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Research Article

A Mathematical Study on the Stages of Cervical Cancer

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ABSTRACT

In this paper, a mathematical model of Cervical Cancer is formulated. The equilibrium analysis is performed. The boundedness and positiveness of cervical cancer model are evaluated. The local and global stability is studied using R.H. criteria and Lyapunov's approach. Numerical simulations are performed to show the flow the variables using MATLAB.

Keywords: Cervical Cancer, Local Stability, Global Stability, Positiveness.

1. INTRODUCTION

In order to predict long-term health outcomes, mathematical models can be utilized to translate short-term results from prevention and reduction experiments. Cervical Cancer is the form of cancer that develops in cervix cells. The bottom part of the uterus, known as the cervix is attached to the vagina. An irregular development of cells into the cervix is the basic cause of Cervical Cancer which may grow towards the nearby parts of the body. An infection with the Human Papillomavirus (HPV) is a common cause of Cervical Cancer. Adenocarcinoma and Squamous cell Carcinoma are two types of Cervical Cancer. Adenocarcinoma type of cancer begins as a column-shaped glandular cells which forms a in the outer part of cervix. Squamous Cell Carcinoma type cancer begins in the thin flat cells (Squamous cells) lining the outer part of the cervix, which projects into the vagina. Most Cervical Cancer are Squamous cell carcinomas.

There are no telltale signs of cervical cancer in its early stages. Pelvic pain, irregular vaginal bleeding, and uncomfortable sex are some of the signs that are discovered as the disease progresses violently over time. Having multiple sexual partners, having set at an early age, weak immune system, smoking are risk factors for Cervical Cancer. Next to breast cancer, Cervical Cancer is the most common cancer that affects women globally. Radiation therapy, surgery and chemotherapy are the known treatment for cervical cancer.

Many authors illustrated mathematical models for Cervical Cancer. Shermita L. Lee [9] studied a mathematical model on Human Papilloma virus and its impact of Cervical Cancer in the United States. Helen C Johnson [6] developed a dynamical model and analyzed the effect of HPV vaccination of Cervical Cancer screening in England. Wenting Wu [10] formulated a mathematical model for Human Papilloma virus and Cervical tumorigenesis. Abdulsamad Engida Sado [1] studied a mathematical model for cervical cancer with vaccination and transmission. Eminugroha Ratna Sari [2] formulated a mathematical model of SIPC age-structured model for Cervical Cancer. E. D. Gurmu [4] developed a mathematical model which showed the impact of COVID-19 on Outcomes of patients with Cervical Cancer in India. In this paper, a model for Cervical Cancer is developed.

2. EQUATIONS OF CERVICAL CANCER MODEL

The system of ODE symbolizes the Cervical Cancer model:

$$\frac{dS}{dt} = \omega N - (\beta + \mu)S$$

$$\frac{dI_E}{dt} = \beta S - (\delta + \gamma_1 + \mu)I_E$$

$$\frac{dI_L}{\mathrm{dt}} = \delta I_E - (\sigma + \gamma_2 + \mu)I_L \qquad (1)$$

$$\frac{dI_A}{dt} = \sigma\, I_L - (\mu + \mathrm{d})I_A$$

$$\frac{\mathrm{dR}}{\mathrm{dt}} = \gamma_1 I_E + \gamma_2 I_L - \mu R$$

With S(t), I_E(t), I_L(t), I_A(t), R(t) ≥ 0.

Also β, μ, δ, σ, ω, γ_1, γ_2 > 0.

Where S(t), I_E(t), I_L(t), I_A(t), R(t) are the Susceptible, Infected at early stage, Infected at locally advanced stage, Infected at advanced stage and Recovery state respectively.

ω – Mean natality rate of female, μ - Mean mortality rate of female, β - Initial infectious rate, δ - Secondary infectious rate, σ - Final infectious rate, γ_1- recovery rate from Initial infectious stage, γ_2- recovery rate from Secondary infectious stage and N(t) - Female Population.

N(t) = S(t)+ I_E(t)+ I_L(t)+I_A(t)+R(t)

The figure shows the Cervical Cancer model:

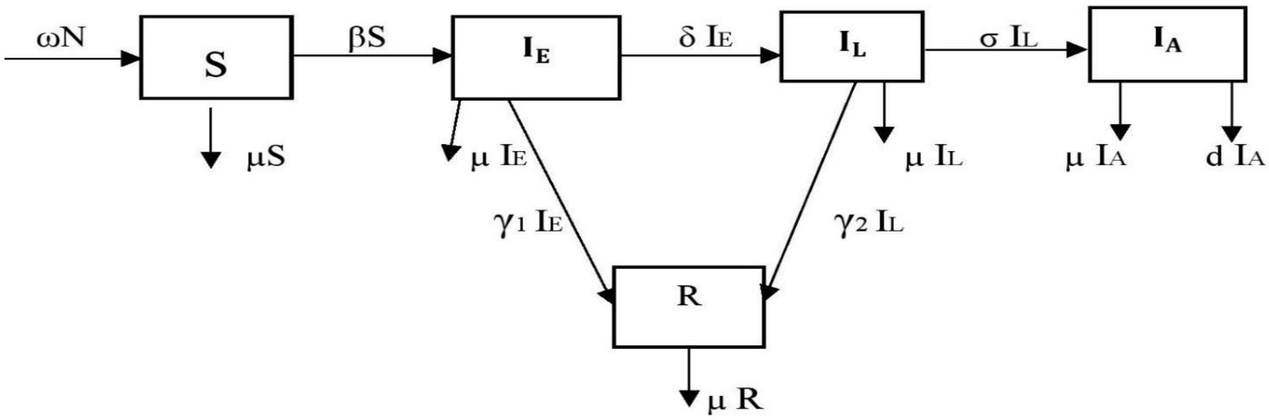

Figure 1: Cervical Cancer Model

3 POSITIVENESS OF THE MODEL

From (1)

$$\frac{\mathrm{dS}}{\mathrm{dt}} \geq -(\beta + \mu)\mathrm{S} \qquad (3)$$

$$\frac{dI_E}{\mathrm{dt}} \geq -(\delta + \gamma_1 + \mu)\mathrm{I_E} \qquad (4)$$

$$\frac{dI_L}{\mathrm{dt}} \geq -(\sigma + \gamma_2 + \mu)\mathrm{I_L} \qquad (5)$$

$$\frac{dI_A}{dt} \geq (\mu + \mathrm{d})\mathrm{I_A} \qquad (6)$$

$$\frac{\mathrm{dR}}{\mathrm{dt}} \geq -\mu R \qquad (7)$$

From (3),

$$\frac{1}{\mathrm{S}}\mathrm{dS} \geq -(\beta + \mu)dt$$

Integrating,

$$\int_0^t \frac{1}{\mathrm{S}}\mathrm{dS} \geq -(\beta + \mu) \int_0^t \mathrm{dt}$$

$$log\, S(t) - log\, S(0) \geq (\beta + \mu)(t - 0)$$
$$log \frac{S(t)}{S(0)} \geq (\beta + \mu)t$$
$$\frac{S(t)}{S(0)} \geq e^{-(\beta+\mu)t}$$

S(t) ≥ S(0) $e^{-(\beta+\mu)t}$

S(t) ≥ 0 as S(0) ≥ 0

From (4),

$$\frac{1}{I_E} \mathrm{d}I_E \geq (\delta + \gamma_1 + \mu) - dt$$

Integrating,

$$\int_0^t \frac{1}{I_E} \mathrm{d}I_E \geq -(\delta + \gamma_1 + \mu) \int_0^t \mathrm{dt}$$
$$log\, I_E(t) - log\, I_E(0) \geq (\delta + \gamma_1 + \mu)(t - 0)$$
$$log \frac{I_E(t)}{I_E(0)} \geq (\delta + \gamma_1 + \mu)t$$
$$\frac{I_E(t)}{I_E(0)} \geq e^{-(\delta+\gamma_1+\mu)t}$$

I_E (t) ≥ I_E (0) $e^{-(\delta+\gamma_1+\mu)t}$

I_E (t) ≥ 0 as I_E (0) ≥ 0

From (5),

$$\frac{1}{I_L} \mathrm{d}I_L \geq (\sigma + \gamma_2 + \mu) - dt$$

Integrating,

$$\int_0^t \frac{1}{I_L} \mathrm{d}I_L \geq -(\sigma + \gamma_2 + \mu) \int_0^t \mathrm{dt}$$
$$log\, I_L(t) - log\, I_L(0) \geq (\sigma + \gamma_2 + \mu)(t - 0)$$
$$log \frac{I_L(t)}{I_L(0)} \geq (\sigma + \gamma_2 + \mu)t$$
$$\frac{I_L(t)}{I_L(0)} \geq e^{-(\sigma+\gamma_2+\mu)t}$$

I_L (t) ≥ I_L (0) $e^{-(\sigma+\gamma_2+\mu)t}$

I_L (t)≥0 as I_L (0)≥0

From (5),

$$\frac{1}{I_A} \mathrm{d}I_A \geq (\mu + \mathrm{d}) - dt$$

Integrating,

$$\int_0^t \frac{1}{I_A} \mathrm{d}I_A \geq -(\mu + \mathrm{d}) \int_0^t \mathrm{dt}$$
$$log\, I_A(t) - log\, I_A(0) \geq (\mu + \mathrm{d})(t - 0)$$
$$log \frac{I_A(t)}{I_A(0)} \geq (\mu + \mathrm{d})t$$

$$\frac{I_A(t)}{I_A(0)} \geq e^{-(\mu+d)t}$$

$$I_A\ (t) \geq I_A\ (0)\ e^{-(\mu+d)t}$$

$$I_A\ (t) \geq 0 \text{ as } I_A\ (0) \geq 0$$

From (7),

$$\frac{1}{R}dR \geq -\mu\, dt$$

Integrating,

$$\int_0^t \frac{1}{R}dR \geq -\mu \int_0^t dt$$

$$\log R(t) - \log R(0) \geq -\mu (t - 0)$$

$$\log \frac{R(t)}{R(0)} \geq \mu t$$

$$\frac{R(t)}{R(0)} \geq e^{-\mu t}$$

$$R(t) \geq R(0)\ e^{-\mu t}$$

$$R(t) \geq 0 \text{ as } R(0) \geq 0$$

Hence, positiveness is proved.

4. BOUNDEDNESS OF THE MODEL

Differentiate (2),

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dI_E}{dt} + \frac{dI_L}{dt} + \frac{dI_A}{dt} + \frac{dR}{dt}$$

$$\frac{dN}{dt} = (\omega - \mu)N$$

$$\frac{1}{dt}dN = (\omega - \mu)dt$$

Integrating,

$$\int_0^t \frac{1}{N}dN \geq \int_0^t (\omega - \mu)\, dt$$

$$\Rightarrow N(t) \leq N(0)\ e^{-(\omega-\mu)t}$$

⇒N(t) is bounded with a positive integer.
S(t), I_E(t), I_L(t), I_A(t) and R(t) are bounded.

5. EQUILIBRIUM ANALYSIS

Disease free equilibrium: $G_1(\tilde{S}, 0,0,0,0)$

$\tilde{S}$ be considered as the positive solution of $\frac{dS}{dt} = 0$

Using (1),

$$\tilde{S} = \frac{\omega N}{(\beta + \mu)}$$

$$G_1(\tilde{S}, 0,0,0,0) = \left(\frac{\omega N}{\beta + \mu}, 0,0,0,0\right)$$

Endemic equilibrium: $G_1(S^*, {I_E}^*, {I_L}^*, {I_A}^*, R^*)$, Let the positive solutions of $\frac{dS}{dt}=0, \frac{dI_E}{dt}=0, \frac{dI_L}{dt}=0, \frac{dI_A}{dt}=0, \frac{dR}{dt}=0$ be $S^*, {I_E}^*, {I_L}^*, {I_A}^*$ and R^*

Using (1),

$$S^* = \frac{\omega N}{\beta+\mu}$$

$${I_E}^* = \frac{\omega N\beta}{(\beta+\mu)(\delta+\gamma_1+\mu)}$$

$${I_L}^* = \frac{\omega N\beta\delta}{(\beta+\mu)(\delta+\gamma_1+\mu)(\sigma+\gamma_2+\mu)}$$

$${I_A}^* = \frac{\omega N\beta\delta\sigma}{(\beta+\mu)(\mu+d)(\delta+\gamma_1+\mu)(\sigma+\gamma_2+\mu)}$$

$$R^* = \left(\frac{\omega N\beta}{\mu(\beta+\mu)(\delta+\gamma_1+\mu)}\right)\left(\frac{\delta\gamma_2+\gamma_1(\sigma+\gamma_2+\mu)}{(\sigma+\gamma_2+\mu)}\right)$$

∴ the cervic equilibrium,

$$G_1(S^*, {I_E}^*, {I_L}^*, {I_A}^*, R^*) = \left(\frac{\omega N}{\beta+\mu}, \frac{\omega N\beta\delta}{(\beta+\mu)(\delta+\gamma_1+\mu)(\sigma+\gamma_2+\mu)}, \frac{\omega N\beta\delta\sigma}{(\beta+\mu)(\mu+d)(\delta+\gamma_1+\mu)(\sigma+\gamma_2+\mu)}, \frac{\omega N\beta}{\mu(\beta+\mu)(\delta+\gamma_1+\mu)}\left(\frac{\delta\gamma_2+\gamma_1(\sigma+\gamma_2+\mu)}{(\sigma+\gamma_2+\mu)}\right)\right)$$

∴ G_1 is positive.

6. STABILITY: LOCAL BEHAVIOR

The Jacobian of (1) is

$$\begin{pmatrix} -(\beta+\mu) & 0 & 0 & 0 & 0 \\ \beta & -(\delta+\gamma_1+\mu) & 0 & 0 & 0 \\ 0 & \delta & -(\sigma+\gamma_2+\mu) & 0 & 0 \\ 0 & 0 & \sigma & -(\mu+d) & 0 \\ 0 & \gamma_1 & \gamma_2 & 0 & -\mu \end{pmatrix} \tag{8}$$

At the interior equilibrium

$$\begin{pmatrix} -\frac{\omega N}{S} & 0 & 0 & 0 & 0 \\ \beta & -\frac{\beta S}{I_E} & 0 & 0 & 0 \\ 0 & \delta & -\frac{\delta I_E}{I_L} & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sigma I_E}{I_A} & 0 \\ 0 & \gamma_1 & \gamma_2 & 0 & -\frac{\gamma_1 I_E+\gamma_2 I_L}{R} \end{pmatrix} \tag{9}$$

The characteristic equation of (9) is

$$\begin{vmatrix} -\frac{\omega N}{S}-\lambda & 0 & 0 & 0 & 0 \\ \beta & -\frac{\beta S}{I_E}-\lambda & 0 & 0 & 0 \\ 0 & \delta & -\frac{\delta I_E}{I_L}-\lambda & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sigma I_E}{I_A}-\lambda & 0 \\ 0 & \gamma_1 & \gamma_2 & 0 & -\frac{\gamma_1 I_E+\gamma_2 I_L}{R}-\lambda \end{vmatrix} = 0$$

$$\lambda^5 + (\frac{\omega N}{S} + \frac{\beta S}{I_E} + \frac{\delta I_E}{I_L} + \frac{\sigma I_L}{I_A} + \frac{\gamma_1 I_E + \gamma_2 I_L}{R})\lambda^4 + (\frac{\omega N\beta}{I_E} + \frac{\omega N}{S}[\frac{\gamma_1 I_E + \gamma_2 I_L}{R}] + \frac{\omega N\sigma I_L}{SI_A} + \frac{\omega N\delta I_E}{SI_L} + \frac{\beta S}{I_E}[\frac{\gamma_1 I_E + \gamma_2 I_L}{R}] + \frac{\beta\sigma S I_L}{I_E I_A} + \frac{\beta\delta S}{I_L} + \frac{\delta I_E}{I_L}[\frac{\gamma_1 I_E + \gamma_2 I_L}{R}] + \frac{\sigma I_L}{I_A}[\frac{\gamma_1 I_E + \gamma_2 I_L}{R}] + \frac{\sigma\delta I_E}{I_A})\lambda^3 + (\frac{\omega N\beta}{I_E}[\frac{\gamma_1 I_E + \gamma_2 I_L}{R}] + \frac{\omega N\beta\sigma I_L}{I_E I_A} + \frac{\omega N\beta\delta}{I_L} + \frac{\omega N\delta I_E}{SI_L}[\frac{\gamma_1 I_E + \gamma_2 I_L}{R}] + \frac{\omega N\delta\sigma I_E}{SI_A} + \frac{\omega N\sigma I_L}{SI_A}[\frac{\gamma_1 I_E + \gamma_2 I_L}{R}] + \frac{\beta\delta S}{I_A}[\frac{\gamma_1 I_E + \gamma_2 I_L}{R}] + \frac{\beta\delta\sigma S I_E}{I_A} + \frac{\beta\sigma S I_L}{I_E I_A} + \frac{\delta\sigma I_E}{I_A}])\lambda^2 + (\frac{\omega N\beta\delta}{I_L}[\frac{\gamma_1 I_E + \gamma_2 I_L}{R}] + \frac{\omega N\beta\delta\sigma I_E}{I_A} + \frac{\omega N\beta\sigma I_L}{I_E I_A}[\frac{\gamma_1 I_E + \gamma_2 I_L}{R}] + \frac{\omega N\delta\sigma I_E}{I_A}[\frac{\gamma_1 I_E + \gamma_2 I_L}{R}] + \frac{\beta\delta\sigma S}{I_A}[\frac{\gamma_1 I_E + \gamma_2 I_L}{R}])\lambda + \frac{\omega N\beta\sigma\delta}{I_A}[\frac{\gamma_1 I_E + \gamma_2 I_L}{R}] = 0$$

This is in the form of $\lambda^5 + G_1\lambda^4 + G_2\lambda^3 + G_3\lambda^2 + G_4\lambda + G_5 = 0$

Where

$$G_1 = \frac{\omega N}{S} + \frac{\beta S}{I_E} + \frac{\delta I_E}{I_L} + \frac{\sigma I_L}{I_A} + \frac{\gamma_1 I_E+\gamma_2 I_L}{R}$$

$$G_2 = \frac{\omega N\beta}{I_E} + \frac{\omega N}{S}[\frac{\gamma_1 I_E+\gamma_2 I_L}{R}] + \frac{\omega N\sigma I_L}{SI_A} + \frac{\omega N\delta I_E}{SI_L} + \frac{\beta S}{I_E}[\frac{\gamma_1 I_E+\gamma_2 I_L}{R}] + \frac{\beta\sigma S I_L}{I_E I_A} + \frac{\beta\delta S}{I_L} + \frac{\delta I_E}{I_L}[\frac{\gamma_1 I_E+\gamma_2 I_L}{R}] + \frac{\sigma I_L}{I_A}[\frac{\gamma_1 I_E+\gamma_2 I_L}{R}] + \frac{\sigma\delta I_E}{I_A}$$

$$G_3 = \frac{\omega N\beta}{I_E}[\frac{\gamma_1 I_E+\gamma_2 I_L}{R}] + \frac{\omega N\beta\sigma I_L}{I_E I_A} + \frac{\omega N\beta\delta}{I_L} + \frac{\omega N\delta I_E}{SI_L}[\frac{\gamma_1 I_E+\gamma_2 I_L}{R}] + \frac{\omega N\delta\sigma I_E}{SI_A} + \frac{\omega N\sigma I_L}{SI_A}[\frac{\gamma_1 I_E+\gamma_2 I_L}{R}] + \frac{\beta\delta S}{I_A}[\frac{\gamma_1 I_E+\gamma_2 I_L}{R}] + \frac{\beta\delta\sigma S I_E}{I_A} + \frac{\beta\sigma S I_L}{I_E I_A} + \frac{\delta\sigma I_E}{I_A}$$

$$G_4 = \frac{\omega N\beta\delta}{I_L}[\frac{\gamma_1 I_E+\gamma_2 I_L}{R}] + \frac{\omega N\beta\delta\sigma I_E}{I_A} + \frac{\omega N\beta\sigma I_L}{I_E I_A}[\frac{\gamma_1 I_E+\gamma_2 I_L}{R}] + \frac{\omega N\delta\sigma I_E}{I_A}[\frac{\gamma_1 I_E+\gamma_2 I_L}{R}] + \frac{\beta\delta\sigma S}{I_A}[\frac{\gamma_1 I_E+\gamma_2 I_L}{R}]$$

$$G_5 = \frac{\omega N\beta\sigma\delta}{I_A}[\frac{\gamma_1 I_E+\gamma_2 I_L}{R}]$$

Here

$G_1 > 0$; $G_2 > 0$; $G_1G_2 - G_3 > 0$; $G_3(G_1G_2 - G_3) - G_1^2G_4) > 0$;

$G_4[(G_1^2G_4 - G_5G_1) - G_5(G_2^2G_1 - G_3G_5) - G_1G_4 - G_5)] > 0$ and

when $\frac{\omega N\beta\delta\sigma}{(\beta+\mu)(\mu+d)(\delta+\gamma_1+\mu)(\sigma+\gamma_2+\mu)} > \frac{1}{(\beta+\mu)(\delta+\gamma_1+\mu)(\sigma+\gamma_2+\mu)}$.

Here, the model is stable locally by R.H. Criteria.

7. STABILITY: GLOBAL BEHAVIOR

Let

$V(S, I_E, I_L, I_A, R) = ((S - S^*) - \ln\frac{S}{S^*}) + l_1((I_E - I_E^*) - I_E^*\ln\frac{I_E}{I_E^*}) + l_2((I_L - I_L^*) - I_L^*\ln\frac{I_L}{I_L^*}) + l_3((I_A - I_A^*) - I_A^*\ln\frac{I_A}{I_A^*}) + l_4((R - R^*) - R^*\ln\frac{R}{R^*}))$ }10

Differentiate (10),

$$\frac{dV}{dt} = \left(\frac{S-S^*}{S}\right)\frac{dS}{dt} + \left(\frac{I_E - I_E^*}{I_E}\right)\frac{dI_E}{dt} + \left(\frac{I_L - I_L^*}{I_L}\right)\frac{dI_L}{dt} + \left(\frac{I_A - I_A^*}{I_A}\right)\frac{dI_A}{dt} + \left(\frac{R-R^*}{R}\right)\frac{dR}{dt}$$

Using (1),

$$\frac{dV}{dt}=\left(\frac{S-S^*}{S}\right)(\omega N-(\beta+\mu)S)+\left(\frac{I_E-I_E^*}{I_E}\right)(\beta S-(\delta+\gamma_1+\mu)I_E)+\left(\frac{I_L-I_L^*}{I_L}\right)(\delta I_E-(\sigma+\gamma_2+\mu)I_L)$$
$$+\left(\frac{I_A-I_A^*}{I_A}\right)(\sigma I_L-(\mu+d))+\left(\frac{R-R^*}{R}\right)(\gamma_1 I_E+\gamma_2 I_L-\mu R)$$

$$\frac{dV}{dt}=(S-S^*)\left(\frac{\omega N}{S}-(\beta S+\mu S)\right)+l_1(I_E-I_E^*)\left(\frac{\beta S}{I_E}-(\delta+\gamma_1+\mu)\right)+l_2(I_L-I_L^*)\left(\frac{\delta I_E}{I_L}-(\sigma+\mu+\gamma_2)\right)$$
$$+l_3(I_A-I_A^*)\left(\frac{\sigma I_L}{I_A}-(\mu+d)\right)+l_4(R-R^*)\left(\frac{\gamma_1 I_E+\gamma_2 I_L}{R}-\mu\right)$$

At (S, I_E, I_L, I_A, R) ,

$$\frac{dV}{dt}=(S-S^*)\left[\frac{\omega N}{S}-\left(\frac{\omega N}{S^*}\right)\right]+l_1(I_E-I_E^*)\left[\frac{\beta S}{I_E}-\left(\frac{\beta S^*}{I_E^*}\right)\right]+l_2(I_L-I_L^*)\left[\frac{\delta I_E}{I_L}-\left(\frac{\delta I_E^*}{I_L^*}\right)\right]+l_3(I_A-I_A^*)\left[\frac{\sigma I_L}{I_A}-\left(\frac{\sigma I_L^*}{I_A^*}\right)\right]$$
$$+l_4(R-R^*)\left[\frac{\gamma_1 I_E+\gamma_2 I_L}{R}-\frac{\gamma_1 I_E^*+\gamma_2 I_E^*}{R^*}\right]$$

$$\frac{dV}{dt}=(S-S^*)\omega N\left(\frac{1}{S}-\frac{1}{S^*}\right)+l_1(I_E-I_E^*)\beta\left(\frac{S}{I_E}-\frac{S^*}{I_E^*}\right)+l_2(I_L-I_L^*)\delta\left(\frac{I_E}{I_L}-\frac{I_E^*}{I_L^*}\right)+l_3(I_A-I_A^*)\sigma\left(\frac{I_L}{I_A}-\frac{I_L^*}{I_A^*}\right)$$
$$+l_4(R-R^*)\left(\gamma_1\left[\frac{I_E}{R}-\frac{I_E^*}{R^*}\right]+\gamma_2\left[\frac{I_L}{R}-\frac{I_L^*}{R^*}\right]\right)$$

Choosing $l_1=\frac{1}{\beta}, l_2=\frac{1}{\delta}, l_3=\frac{1}{\sigma}, l_4=\frac{1}{\gamma_1\gamma_2}$

$$\frac{dV}{dt}=\frac{-\omega N(S-S^*)^2}{SS^*}+(I_E-I_E^*)\left(\frac{I_E^*S-S^*I_E}{I_E I_E^*}\right)+(I_L-I_L^*)\left(\frac{I_E I_L^*-I_L I_E^*}{I_L I_L^*}\right)+(I_A-I_A^*)\left(\frac{I_L I_A^*-I_A I_L^*}{I_A I_A^*}\right)$$
$$+\frac{(R-R^*)}{\gamma_1}\left(\frac{I_E R^*-R I_E^*}{RR^*}\right)+\frac{(R-R^*)}{\gamma_2}\left(\frac{I_L R^*-R I_L^*}{RR^*}\right)$$

$$\frac{dV}{dt}=\frac{-\omega N(S-S^*)^2}{SS^*}+\frac{1}{I_E I_E^*}\left[I_E^* I_E S-S I_E^2-S I_E^{*2}+I_E I_E^* S^*\right]+\frac{1}{I_L I_L^*}\left[I_L^* I_L I_E-I_E I_L^2-I_E I_L^{*2}+I_L I_L^* I_E^*\right]$$
$$+\frac{1}{I_A I_A^*}\left[I_A^* I_A I_L-I_L I_A^2-I_L I_A^{*2}+I_A I_A^* I_L^*\right]+\frac{1}{\gamma_1 RR^*}\left[I_E R^* R-I_E^* R^{*2}-I_E R^2+I_E^* RR^*\right]$$
$$+\frac{1}{\gamma_2 RR^*}\left[I_L R^* R-I_L^* R^{*2}-I_L R^2+I_L^* RR^*\right]$$

$$\frac{dV}{dt}=\frac{-\omega N(S-S^*)^2}{SS^*}+\left[S-\frac{I_E S^*}{I_E^*}-\frac{I_E^* S}{I_E}+S^*\right]+\left(I_E-\frac{I_L I_E^*}{I_L^*}-\frac{I_L^* I_E}{I_L}+I_E^*\right)+\left(I_L-\frac{I_A I_L^*}{I_A^*}-\frac{I_A^* I_L}{I_A}+I_L^*\right)$$
$$+\frac{1}{\gamma_1}\left(I_E-\frac{R I_E^*}{R^*}-\frac{R^* I_E}{R}+I_E^*\right)+\frac{1}{\gamma_2}\left(I_L-\frac{R I_L^*}{R^*}-\frac{R^* I_L}{R}+I_L^*\right)$$

$$\frac{dV}{dt}=\frac{-\omega N(S-S^*)^2}{SS^*}+\left[(S+S^*)-\left(\frac{I_E S^*}{I_E^*}+\frac{I_E^* S}{I_E}\right)\right]+\left[(I_E+I_E^*)-\left(\frac{I_L I_E^*}{I_L^*}+\frac{I_L^* I_E}{I_L}\right)\right]+\left[(I_L+I_L^*)-\left(\frac{I_A I_L^*}{I_A^*}-\frac{I_A^* I_L}{I_A}\right)\right]+\frac{1}{\gamma_1}\left[(I_E+I_E^*)-\left(\frac{R I_E^*}{R^*}-\frac{R^* I_E}{R}\right)\right]+\frac{1}{\gamma_2}\left[(I_L+I_L^*)-\left(\frac{R I_L^*}{R^*}-\frac{R^* I_L}{R}\right)\right]$$

Therefore $\frac{dV}{dt}<0$, when $\frac{S}{I_E}<\frac{S^*}{I_E^*}, \frac{I_E}{I_L}<\frac{I_E^*}{I_L^*}, \frac{I_L}{I_A}<\frac{I_L^*}{I_A^*}, \frac{I_E}{R}<\frac{I_E^*}{R^*}, \frac{I_L}{R}<\frac{I_L^*}{R^*}$

Here, the model is stable globally by Lyapunov's approach.

8. NUMERICAL SIMULATIONS

We have chosen the parameter values are taken as $\delta = 0.30$, $\omega = 0.092$, $\mu = 0.018$, $\beta = 0.75$, $\gamma_1 = 0.04$, $\gamma_2=0.004$.

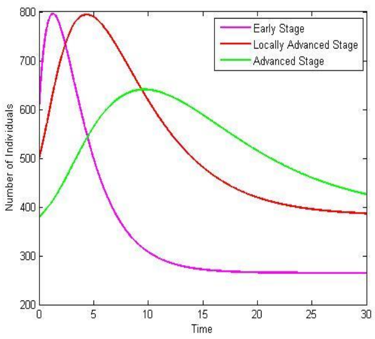

Figure 2: Cervical Cancer Model

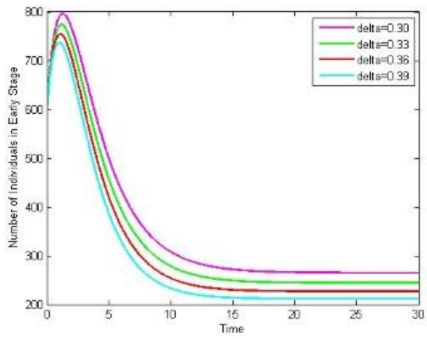

Figure 3: Different values of δ in I_E

Figure (2) represents the three stages namely, Infected at early stage, Infected at Locally advanced stage and Infected at Advanced stage class of the Cervical Cancer model.

Figure (3) shows the number of women of Infected at early stage class with various values of secondary infectious rate. The number of women decreases in the Infected at early stage class whenever the secondary infectious rate δ increases.

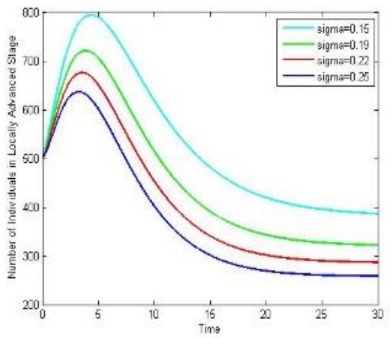

Figure 4: Different values of σ in I_L

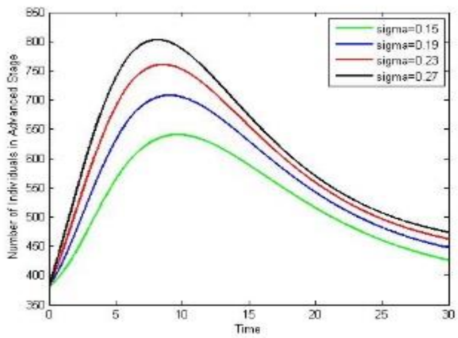

Figure 5: Different values of σ in I_A

Figure (4) shows the number of women of Infected at locally advanced stage class with various values of final infectious rate σ. In Infected at locally advanced stage class, the number of women decreases whenever the final infectious rate σ increases.

Figure (5) represents that the flow of number of individuals of Infected at advanced stage class with different values of final infectious rate σ. In the Infected at advanced stage class, the final infectious rate σ increases the Infected at advanced stage individuals increases.

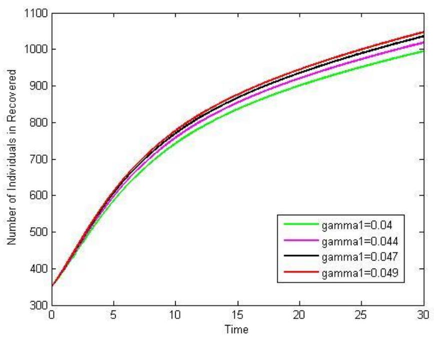

Figure 6: Different values of γ_1

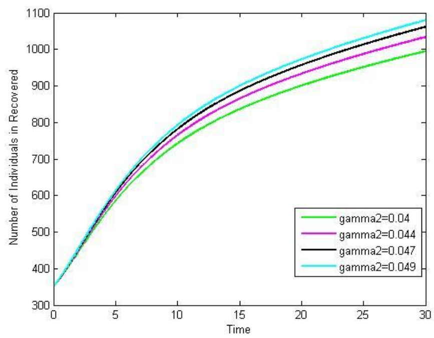

Figure 7: Different values of γ_2

Figure (6) shows the number of women recovered class for various values of recovery rate of Initial infectious stage class and the number of women of recovered class increases whenever the recovery rate of women of Infected at early stage γ_1 increases.

Figure (7) represents the number of women of recovered class for various values of recovery rate of secondary infectious stage class and the number of women of recovered class increases whenever the recovery rate of Infected at locally advanced stage γ_2 increases.

9. CONCLUSION

This paper narrates the dynamical behaviour of a mathematical model of Cervical Cancer with five compartments namely Susceptible, Infected at early stage, Infected at locally advanced stage, Infected at advanced stage and Recovered class. The positivity and boundedness of the model are derived. The disease free equilibrium and endemic equilibrium points are found. The system is found to be locally asymptotically stable around the endemic equilibrium through the R.H. criteria and globally asymptotically stable under a particular condition using Lyapunov theorem. Numerical simulations show the flow of dynamical behaviour of different parameters such as initial infectious rate β, secondary infectious rate δ, final infectious rate σ, recovery rate from Infected at early stage class γ_1 and recovery rate from Infected at locally advanced stage class γ_2. The number of individuals increases in the Infected at early stage, when the secondary infection rate δ decreases. When final infectious rate σ increases, the number of individuals in Infected at locally advanced stage decreases. Recovery is possible only in Infected at early stage and Infected at locally advanced stage. There is no recovery in Infected at advanced stage.

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