

# Optimizing Intuitionistic Trapezoidal Fuzzy Transportation Systems: An Earth Mover's Distance-Based Methodology

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## ARTICLE INFO

Received: 24 Dec 2024

Revised: 12 Feb 2025

Accepted: 26 Feb 2025

## ABSTRACT

This paper proposes a novel technique for ranking Intuitionistic Trapezoidal Fuzzy Numbers (ITFNs) using the Earth Mover's Distance (EMD) for transportation optimization. Traditional methods for solving transportation problems often struggle to address the inherent uncertainty and vagueness present in real-world scenarios. By incorporating ITFNs, which capture both non-membership and membership degrees, we can better model such uncertainties. Our methodology employs the EMD to quantify the dissimilarity between ITFNs, providing a robust mechanism for ranking and comparison. This ranking is then used to prioritize and optimize transportation routes and allocations. The proposed approach enables more accurate and reliable decision-making, ultimately enhancing the overall efficiency of transportation systems.

**Keywords:** Intuitionistic Trapezoidal Fuzzy Numbers; Earth Mover's Distance; Transportation Problems; Ranking Function; Uncertainty

## INTRODUCTION

The transportation problem [1], a fundamental challenge in operations research and management science, involves optimizing resource distribution while minimizing costs. However, traditional approaches to solving transportation problems often rely on crisp data and neglect the inherent uncertainty and ambiguity present in real-world scenarios. Fuzzy set theory [2–10], presenting a mathematical framework for modeling and addressing imprecision and uncertainty in decision-making processes, has been used by researchers to overcome this constraint. In 1965, Zadeh introduced fuzzy sets [13] and Bellman and Zadeh [14] provided methods to quantitatively handle imprecise information in decision-making. Researchers investigating intuitionistic, fuzzy, and interval-valued intuitionistic fuzzy optimization approaches have since built upon the intuitionistic fuzzy optimization techniques that Angelov [15] pioneered.

In recent years, Intuitionistic Trapezoidal Fuzzy Numbers (ITFNs) have emerged as a powerful tool for capturing the complexities of uncertain data. By integrating non-membership and membership degrees, ITFNs offers greater complexity and thorough depiction of uncertainty [11]. Nevertheless, the challenge of ranking and comparing ITFNs remains, as conventional methods often fail to adequately address the subtleties inherent in intuitionistic fuzzy sets.

This paper introduces an innovative approach to this problem by using EMD as a ranking mechanism for ITFNs in the context of transportation problems. EMD, a metric originally developed in computer vision, measures the dissimilarity between two probability distributions, providing a robust mechanism for quantifying the differences between ITFNs [12]. By combining ITFNs with EMD, we provide a robust framework for solving transportation problems characterized by uncertainty, enabling more informed and dependable decision-making.

The organisation of the study is: Section 2 provides the preliminaries of fuzzy notions, such as arithmetic operations and ITFN. Section 3 propose the mathematical formulation of transportation model using Linear Programming Problem (LPP). Calculations of EMD for GITFNs and the modified EMD for GITFNs in transportation problems are established in section 4. In Section 5, proposed solution to the transportation problem using modified EMD for ranking GITFNs is presented. A numerical example utilizing EMD to apply this concept io a transportation problem integrating with ITFN is presented in Section 6. Section 7 presents a comparative study. The paper concludes with results and discussion in Section 8.

## PRELIMINARIES

Several basic descriptions, as stated in reference [16], are included in this section.

### Intuitionistic Fuzzy Set

In the universal set  $X$ , the intuitionistic fuzzy set  $\tilde{A}$  is defines as  $\tilde{A} = \{(x, f_{\tilde{A}}(x), g_{\tilde{A}}(x)) : x \in X\}$  where the degree of membership and non-membership is represented as the function  $f_{\tilde{A}} : X \rightarrow [0, 1]$  and  $g_{\tilde{A}} : X \rightarrow [0, 1]$  respectively and  $0 \leq f_{\tilde{A}}(x) + g_{\tilde{A}}(x) \leq 1$ , for every  $x \in X$ .

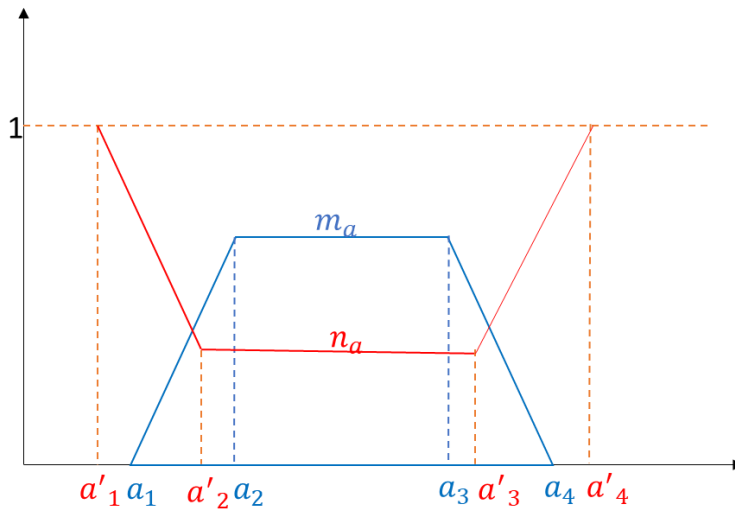
### Generalized Trapezoidal Intuitionistic Fuzzy Number (GTIFN)

If  $R$  is a set of real numbers, the fuzzy set on it is represented by the GTIFN  $\tilde{A} = \{(a_1, a_2, a_3, a_4, a'_1, ia'_2, iia'_3, ia'_4); m_a, n_a\}$  whose non-membership and membership function are given as (Figure 1).

$$f_{\tilde{A}} = \begin{cases} 0, & x < a_1 \\ \frac{x-a_1}{a_2-a_1} m_a, & a_1 \leq x \leq a_2 \\ m_a, & ai_2 \leq xi \leq ai_3 \\ \frac{a_4-x}{a_4-a_3} m_a, & a_3 \leq x \leq a_4 \\ 0, & a_4 < x \end{cases}$$

$$g_{\tilde{A}} = \begin{cases} 1, & x < ai'_1 \\ \frac{(a'_2-x)+n_a(x-a'_1)}{(a'_2-a'_1)}, & ai'_1 \leq xi \leq ai'_2 \\ n_a, & a'_2 \leq x \leq a'_3 \\ \frac{(xi-a'_3)+n_a(a'_4-x)}{a'_4-a'_3}, & a'_3 \leq xi \leq ai'_4 \\ 1, & ai'_4 < xi \end{cases}$$

The constraints  $0 \leq ni_a \leq 1$ ,  $0 \leq mi_a \leq 1$ , and  $0 \leq mi_a + ni_a \leq 1$ , are satisfied by the numbers  $m_a$  and  $n_a$ , which stand for the minimum degree of non-membership and maximum degree of membership function. The non-confidence and confidence levels of the elements  $x$  in  $\tilde{A}$  are reflected in the parameters  $m_a$  and  $n_a$ .



**Figure 1:** GTIFN of  $\tilde{A} = \{(a_1, ai_2, ai_3, ai_4, ai'_1, ai'_2, ai'_3, ai'_4); m_a, n_a\}$

### Arithmetic operations:

Let  $\tilde{A} = \{(a'_1, a'_2, a'_3, a'_4, a_1, a_2, a_3, a_4); n_a, m_a\}$

and  $\tilde{B} = \{(b'_1, b'_2, b'_3, b'_4, b_1, b_2, b_3, b_4); n_b, m_b\}$  be two GTIFN's.

Then the arithmetic operations are

(i) Addition:

$$\tilde{A} \oplus \tilde{B} = \{(a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a'_1 + b'_1, a'_2 + b'_2, a'_3 + b'_3, a'_4 + b'_4); (m_a, m_b), \max(n_a, n_b)\}$$

(ii) Subtraction:

$$\tilde{A} \tilde{-} \tilde{B} = \{(ai_1 - bi_4, ai_2 - bi_3, ai_3 - bi_2, ai_4 - bi_1, ai'_1 - bi'_4, ai'_2 - bi'_3, ai'_3 - bi'_2, ai'_4 - bi'_1); (m_a, m_b), \max(n_a, n_b)\}$$

(iii) Scalar Multiplication:

$$k \tilde{A} = \{ \{ (k a_{i4}, k a_{i3}, k a_{i2}, k a_{i1}, i k a_{i'4}, i k a_{i'3}, i k a_{i'2}, i k a_{i'1}); m_a, n_a \} k < 0 \{ (k a_{i1}, k a_{i2}, k a_{i3}, k a_{i4}, i k a_{i'1}, i k a_{i'2}, i k a_{i'3}, i k a_{i'4}); m_a, n_a \} k \geq 0 \}$$

### MATHEMATICAL FORMULATION OF TRANSPORTATION MODEL USING LINEAR PROGRAMMING (LPP)

Mathematically, the fuzzy transportation problem in Table 1 depicted as:

$$\text{Minimize } z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij},$$

$$\text{subject to } \sum_{j=1}^n x_{ij} \leq a_i, i = 1, 2, \dots, m$$

$$\sum_{i=1}^m x_{ij} \geq b_j, \quad j = 1, 2, \dots, n$$

and the non-negative restrictions,  $x_{ij}$  ( $i=1,2,\dots,m$ ;  $j=1,2,\dots,n$ ) are vectors.

**Table 1:** Transportation problem

Destination Sources	D <sub>1</sub>	D <sub>2</sub>	...	D <sub>n</sub>	Supply $a_i$
S <sub>1</sub>	$c_{11}$	$c_{12}$	...	$c_{1n}$	$a_1$
S <sub>2</sub>	$c_{21}$	$c_{22}$	...	$c_{2n}$	$a_2$
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
S <sub>m</sub>	$c_{m1}$	$c_{m2}$	...	$c_{mn}$	$a_m$
Demand $b_j$	$b_1$	$b_2$	...	$b_n$	$\sum_i a_i = \sum_j b_j$

where,  $a_i$  : Quantity of sources of materials availability at Source (S<sub>i</sub>,  $i=1,2,\dots,m$ )

$b_j$  : Quantity of sources of material required at destination (D<sub>j</sub>,  $j=1,2,\dots,n$ )

$c_{ij}$  : Transformation cost per unit from source S<sub>i</sub> to destination D<sub>j</sub>.

### EARTH MOVER'S DISTANCE (EMD)

The computation of EMD between two generalized intuitionistic trapezoidal fuzzy numbers and its use in a transportation problem are described in this section.

#### 4.1 Earth Mover's Distance between Two GITFN [12]

EMD between two GITFN's  $\tilde{A}$  and  $\tilde{B}$  can be calculated as follows:

1. Compute non-membership and membership function for both  $\tilde{A}$  and  $\tilde{B}$ .
2. Compute cumulative distribution functions for both  $f_{\tilde{A}}$  and  $g_{\tilde{A}}$  and  $f_{\tilde{B}}$  and  $g_{\tilde{B}}$

$$\text{i.e., } iF_{f_{\tilde{A}}}(xi) \text{ and } iF_{g_{\tilde{A}}}(xi)$$

$$iF_{f_{\tilde{B}}}(xi) \text{ and } iF_{g_{\tilde{B}}}(xi)$$

3. Compute EMD for both  $\tilde{A}$  and  $\tilde{B}$

$$EMD_g(\tilde{A}, \tilde{B}) = \int_{-\infty}^{\infty} |iF_{g_{\tilde{A}}}(x) - iF_{g_{\tilde{B}}}(x)| dx$$

$$EMD_f(\tilde{A}, \tilde{B}) = \int_{-\infty}^{\infty} |iF_{f_{\tilde{A}}}(x) - iF_{f_{\tilde{B}}}(x)| dx$$

$$4. \text{ Total EMD is calculated as } \text{EMD}(\tilde{A}, \tilde{B}) = \frac{\text{EMD}_f(\tilde{A}, \tilde{B}) + \text{EMD}_g(\tilde{A}, \tilde{B})}{2}$$

In the context of transportation problem, EMD can be employed to measure the dissimilarity between supply and demand distributions, ensuring minimum transportation. EMD is particularly useful when dealing with fuzzy numbers, as it can effectively handle the inherent uncertainty and variability.

#### 4.2 Modified Earth Mover's Distance (EMD) Calculation for GITFNs in Transportation Problems

Consider GITFN represented by  $\tilde{A} = (a_{i1}, a_{i2}, a_{i3}, a_{i4}; m_a)(a'_{i1}, a_{i2}, a_{i3}, a'_{i4}; n_a)$  with non-membership function  $g_A$  and membership function  $f_A$  (Definition 2.5). Compute the cumulative distribution functions for both non-membership function  $F_{g_{\tilde{A}}}$  and membership function  $F_{f_{\tilde{A}}}$  and calculate for each range using the formula  $\int_{-\infty}^{\infty} |F_{f_{\tilde{A}}}(x) - F_{g_{\tilde{A}}}(x)| dx$  and then calculate the EMD using the formula  $\text{EMD}(\tilde{A}) = \int_{-\infty}^{\infty} |F_{f_{\tilde{A}}}(x) - F_{g_{\tilde{A}}}(x)| dx \cdot n_a \cdot \frac{a_1 + a_2 + a_3 + a_4}{4}$  for each cell in the transportation problem.

#### Comparison of EMD for Each Cell in the Transportation Problem

To compare the EMD values of two Generalized Intuitionistic Trapezoidal Fuzzy Numbers (GITFNs), GITFN1 and GITFN2, compute their respective EMD values.

The comparison can be made as follows:

- (i) If  $\text{EMD}(\text{GITFN1}) - \text{EMD}(\text{GITFN2}) > 0$ , then GITFN1 is considered greater than GITFN2.
- (ii) If  $\text{EMD}(\text{GITFN1}) - \text{EMD}(\text{GITFN2}) < 0$ , then GITFN1 is considered less than GITFN2.
- (iii) If  $\text{EMD}(\text{GITFN1}) - \text{EMD}(\text{GITFN2}) = 0$ , then GITFN1 and GITFN2 are considered equal.

In the context of a transportation problem, the EMD values can be used to rank different transportation options or routes. A lower EMD value indicates a more reliable or efficient transportation option.

### PROPOSED SOLUTION TO THE TRANSPORTATION PROBLEM USING EARTH MOVER'S DISTANCE FOR RANKING GENERALIZED INTUITIONISTIC TRAPEZOIDAL FUZZY NUMBERS (GITFN)

The efficient fuzzy transportation solution of generalized intuitionistic transportation problem is achieved using the suggested approach. The proposed method's steps are as follows:

#### Step 1: Formulate the Generalized Intuitionistic Transportation Problem

Assume a generalized intuitionistic transportation problem with  $n$  destinations and  $m$  sources, where each source  $i$  has an availability (supply) of  $a_i$  ( $i = 1, 2, \dots, m$ ) and each destination  $j$  has a demand of  $b_j$  ( $j = 1, 2, \dots, n$ ). If the transportation problem is balanced ( $\sum_i a_i = \sum_j b_j$ ), proceed to Step 2. Alternatively, to generate a balanced transportation problem, add dummy rows or columns with zero intuitionistic fuzzy costs.

#### Step 2: Convert to Linear Programming Problem (LPP)

Convert the intuitionistic transportation problem to LPP:

$$\text{Minimize } z_i = \sum_{i=1}^m \sum_{j=1}^n \tilde{c}_{ij} \otimes x_{ij},$$

$$\text{subject to } \sum_{j=1}^n x_{ij} \leq a_i, i = 1, 2, \dots, m$$

$$\sum_{i=1}^m x_{ij} \geq b_j, \quad j = 1, 2, \dots, n$$

and the non-negative restrictions,  $x_{ij}$  ( $i = 1, 2, \dots, m; j = 1, 2, \dots, n$ ) are vectors.

Where  $\tilde{c}_{ij}$  ( $i = 1, 2, \dots, m; j = 1, 2, \dots, n$ ) represents fuzzy transportation costs expressed as generalized intuitionistic trapezoidal fuzzy numbers.

### Step 3: Apply EMD as a Ranking Function

Using the Earth Mover's Distance (EMD) in section 4, transform the LPP from Step 2 into a new transportation problem applying the ranking function EMD:

$$\text{Minimize } z = \sum_{i=1}^m \sum_{j=1}^n \text{EMD}(\tilde{c}_{ij})x_{ij},$$

$$\text{subject to } \sum_{j=1}^n x_{ij} \leq a_i, i = 1, 2, \dots, m$$

$$\sum_{i=1}^m x_{ij} \geq b_j, j = 1, 2, \dots, n$$

and the non-negative restrictions,  $x_{ij} (i = 1, 2, \dots, m; j = 1, 2, \dots, n) \geq 0$

For each ITFN in the cell of a transportation problem, we need to identify the intervals where the membership function changes. We can take the midpoints of each range and the associated membership values. For each discrete point (x), the cumulative distribution function (CDF) is the sum of all previous membership values up to that point.

### Step 4: Convert to Crisp Numbers

The generalized intuitionistic fuzzy numbers can be transformed into crisp numbers by using the EMD.

### Step 5: Solve LPP

Employ the linear programming techniques to solve the LPP from Step 4 to obtain the values  $x_{ij}$  ( $i = 1, 2, \dots, m$ ;  $j = 1, 2, \dots, n$ ). Non-zero values  $x_{ij}$  represents allocations, while zero values indicate no allocation. Also, obtain the minimum total cost.

### Step 6: Obtain Minimum Cost in Generalized Intuitionistic Fuzzy Form

Using the allocations obtained in Step 5, evaluate the minimum cost using arithmetic operations in terms of generalized intuitionistic fuzzy numbers.

## NUMERICAL EXAMPLE

The suggested approach is used to solve a generalized ITFN transportation problem that was first introduced by Agarwal et al. [19] (Table 2).  $D_1, D_2$ , and  $D_3$  are the three destinations and  $S_1, S_2$ , and  $S_3$  are the three sources in this problem.

### Step 1: Formulate “the Intuitionistic Transportation Problem (Table 2)”

**Table 2 : GITFN – Transportation Problem**

Supply Demand	$D_1$	$D_2$	$D_3$	Supply $s_i$
$S_1$	(2,4,8,15;0.6) (1,4,8,18;0.3)	(3,5,7,12; 0.5) (1,5,7,15;0.3)	(2,5,9,16;0.7) (1,5,9,18;0.3)	25
$S_2$	(2,5,8,10;0.6) (1,5,8,12;0.2)	(4,8,10,13;0.4) (3,8,10,15,0.3)	(4,8,10,13;0.4) (3,8,10,15;0.3)	30
$S_3$	(2,7,11,15;0.5) (1,7,11,18;0.3)	(5,9,12,16;0.7) (3,9,12,19;0.2)	(4,6,8,10;0.6) (3,6,8,12;0.3)	40
Demand $d_j$	35	45	15	<b>95</b>

### Step 2 : Convert to Linear” Programming Problem

$$\text{Minimize } z = \tilde{c}_{11} \otimes x_{11} \oplus \tilde{c}_{21} \otimes x_{21} \oplus \tilde{c}_{31} \otimes x_{31} \oplus \tilde{c}_{12} \otimes x_{12} \oplus \tilde{c}_{22} \otimes x_{22} \oplus \tilde{c}_{32} \otimes x_{32} \oplus \tilde{c}_{13} \otimes x_{13} \oplus \tilde{c}_{23} \otimes x_{23} \oplus \tilde{c}_{33} \otimes x_{33}$$

subject to

$$x_{11} \oplus x_{12} \oplus x_{13} \leq a_1$$

$$x_{21} \oplus x_{22} \oplus x_{23} \leq a_2$$

$$x_{31} \oplus x_{32} \oplus x_{33} \leq a_3$$

$$x_{11} \oplus x_{21} \oplus x_{31} \geq b_1$$

$$x_{12} \oplus x_{22} \oplus x_{32} \geq b_2$$

$$x_{13} \oplus x_{23} \oplus x_{33} \geq b_3$$

Using Table 2,

$$\begin{aligned} \text{Minimize } z = & (2,4,8,15;0.6)(1,4,8,18;0.3) \otimes x_{11} \oplus (3,5,7,12; 0.5)(1,5,7,15;0.3) \otimes x_{12} \oplus (2,5,9,16;0.7) (1,5,9,18;0,3) \otimes x_{13} \oplus \\ & (2,5,8,10;0.6) (1,5,8,12;0.2) \otimes x_{21} \oplus (4,8,10,13;0.4)(3,8,10,15,0.3) \otimes x_{22} \oplus (4,8,10,13;0.4)(3,8,10,15;0.3) \otimes x_{23} \oplus \\ & (2,7,11,15;0.5)(1,7,11,18;0.3) \otimes x_{31} \oplus (5,9,12,16;0.7)(3,9,12,19;0.2) \otimes x_{32} \oplus (4,6,8,10;0.6)(3,6,8,12;0.3) \otimes x_{33} \end{aligned}$$

subject to

$$x_{11} \oplus x_{12} \oplus x_{13} \leq 25$$

$$x_{21} \oplus x_{22} \oplus x_{23} \leq 30$$

$$x_{31} \oplus x_{32} \oplus x_{33} \leq 40$$

$$x_{11} \oplus x_{21} \oplus x_{31} \geq 35$$

$$x_{12} \oplus x_{22} \oplus x_{32} \geq 45$$

$$x_{31} \oplus x_{23} \oplus x_{33} \geq 15$$

### Step 3: Apply Ranking Function as EMD

$$\begin{aligned} \text{Minimize } z = & \text{EMD}(\tilde{c}_{11}) x_{11} + \text{EMD}(\tilde{c}_{12}) x_{12} + \text{EMD}(\tilde{c}_{13}) x_{13} + \text{EMD}(\tilde{c}_{21}) x_{21} + \text{EMD}(\tilde{c}_{22}) x_{22} + \text{EMD}(\tilde{c}_{23}) x_{23} + \text{EMD} \\ & (\tilde{c}_{31}) x_{31} + \text{EMD}(\tilde{c}_{32}) x_{32} + \text{EMD}(\tilde{c}_{33}) x_{33} \end{aligned}$$

subject to

$$x_{11} + x_{12} + x_{13} \leq a_1$$

$$x_{21} + x_{22} + x_{23} \leq a_2$$

$$x_{31} + x_{32} + x_{33} \leq a_3$$

$$x_{11} + x_{21} + x_{31} \geq b_1$$

$$x_{12} + x_{22} + x_{32} \geq b_2$$

$$x_{13} + x_{23} + x_{33} \geq b_3$$

It becomes,

$$\begin{aligned} \text{Minimize } z = & \text{EMD}[(2,4,8,15;0.6)(1,4,8,18;0.3)] \otimes x_{11} \oplus \text{EMD}[(3,5,7,12; 0.5)(1,5,7,15;0.3)] \otimes x_{12} \oplus \text{EMD}[(2,5,9,16;0.7) \\ & (1,5,9,18;0,3)] \otimes x_{13} \oplus \text{EMD}[(2,5,8,10;0.6) (1,5,8,12;0.2)] \otimes x_{21} \oplus \text{EMD}[(4,8,10,13;0.4)(3,8,10,15,0.3)] \otimes x_{22} \oplus \\ & \text{EMD}[(4,8,10,13;0.4)(3,8,10,15;0.3)] \otimes x_{23} \oplus [(2,7,11,15;0.5)(1,7,11,18;0.3)] \otimes x_{31} \oplus [(5,9,12,16;0.7)(3,9,12,19;0.2)] \otimes x_{32} \oplus \\ & \text{EMD}[(4,6,8,10;0.6)(3,6,8,12;0.3)] \otimes x_{33} \end{aligned}$$

subject to

$$x_{11} \oplus x_{12} \oplus x_{13} \leq 25$$

$$x_{21} \oplus x_{22} \oplus x_{23} \leq 30$$

$$x_{31} \oplus x_{32} \oplus x_{33} \leq 40$$

$$x_{11} \oplus x_{21} \oplus x_{31} \geq 35$$

$$x_{12} \oplus x_{22} \oplus x_{32} \geq 45$$

$$x_{31} \oplus x_{23} \oplus x_{33} \geq 15$$

#### Step 4: Convert to Crisp Numbers Using EMD

$$\text{Minimize } z = 13.37 x_{11} + 10.89 x_{12} + 13.98 x_{13} + 5.20 x_{21} + 7.99 x_{22} + 12.89 x_{23} + 12.77 x_{31} + 14.53 x_{32} + 7.85 x_{33}$$

subject to

$$x_{11} + x_{12} + x_{13} \leq 25$$

$$x_{21} + x_{22} + x_{23} \leq 30$$

$$x_{31} + x_{32} + x_{33} \leq 40$$

$$x_{11} + x_{21} + x_{31} \leq 35$$

$$x_{12} + x_{22} + x_{32} \leq 45$$

$$x_{13} + x_{23} + x_{33} \leq 15$$

#### Step 5: Solve LPP

Solving linear programming in step 4,

we get  $x_{11} = 0$ ,  $x_{21} = 30$ ,  $x_{31} = 5$ ,  $x_{12} = 25$ ,  $x_{22} = 0$ ,  $x_{32} = 20$ ,  $x_{13} = 0$ ,  $x_{23} = 0$ ,  $x_{33} = 15$ .

$m+n-1 = 3+3-1 = 5$  allocations,  $x_{12} = 25$ ,  $x_{21} = 30$ ,  $x_{31} = 5$ ,  $x_{32} = 20$ ,  $x_{33} = 15$ ."

#### Step 6: Obtain Minimum Cost in Intuitionistic Fuzzy Form

Minimum Total Cost =

$$\sum_{i=1}^m \sum_{j=1}^n \tilde{c}_{ij} \otimes x_{ij} = \tilde{c}_{12} \otimes x_{12} \oplus \tilde{c}_{21} \otimes x_{21} \oplus \tilde{c}_{31} \otimes x_{31} \oplus \tilde{c}_{32} \otimes x_{32} \oplus \tilde{c}_{33} \otimes x_{33}$$

$$= (3,5,7,12; 0.5)(1,5,7,15;0.3) \otimes 25 \oplus (2,5,8,10;0.6)(1,5,8,12;0.2) \otimes 30 \oplus (2,7,11,15;0.5)(1,7,11,18;0.3) \otimes 5 \oplus$$

$$(5,9,12,16;0.7)(3,9,12,19;0.2) \otimes 20 \oplus (4,6,8,10;0.6)(3,6,8,12;0.3) \otimes 15$$

$$= (305,580,830,1145;0.5)(165,580,830,1385;0.3)$$

#### COMPARATIVE STUDY

The Indira and Shankar [18] technique and the suggested way yield the same fuzzy optimum cost. Table 3 presents a comparative analysis.

**Table 3:** Table of Comparisons

Example	Ranking Procedure	Fuzzy Transportation Method	Fuzzy Optimal Cost
Agarwal et al.[19]	EMD method	Proposed Method	(305,580,830,1145;0.5) (165,580,830,1385;0.3)
Agarwal et al.[19]	Pardha Saradhi et al. [20]	Indira and Shankar [18]	(305,580,830,1145;0.5) (165,580,830,1385;0.3)

#### RESULTS AND DISCUSSION

The transportation problem has been resolved by implementing the proposed optimization technique that makes use of Modified Earth Mover's Distance (EMD) and Generalized Intuitionistic Trapezoidal Fuzzy Numbers (GITFNs). The suggested approach and the current methodology were compared in order to assess transportation costs. The findings for both approaches were the same. These results show that the suggested GITFN-based optimization approach is both legitimate and successful in resolving transportation-related issues.

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