a \leq 1$.

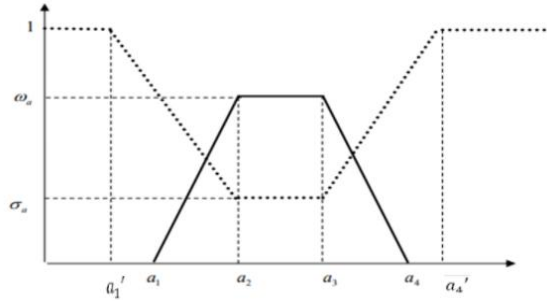


Figure 2 : Generalized ITFN

Arithmetic operations:

Let $A = \langle (a_1, a_2, a_3, a_4; \omega_a)(a'_1, a_2, a_3, a'_4; \sigma_a) \rangle$ and $B = \langle (b_1, b_2, b_3, b_4; \omega_b)(b'_1, b_2, b_3, b'_4; \sigma_b) \rangle$ be Generalized trapezoidal intuitionistic fuzzy numbers. Then the arithmetic operations are

(i) Addition:

$$A \oplus B = \left\{ (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4; \min(\omega_a, \omega_b)), (a'_1 + b'_1, a_2 + b_2, a_3 + b_3, a'_4 + b'_4; \max(\sigma_a, \sigma_b)) \right\}$$

(ii) Subtraction:

$$A \ominus B = \left\{ (a_1 - b_4, a_2 - b_2, a_3 - b_3, a_4 - b_1; \min(\omega_a, \omega_b)), (a'_1 - b'_4, a_2 - b_2, a_3 - b_3, a'_4 - b'_1; \max(\sigma_a, \sigma_b)) \right\}$$

(iii) Scalar Multiplication:

$$k \otimes A = \begin{cases} (ka_1, ka_2, ka_3, ka_4; \omega_a)(ka'_1, ka_2, ka_3, ka'_4; \sigma_a) & \text{if } k \geq 0 \\ (ka_4, ka_3, ka_2, ka_1; \omega_a)(ka'_4, ka_3, ka_2, ka'_1; \sigma_a) & \text{if } k < 0 \end{cases}$$

3. EXISTING GENERALIZED ITFN RANKING METHOD [25]

Consider a Generalized Intuitionistic Trapezoidal Fuzzy Number (GITFN) represented as: $A = (a_1, a_2, a_3, a_4; \omega_a)(a'_1, a_2, a_3, a'_4; \sigma_a)$ with membership function f_A and non-membership function g_A as shown in Figure 2. The centroid of the trapezoid is considered its balancing point. Divide the trapezoid into three triangular regions corresponding to the membership function and non-membership functions. Calculate the centroids of each triangle, then compute the overall centroid.

For the membership function, the centroid is :

$$G(x_0, y_0) = \left(\frac{2a_1 + 7a_2 + 7a_3 + 2a_4}{18}, \frac{7\omega_a}{18} \right).$$

The area of the triangle is $x_0.y_0$. denoted as :

$$S_{f_A} = \frac{2a_1+7a_2+7a_3+2a_4}{18} \cdot \frac{7\omega_a}{18}$$

For non-membership function the centroid be $G(x_0, y_0) = \left(\frac{2a'_1+7a'_2+7a'_3+2a'_4}{18}, \frac{11+7\sigma_a}{18} \right)$

Area of the triangle is $x_0.y_0$. and it is denoted by $S_{g_A} = \frac{2a'_1+7a'_2+7a'_3+2a'_4}{18} \cdot \frac{11+7\sigma_a}{18}$

Rank of A is defined as $R(A) = \frac{\omega_a S_{f_A} + \sigma_a S_{g_A}}{\omega_a + \sigma_a}$.

Comparing Generalized intuitionistic trapezoidal fuzzy numbers using the Ranking function :

Let A and B be two GITFNs. Then :

- (i) If $R(A) > R(B)$ then $A > B$
- (ii) If $R(A) < R(B)$ then $A < B$
- (iii) If $R(A) = R(B)$ then $A = B$

4. PROPOSED METHODOLOGY

The proposed method aims to determine the optimal solution $\{x_{ij}\}$ ($i=1,2,\dots,m$; $j=1,2,\dots,n$) and the generalized intuitionistic trapezoidal fuzzy optimal value z of the transportation problem with m sources (S_i , $i=1,2,\dots,m$) and n destinations (D_j , $j=1,2,\dots,n$). The supply and demand parameters are represented as real numbers, while the transportation cost, c_{ij} ($i=1,2,\dots,m$; $j=1,2,\dots,n$), from the i^{th} source to the j^{th} destination is modelled as a generalized intuitionistic trapezoidal fuzzy number, as shown in Table 1.

Step 1: Obtaining Row Reduced Form Using Existing Ranking Method for Generalized ITFNs (Section 3)

Using the ranking of each cell in Table 1, select the minimum Generalized ITFN from each row of the Generalized Intuitionistic Trapezoidal Fuzzy Cost Matrix (Table 1) and subtract it from each corresponding Intuitionistic Trapezoidal Fuzzy Numbers in that row.

Table 1: Transportation problem

Destination Sources	D_1	D_2	...	D_n	Supply a_i
S_1	c_{11}	c_{12}	...	c_{1n}	a_1
S_2	c_{21}	c_{22}	...	c_{2n}	a_2
.
.
.
S_m	c_{m1}	c_{m2}	...	c_{mn}	a_m
Demand b_j	b_1	b_2	...	b_n	$\sum_i a_i = \sum_j b_j$

where, a_i : Quantity of sources of materials availability at Source (S_i , $i=1,2,\dots,m$)

b_j : Quantity of sources of material required at destination (D_j , $j=1,2,\dots,n$)

c_{ij} : Unit cost of transformation from sources S_i to destination D_j .

Step 2: Obtaining Column Reduced Form Using Existing Ranking Method for Generalized ITFNs (Section 3)

Based on Section3, identify the minimum Intuitionistic Fuzzy Number in each column of the Generalized Intuitionistic Fuzzy Cost Matrix derived in Step 1. Then, subtract this minimum value from all Intuitionistic Fuzzy Numbers within the same column.

Step 3 : Zero-Centering Generalized Intuitionistic Fuzzy Numbers

Check whether that each row and each column has at least one generalized intuitionistic fuzzy number whose rank is zero. If it is not so, then repeat Step 1 and Step 2. Otherwise, calculate the intuitionistic fuzzy zero centered value using the formula

$$Z_{ij} = \frac{\text{Total Generalized Intuitionistic Trapezoidal fuzzy cost for cells with zero rank}}{\text{Count of Generalized Intuitionistic Trapezoidal Fuzzy costs with non - zero ranks}}$$

Step 4: Cell Assignment

Select the cell (i,j) with the maximum rank of fuzzy zero-centered value Z_{ij} . Assign the maximum possible quantity to this cell. Then, delete either the i^{th} row or j^{th} column, whichever has its quantity fully assigned.

Step 5 : Iterative Assignment

Repeat steps 3 and 4 until all assignments are completed.

Step 6: Determining Optimum Solution and Generalized Intuitionistic Trapezoidal Fuzzy Optimal Value

The optimum solution $\{x_{ij}\}$ and the generalized intuitionistic trapezoidal fuzzy optimal value, represented as $\sum_{i=1}^m \sum_{j=1}^n c_{ij} \otimes x_{ij}$ are obtained from step 5.

5. NUMERICAL EXAMPLE

A Generalized ITFN transportation problem, originally presented by Agarwal et al.[27] (Table 2(a)) is addressed using the proposed method. This problem consists of three sources S_1, S_2 , and S_3 and three destinations D_1, D_2 , and D_3 . Ranks of each cell in Table 2(b) is obtained using ranking method [25]

Table 2 (a) : Generalized Intuitionistic Trapezoidal Fuzzy Transportation Problem

Destinations → Sources ↓	D_1	D_2	D_3	Supply s_i
S_1	(2,4,8,15;0.6) (1,4,8,18;0.3)	(3,5,7,12; 0.5) (1,5,7,15;0.3)	(2,5,9,16;0.7) (1,5,9,18;0.3)	25
S_2	(2,5,8,10;0.6) (1,5,8,12;0.2)	(4,8,10,13;0.4) (3,8,10,15,0.3)	(4,8,10,13;0.4) (3,8,10,15;0.3)	30
S_3	(2,7,11,15;0.5) (1,7,11,18;0.3)	(5,9,12,16;0.7) (3,9,12,19;0.2)	(4,6,8,10;0.6) (3,6,8,12;0.3)	40
Demand d_j	35	45	15	95

Table 2(b) : Ranks of each cell of table2 (a) using ranking method [25]

Destinations Sources	D_1	D_2	D_3	Supply s_i
S_1	2.6639	2.52845	3.0682	25
S_2	2.2374	3.5976	3.5976	30
S_3	3.5667	3.8476	2.814	40
Demand d_j	35	45	15	95

Step 1: Section 3 describes the existing ranking method for GITFNs used to obtain the row reduced form presented in Table 3.

Table 3 : Row Reduced Form by Step 1

Destinations Sources	D_1	D_2	D_3	Supply s_i
S_1	(-13,-4,4,13;0.6) (-17,-4,4,17;0.3)	(-12,-3,3,10;0.5) (-17,-3,3,14;0.3)	(-13,-3,5,14;0.6) (-17,-3,15,17;0.3)	25
S_2	(-8,-3,3,8;0.6) (-11,-3,3,11;0.2)	(-6,0,5,11;0.4) (-9,0,5,14;0.3)	(-6,0,5,11;0.4) (-9,0,5,14;0.3)	30
S_3	(-8,-1,5,11;0.5) (-11,-1,5,15;0.3)	(-5,1,6,12;0.6) (-9,1,6,16;0.3)	(-6,-2,2,6;0.6) (-9,-2,2,9;0.3)	40
Demand d_j	35	45	15	95

Step 2: Table 4 presents the column reduced form obtained using the existing ranking method for Generalized ITFNs, as described in Section 3.

Table 4 : Column Reduced Form by Step2

Destinations Sources	D_1	D_2	D_3	Supply s_i
S_1	(-13,-4,4,13;0.6) (-17,-4,4,17;0.3)	(-22,-6,9,22;0.5) (-31,-6,9,26;0.3)	(-13,-3,5,14;0.6) (-17,-3,15,17;0.3)	25
S_2	(-8,-3,3,8;0.6) (-11,-3,3,11;0.2)	(-16,-3,8,23;0.4) (-23,-3,8,31;0.3)	(-6,0,5,11;0.4) (-9,0,5,14;0.3)	30
S_3	(-8,-1,5,11;0.5) (-11,-1,5,15;0.3)	(-15,-2,9,24;0.5) (-23,-2,9,33;0.3)	(-6,-2,2,6;0.6) (-9,-2,2,9;0.3)	40
Demand d_j	35	45	15	95

Step 3 : The Zero-Centering Generalized Intuitionistic Trapezoidal Fuzzy Numbers are computed and summarized in Table 5.

Table 5 : Zero-Centering by Step 3 .

Destinations Sources	D_1	D_2	D_3	Supply s_i
S_1	$(-13,-4,4,13;0.6)$ $(-17,-4,4,17;0.3)$	$(-44,-15,15,44;0.5)$ $(-57,-15,15,57;0.3)$	$(-13,-3,5,14;0.6)$ $(-17,-3,15,17;0.3)$	25
S_2	$(-8,-3,3,8;0.6)$ $(-11,-3,3,11;0.2)$	$(-38,-12,14,45;0.4)$ $(-49,-12,14,62;0.3)$	$(-6,0,5,11;0.4)$ $(-9,0,5,14;0.3)$	30
S_3	$(-8,-1,5,11;0.5)$ $(-11,-1,5,15;0.3)$	$(-27,-8,12,34;0.5)$ $(-35,-8,12,47;0.3)$	$(-6,-2,2,6;0.6)$ $(-9,-2,2,9;0.3)$	40
Demand d_j	35	45	15	95

Step 4: Cell Assignment was calculated using proposed method and the results are presented in Table 6.

Table 6 : Cell Assignment Using Step 4

Destinations Sources	D_1	D_2	Supply s_i
S_1	$(-13,-4,4,13;0.6)$ $(-17,-4,4,17;0.3)$	$(-44,-15,15,44;0.5)$ $(-57,-15,15,57;0.3)$	25
S_2	$(-8,-3,3,8;0.6)$ $(-11,-3,3,11;0.2)$	$(-38,-12,14,45;0.4)$ $(-49,-12,14,62;0.3)$	30
S_3	$(-8,-1,5,11;0.5)$ $(-11,-1,5,15;0.3)$	$(-27,-8,12,34;0.5)$ $(-35,-8,12,47;0.3)$	40
Demand d_j	35	45	95

Step 5 : Table 7 presents the iterative assignment results obtained using the proposed method described in Section 4.

Table 7 : New reduced table

Destinations Sources	D_1	D_2	Supply s_i
S_1	(-13,-4,4,13;0.6) (-17,-4,4,17;0.3)	(-44,-15,15,44;0.5) (-57,-15,15,57;0.3)	25
S_2	(-8,-3,3,8;0.6) (-11,-3,3,11;0.2)	(-38,-12,14,45;0.4) (-49,-12,14,62;0.3)	30
S_3	(-19,-6,6,19;0.5) (-26,-6,6,26;0.3)	(-31,-13,13,42;0.5) (-50,-13,13,58;0.3)	40
Demand d_j	35	45	95

Step 6: The optimum solution and Generalized Intuitionistic Trapezoidal Fuzzy Optimal Value are presented in Table 8, which also illustrates the obtained allocation.

Table 8 : Optimal Solution by Step 6

Destinations Sources	D_1	D_2	D_3	Supply s_i
S_1	(2,4,8,15;0.6) (1,4,8,18;0.3)	(3,5,7,12; 0.5) (1,5,7,15;0.3) (25)	(2,5,9,16;0.7) (1,5,9,18;0.3)	25
S_2	(2,5,8,10;0.6) (1,5,8,12;0.2) (30)	(4,8,10,13;0.4) (3,8,10,15,0.3)	(4,8,10,13;0.4) (3,8,10,15;0.3)	30
S_3	(2,7,11,15;0.5) (1,7,11,18;0.3) (5)	(5,9,12,16;0.7) (3,9,12,19;0.2) (20)	(4,6,8,10;0.6) (3,6,8,12;0.3) (15)	40
Demand d_j	35	45	15	95

$$x_{12} = 25, \quad x_{21} = 30, \quad x_{31} = 5, \quad x_{32} = 20, \quad x_{33} = 15$$

$$\sum_{i=1}^m \sum_{j=1}^n c_{ij} \otimes x_{ij} = c_{12} \otimes x_{12} \oplus c_{21} \otimes x_{21} \oplus c_{31} \otimes x_{31} \oplus c_{32} \otimes x_{32} \oplus c_{33} \otimes x_{33}$$

$$= (3,5,7,12; 0.5)(1,5,7,15;0.3) \otimes 25 \oplus (2,5,8,10;0.6)(1,5,8,12;0.2) \otimes 30 \oplus (2,7,11,15;0.5)(1,7,11,18;0.3) \otimes 5 \oplus (5,9,12,16;0.7)(3,9,12,19;0.2) \otimes 20 \oplus (4,6,8,10;0.6)(3,6,8,12;0.3) \otimes 15$$

$$= (305,580,830,1145;0.5)(165,580,830,1385;0.3)$$

6. COMPARATIVE STUDY

The fuzzy optimal cost obtained using the proposed method and the method by Indira and Shankar [28] are identical. A comparative study is presented in Table 9.

Table 9: Comparative Table

Example	Ranking Procedure	Fuzzy Transportation Method	Fuzzy Optimal Cost
Agarwal et al.[27]	Pardha Saradhi et al. [28]	Indira and Shankar [26]	(305,580,830,1145;0.5) (165,580,830,1385;0.3)
Agarwal et al.[27]	Gani and Mohammed [25]	Proposed	(305,580,830,1145;0.5) (165,580,830,1385;0.3)

7. RESULTS AND DISCUSSION

The proposed optimization method using Generalized Intuitionistic Trapezoidal Fuzzy Numbers (GITFNs) was applied to transportation problem. A comparative study was conducted to evaluate transportation costs using the proposed method and existing methodology. The results showed identical outcomes for both methods. These findings demonstrate the effectiveness and validity of the proposed GITFN-based optimization method in solving transportation problems.

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