

Minimising the Cipher Length of Elliptic Curve Cryptography Basing on Chebyshev Polynomial

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ABSTRACT

Introduction: Public Key Cryptography (PKC) is one of the key components of contemporary cryptographic systems, mainly employed for securing digital communications using protocols such as SSL and TLS. PKC relies on trapdoor functions, which are computationally easy in one direction but impossible to invert without access to a secret key. RSA and the Diffie-Hellman key exchange protocol are classical algorithms that have been extensively employed for secure data transfer. More recently, Elliptic Curve Cryptography (ECC) has been in the limelight because of its high security with compact key sizes and fast computation, based on the algebraic structure of elliptic curves.

Objectives: This study will seek to alleviate one of the most significant weaknesses of ECC-based encryption—namely the expansion of ciphertext size in ElGamal encryption. The goal is to create a revised ECC-based encryption scheme that is highly secure while reducing ciphertext size. Furthermore, the proposed approach will seek to include digital signature functionality for improved data authentication and integrity.

Methods: The new encryption scheme is based on the ElGamal encryption scheme but with an addition involving Chebyshev polynomials of the first kind as a random value generator. This inclusion enables secure randomization during encryption. The new scheme guarantees that the ciphertext size is equal to the plaintext size, essentially compressing the output without loss of security. The scheme also includes digital signature functionality to ensure authenticity and non-repudiation.

Results: The revised ECC-based encryption method proves to be an enormous advancement compared to conventional ElGamal encryption in terms of ciphertext length. Without compromising the strong security attributes of ECC, the new method attains ciphertext compression, maintaining the encrypted message size equal to the plaintext size. Additionally, the scheme efficiently accommodates digital signature operations, providing improved message integrity and verification.

Conclusions: The improved encryption scheme overcomes a significant obstacle to ECC-based cryptography by decreasing ciphertext length at no cost in security. Combining Chebyshev polynomials and digital signature capability is a step towards better secure and efficient cryptographic protocols. The scheme has special application value for any scenarios involving light encryption with tight security assurances, e.g., secure messaging, cloud data encryption, and IoT transmission.

Keywords: Elliptic Curve Cryptography (ECC), Public Key Cryptography (PKC), Secure Communication, Chebyshev Polynomial, Digital Signature, Text-Based Encryption, Ciphertext Optimization, ElGamal Encryption

INTRODUCTION

The elliptic curve equation for cryptographic purposes is given by: $y^2 = x^3 + px + q$ over a finite field. The condition for the curve to be non-singular is: $4p^3 + 27q^2 \neq 0$. The plot of an elliptic curve is symmetric along the x-axis. The points on the finite curve, along with the infinity point, form an abelian group with respect to the binary

composition addition of points. The addition operation of points is defined as follows: For two points $P(a_1, b_1)$ and $Q(a_2, b_2)$ on the elliptic curve, the sum $P + Q$ is the image of the point obtained by the intersection of the line PQ with the curve. This resulting point is denoted as R .

$$R = (a_3, b_3)$$

$$a_3 = \left(\frac{b_2 - b_1}{a_2 - a_1} \right)^2 - a_1 - a_2$$

$$b_3 = \frac{b_2 - b_1}{a_2 - a_1} (a_1 - a_2) - b_1$$

❖ Point doubling or Point multiplication

For $p \neq -p$, $2p$ is the intersection of the curve with the line that goes through P gives the point.

$$2p = (a_3, b_3)$$

$$a_3 = \left(\frac{3a_1^2 + p}{b_1} \right)^2 - 2a_1$$

$$b_3 = \frac{3a_1^2 + p}{2b_1} (a_1 - a_3) - b_1$$

❖ Elliptic Curve Discrete Logarithmic Problem

ECC is strong against several active and passive attacks because the resistance of (ECDLP) Discrete Logarithmic Problem. Consider points P, Q of the curve and for a random number a it is difficult to calculate the value of a , taken P and Q such that $Q = aP$

The present algorithm uses chebyshev polynomial of first kind and simple bit rotation operation to compute some of the parameters used in the technique.

❖ Conventional ECC Technique Similar to El Gamal

In conventional encryption technique of ECC the entries of the message are mapped to points on the curve. Every point says P_m maps to two points (C_1, C_2) .

$$C_1 = K \times Q, C_2 = P_m + K \times PB$$

Where Q is generator, K is randomly big number, qB are large primes less than the order q of the curve $E_q(a, b)$

$$PB = qB \times Q$$

qB is also randomly big number and Q is generator of the group of elliptic curve points. Basing on conventional ECC scheme, other schemes were developed by researchers modifying the conventional ECC scheme.

❖ First modified scheme of ECC

In this modified scheme [17], points of the curve are public. English alphabets and the numbers 0 to 9 are mapped to 35 points. Sender and receiver agree to use a shared key (private) K . For each message character m , x is computed

$$x = m \times K + I, \text{ for any random integer } I.$$

If the computed x value matches to any point (x, y) of the curve then the message point m is assigned to that point. If no point is there I is incremented by 1 at each stage and repeat the same computing process until the point (x, y) with computed x value occurs. This encryption procedure is extended to the entire message characters. Decryption is

$$m = (x - I)/K.$$

❖ Second modified Scheme of ECC

Improved Encryption Scheme: This method is a modification of Scheme 1, in which the message is broken into blocks of size M . These blocks are then translated into ASCII binary values and expressed as a string of $M \times 8$ bits. The decimal value of this number is taken as x . The corresponding y value is calculated from the following elliptic curve equation:

$$Y^2 = x^3 + ax + b$$

When the above-calculated (x, y) pair is within the given range, it is taken as the cipher for the character of the message m . Else, x is incremented, and the loop repeats. The recipient is only interested in the x values when decrypting. However, these modified methods pose a significant computational challenge. The process of calculating y has to be run multiple times, resulting in excessive time complexity. Furthermore, cipher block collisions are a serious concern. In an attempt to reduce this, some proposals suggest increasing the number of iterations, which is impractical for real-world applications.

ELLIPTIC CURVE CRYPTOGRAPHY LITERATURE REVIEW

Some researchers have investigated text encryption based on elliptic curve cryptography (ECC), as noted below:

P. L. Sharma et al. [1] suggested a text encryption technique based on elliptic curves for increased security. The authors introduced a tailored permuted ASCII table for using elliptic curve coordinates as keys, which were utilized in a random number generator. Younes Lahraoui et al. [2] designed a message mapping hash algorithm with ECC. Their model provides message integrity by adding a tag to the ciphertext through hash functions and elliptic curve points. Shamsheer Ullah et al. [3] discussed a comprehensive overview of ECC uses in data transmission. Their work explains the main advantages of ECC in distributed computing. Nadine Nibigira et al. [4] proposed a secure method for protecting sensitive data through elliptic curves. Their work emphasizes securing sensitive data by means of tools such as PyCryptodome and PyQt5. Yohan Yan [5] elaborated on ECC, comparing it with other public-key cryptosystems. He concluded that ECC plays a crucial role in the creation of new cryptography. David et al. [6] developed an ECC-based text encryption algorithm with several rounds of encryption and a 256-bit key size. L. D. Singh & K. M. Singh [7] proposed an encryption scheme in which ASCII values of characters are matched rather than the conventional affine point mapping. S. M. Celestin & K. Muneeswaran [8] proposed an ECC-based encryption technique that maps ASCII values into affine points and adds them to the generator point. Balamurugan et al. [9] introduced a rapid mapping method, encrypting message characters via ElGamal encryption with one more non-singular matrix parameter. Keerthi K & B. Surendiran [10] proposed an ECC encryption scheme wherein ASCII hexadecimal message values are coupled to create (x, y) coordinates and encrypted in reversed order to avert security attacks. Obaidur Rahaman [11] studied the function of ECC in contemporary data and information security in lieu of classical character-to-affine plane mapping algorithms. Omar Reyad [12] presented an encoding method of text messages utilizing computational operations on affine points over the elliptic curve. Sravana Kumar D et al. [13] designed an encryption algorithm utilizing ECC combined with public-key cryptosystems. G. J. Fee et al. [14] employed Chebyshev polynomials $T_n(x)$ to substitute monomials x^n in Diffie-Hellman and RSA schemes, incorporating more security parameters. Kai-Yuen Cheong [15] compared chaotic encryption systems with one-way functions through the use of Chebyshev polynomials and gave insight into chaos-based cryptosystems. K. Prasad et al. [16] proposed a public-key encryption system on Chebyshev polynomials. This review underlines progress in ECC-based encryption methods, highlighting their potential in reinforcing security while overcoming computational hurdles.

MATHEMATICAL MODEL EMPLOYED IN THE ALGORITHM

During this setup phase, the elliptic curve, generator, and uniform lookup table are known to everyone. A special user in the communication group picks a suitable curve, initializes a generator, and encodes ASCII characters into particular points on the curve via the uniform lookup table-1. He or she publishes these items for access to all the members. As mentioned above, the Chebyshev polynomial is a tool for a random number generator used in finding a crucial value involved in the mechanism. For this, the genuine user agrees to use a random function $f(x, y)$. The value is different for encrypting different message characters. Each plaintext character is encrypted to two points of the curve. As given in the abstract, this paper not only addresses security issues but also focuses on bandwidth. For this, the integer multiples of the generator for all cipher points are consecutively arranged as a big binary stream.

Then, the complete binary string is right-rotated L times, which acts as the ciphertext. The number of right rotations is also confidential between the users. At the other end, during the decryption stage, the big binary string is left-rotated L times to proceed with decryption. The entire encryption and decryption procedure is explained in detail below.

METHODOLOGY

The curve is compatible with encryption and possesses all required features Equation Eq(a,b) has a large order, and for a prime q , the generator Q and the points $2Q, 3Q, \dots$ are public. Either user can construct a uniform lookup table by assigning ASCII characters to the curve points, which is also public. This phase, known as the setup phase, is conducted by either one of the users or by a trusted third party. In this encryption scheme, ' γ ' is a crucial integer obtained through a consented function $f(x,y)$ agreed upon by the users.

❖ Calculating ' γ ' Value

- ❖ User 1 selects a random point
- ❖ $A_1 = \mu * Q$ for $\mu < \mathcal{E}$, sends to user 2 via public channel.

User 2 selects a random point

- ❖ $A_2 = \epsilon * Q$ for some integer $\epsilon < \mathcal{E}$ and communicates to user1

User 1 computes

- ❖ $\mu * A_2 + B = CB$ sends to user1 with random point B selected by him

User 2 retrieves B as

- ❖ $B = CB - \beta A_1$

Now both the users are possessing B . Then they compute the γ value as

- ❖ $\gamma = f[xB, yB]$

Where xB, yB are B point coordinates $f[x, y]$ is consented function.

If 2 authorized users Raechel and Samuel wish to bring the messages

- ❖ Raechel selects a point R on the curve, calculates the point
- ❖ $R_1 = [\alpha (Q+R)] \bmod q$ and $R_2 = [\alpha R] \bmod q$ and declares them as her overall public keys. Likewise,
- ❖ Samuel choose a point S on the curve, calculates
- ❖ $S_1 = [\beta (Q+S)] \bmod q$ and $S_2 = [\beta S] \bmod q$ and declares as his overall public keys.
- ❖ Again Raechel computes $RS = [\alpha S_2] \bmod q$,
- ❖ Samuel computes $SR = [\beta R_2] \bmod q$ and publish as the particular public keys to each one.

❖ Encryption

Let us assume **Samuel** wants to convey **M** as a message in the form of $m_1, m_2, m_3, \dots, m_n$.

He first converts it into affine points $P_1, P_2, P_3, \dots, P_n$ on the curve with the help of a standard lookup table.

Then each message point is encrypted into a corresponding pair of cipher points E_1, E_2 .

The first point P_1 is encrypted as

- ❖ $E_{11} = [\gamma_1 Q] \bmod 256, E_{12} = [P_1 + (\beta + \gamma_1) R_1 - \gamma_1 R_2 + RS] \bmod 256$

The second point P_2 is encrypted as

- ❖ $E_{21} = [\gamma_2 Q] \bmod 256, E_{22} = [P_2 + (\beta + \gamma_2) R_1 - \gamma_2 R_2 + RS] \bmod 256$

In general,

- ❖ $E_{k1} = [\gamma_k Q] \bmod 256, E_{k2} = [P_k + (\beta + \gamma_k) R_1 - \gamma_k R_2 + RS] \bmod 256, k = 1 \text{ to } n.$

γ_k is calculated from the Chebyshev polynomial as

- ❖ $\gamma_1 = T_1(\gamma)$
- ❖ $\gamma_2 = T_2(\gamma) = 2 \gamma T_1(\gamma) - T_0(\gamma) = 2 \gamma_2 - 1$
- ❖ $\gamma_k = T(k+1)(\gamma) = 2 \gamma T_k(\gamma) - T(k-1)(\gamma)$, $k = 1$ to n

The points $E_{11}, E_{12}; E_{21}, E_{22}; E_{k1}, E_{k2}$ are integer multiples of the generator point Q say

$I_1, I_{12}; I_{21}, I_{22}; \dots, I_{k1}, I_{k2}$.

For instance,

- ❖ The point $E_{11} = 15Q$ then $I_{11}=15$;
- ❖ $E_{12} = 24Q$ then $I_{12}=24$.

The integers $I_{11}, I_{12}; I_{21}, I_{22}; I_{k1}, I_{k2}$ are converted to 8 bit ASCII binaries, arranged as a lengthy binary string of length $8 \times 2k$. Then the bits of the binary string B are right rotated the number of times say L ($L < 2k \times 8$) to get a new binary stream B_1

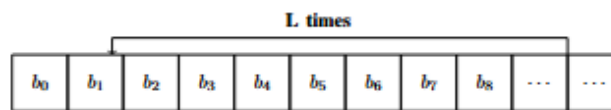


Fig. 1. Encryption Bit sequence representation repeated L times

This big binary stream B_1 is converted to decimal equivalent to obtain a lengthy decimal number which acts as the cipher text C . C is sent to the receiver through public channel

❖ Decryption

The receiver after receiving C first converts to a big binary number B_1 . The bits of B_1 are left rotated L times to get B

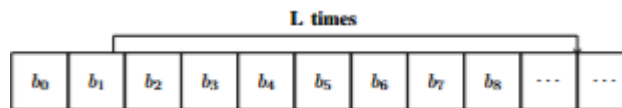


Fig. 2. Decryption Bit sequence representation repeated L times

Then the binary number is split into blocks of 8 bits each from the left and converted to decimal equivalents:

$I_{11}, I_{12}; I_{21}, I_{22}; \dots, I_{k1}, I_{k2}$.

The affine points:

$E_{11}, E_{12}; E_{21}, E_{22}; \dots, E_{k1}, E_{k2}$, $k = 1$ to n , are decoded to the message coded affine points P_1, P_2, \dots as

- ❖ $P_1 = E_{12} - (\alpha E_{11} + \alpha S_1 + SR)$
- ❖ $P_2 = E_{22} - (\alpha E_{21} + \alpha S_1 + SR)$
- ❖ $P_k = E_{k2} - (\alpha E_{k1} + \alpha S_1 + SR)$ $k = 1$ to n .

1. Decryption works out properly:

- ❖ $P_k = E_{k2} - (\alpha E_{k1} + \alpha S_1 + SR)$
- ❖ $= [P_k + (\beta + \gamma_k) R_1 - \gamma_k R_2 + RS] - (\alpha E_{k1} + \alpha S_1 + SR)$
- ❖ $= [P_k + \{(\beta + \gamma_k) \alpha(Q+R)\} - \gamma_k \alpha R + \alpha \beta S] - [\alpha \gamma_k Q + \alpha \beta(Q+S) + \beta \alpha R]$
- ❖ $= P_k + \alpha \beta Q + \alpha \beta R + \alpha \gamma_k Q + \alpha \gamma_k R - \alpha \gamma_k R + \alpha \beta S - \alpha \gamma_k Q - \alpha \beta Q - \alpha \beta R = P_k$

Corresponding text characters of the points from the ordinary look up table yields the primary message.

Block diagram of the proposed algorithm:

2. Algorithm for computing 'γ' value: The table-1 depicts an algorithm to calculate a certain value based on elliptic curve cryptography concepts. It illustrates procedures for two users, User1 and User2, to generate and

share cryptographic keys. Steps include random number selection, calculation of points on a curve, modular arithmetic operation, and public and private key generation.

Table I Algorithm For Computing γ Value

User1	User2
Step1: Begin Step2: i: Find $A_1 = \mu * Q$ ii: Consider random point B iii: $CB = (\mu * A_2 + B) \bmod q$ Step3: Find $\gamma = f(x_B, y_B)$	Step1: Begin Step2: i: $A_2 = \epsilon * Q$ ii: $B = (CB - \epsilon * A_1) \bmod q$ Step3: After obtaining B find $\gamma = f(x_B, y_B)$
$\gamma_k = T(k+1)(\gamma) = 2\gamma T_k(\gamma) - T(k-1)(\gamma) \quad k = 1 \text{ to } n \text{ and } \gamma_1 = \gamma$	
Selects α as a random number, a point R which is on the curve Calculates $R_1 = [\alpha * (Q + R)] \bmod q$, $R_2 = [\alpha * R] \bmod q$ and publishes as general public keys Again computes $RS = [\alpha * S_2] \bmod q$ and publishes as particular public key to user2	Selects β as a random number, a point S which is on the curve Calculates $S_1 = [\beta * (Q + S)] \bmod q$, $S_2 = [\beta * S] \bmod q$ and publishes as general public keys Again calculates $SR = [\beta * R_2] \bmod q$ and publishes as particular public key to user1z

3. Encryption Algorithm: The encryption algorithm uses Chebyshev polynomials and elliptic curve cryptography to convert text into encrypted decimal ciphertext

Input: Plain text, Chebyshev polynomial

Output: Cipher text in form of decimal number

Step 1: Begin

Step 2:

- ❖ (i). Convert the message characters to points using common look up table
- ❖ (ii). Encrypt each point P_k to pair of points E_{k1}, E_{k2}
- ❖ $E_{k1} = [\gamma_k Q] \bmod 256$, $E_{k2} = [P_k + (\beta + \gamma_k) R_1 - \gamma_k R_2 + RS] \bmod 256$,
- ❖ $k = 1 \text{ to } n$

Step 3:

- ❖ (i). The integer values of $E_{k1} = I_{k1} * Q$, $E_{k2} = I_{k2} * Q$ where $k = 1 \text{ to } n$ are converted to 8 bit binary numbers and written side by side to get the big binary number B
- ❖ (ii). Bits of B are right rotated L times to get B1

Step 4: Decimal equivalent of B1 is the cipher text C

Upon receiving C, Raechel converts it back into binary, left rotates it 10 times to recover B, and then splits B into blocks of 8 bits to obtain the integers I_{11} , I_{12} ; I_{21} , I_{22} ; After determining the affine points E_{11} , E_{12} ; E_{21} , E_{22} ; ..., she decrypts them into P_1 , P_2 , P_3 , P_4 , P_5 . Finally, using a common lookup table, these points are converted back into the original text: "SSSSS".

PERFORMANCE ANALYSIS

The encryption time for different sizes of data is calculated using Intel core I5 10th generator computer and MATLAB 2020, the time of encryption and decryption are in table 1 fig.3 as shown

TABLE II EXECUTION TIME TABLE

S.No	Data Size (kb)	Encryption Time (ms)	Decryption Time (ms)
1	1kb	0.71	0.63
2	2kb	0.89	0.79
3	3 kb	0.99	0.82
4	4 kb	1.02	1.99
5	5 kb	1.13	1.02
6	6 kb	1.21	1.15
7	7 kb	1.34	1.29
8	8 kb	1.45	1.34
9	9 kb	1.56	1.62
10	10 kb	1.72	1.85

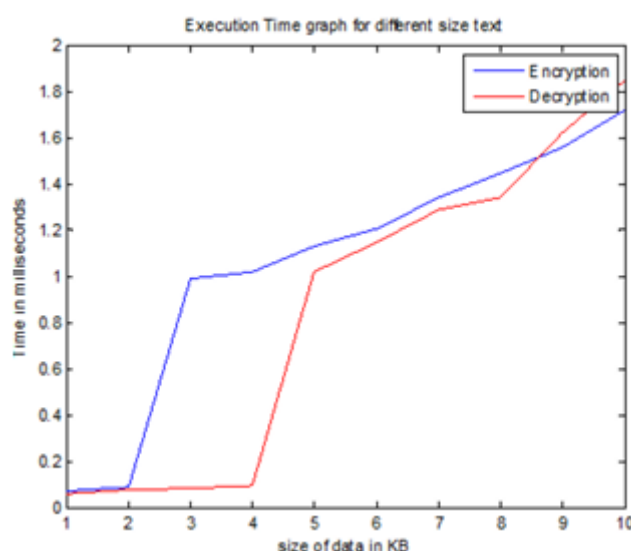


Fig. 3. The repetitions of the numbers

SECURITY ANALYSIS

Strength of the cryptographic primitive depends on different security attributes. In the present algorithm each character is encrypted using sender's secret keys, receiver's public keys and γ . γ Value is different for each character which is calculated from the shared secret point, agreed upon permutation function (x,y) and Chebyshev polynomial of I kind. The data encryption as well as cipher reduction technique presented in this paper relays on a special γ value and binary bit rotation operation. Though the elliptic curve points, generators are public, the present cipher attains extreme security like a secret key mechanism. This is because the γ value is different for different character encryption the value of γ is obtained by calling a random function. Because γ is changing with each encryption step, even duplicate text has different cipher characters assigned to it. The cipher produced is a

stream of decimal digits, thus the resulting proposed encryption protocol is very robust against both passive and active attacks. Linear and differential cryptanalysis is difficult to implement on this cipher. These classical plaintext and ciphertext attacks assume a one-to-one correspondence between the cipher and the message, but in this scheme, there is no direct relationship between the length and type of the message and its cipher. The encrypted output is composed solely of integers, independent of the character make-up of the original message. The frequencies of decimal digits from 0 to 9 are estimated to the message with 1 kb size. The repetition of the numbers are shown as the following histogram fig. 4

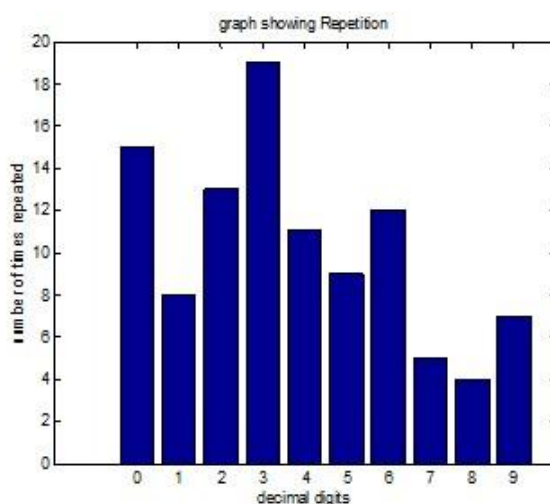


Fig. 4. The repetitions of the numbers

CONCLUSION

The encryption and cipher reduction method suggested in this paper provides confidentiality, authentication, and integrity. Most attacks on ECC, like Pollard-Rho and Pollig-Hellman, aim at the Elliptic Curve Discrete Logarithm Problem (ECDLP). Pollard-Rho, which is an exhaustive search algorithm, is a serious threat. But in the suggested algorithm, the parameter γ changes with every character encryption, so even the same characters are encrypted differently. This adaptive change of γ increases resistance against Pollard-Rho attacks. Furthermore, both active and passive attacks, e.g., ciphertext and plaintext attacks are difficult to perform because the cipher is in the form of a sequence of decimal digits. The algorithm not only enhances security but also solves the issue of cipher bandwidth. One message point was traditionally mapped to two points on the elliptic curve in a cipher. The suggested method, though, uses a binary bit rotation mechanism, which changes the structure of the cipher and minimizes communication hazards. More advancements aim at optimizing the cipher reduction method to correspond each message point with a single cipher point. This advancement optimizes efficiency without sacrificing security. In comparison with standard ECC implementations, the new method presents better safeguarding against cryptanalysis while maximizing encryption efficiency. Therefore, the adapted encryption method offers a secure and efficient alternative to standard ECC-based cryptography schemes.

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