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Research Article

Estimation of Thermal Conductivity and Volumetric Heat Capacity in a Solid Plate using the Levenberg-Marquardt Method.

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ABSTRACT

The city of Ouargla is characterized by extremely high summer temperatures, which pose substantial challenges to achieving indoor thermal comfort, as conventional cooling systems often prove insufficient. This study seeks to mitigate this issue by investigating the thermal behavior of building materials and optimizing their properties to reduce cooling energy demand. An inverse analysis based on the Levenberg-Marquardt technique is employed to estimate key thermal properties-namely density, specific heat capacity, thermal conductivity, and thermal diffusivity-which are generally difficult to determine through direct measurement. To validate the proposed algorithm, a numerical simulation is performed on a solid plate with known thermophysical properties, enabling the computation of transient temperature fields. Sensitivity analysis is used to assess the influence of sensor placement and experiment duration on parameter estimation accuracy. To better represent real experimental conditions, the simulated temperature data are deliberately perturbed with random noise of amplitude 0.01, simulating measurement uncertainties. The results indicate excellent accuracy when using exact temperature data and only minor deviations in the presence of noise, highlighting the robustness and efficiency of the proposed inverse approach. These outcomes offer valuable insights for enhancing building design in Ouargla, improving thermal performance, and alleviating the load on cooling systems during extreme summer periods.

Keywords: parameters identification, Inverse Methods, inverse heat conduction, finite difference, Levenberg-Maquardt Method.

INTRODUCTION

Inverse problems for parameter estimation refer to the process of determining the values of parameters in a mathematical model or system based on observed data or outcomes. These problems are termed "inverse" because they involve working backward from the effects (data) to the causes (parameters), which is the opposite of traditional direct problems where parameters are known and used to predict outcomes.

Inverse heat conduction problems are a scope in which inverse problems are used for estimating parameters or characteristics of a heat transfer process based on temperature measurements over time and possibly other available data, when the measurement of these parameters is impossible. In heat conduction, the direct problem involves calculating how temperature changes over time within a material when the heat sources, boundary conditions, and material properties are known. Conversely, inverse heat conduction problems (IHCPs) focus on determining these properties or boundary conditions when temperature measurements are taken [1,2,4].

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In this paper unknown Parameters thermal conductivity and volumetric heat capacity as a thermophysical properties of a solid plate materials are to be estimated using the method of Levenberg-Marquardt for parameters estimation.

In the first step the Levenberg-Marquardt method using temperature measurements are simulated in the location of two sensors at two points in the slab. The location of the sensors and the optimisation of the experience are the result of the sensitivity problem study to the variation of the parameters under investigation, then a simulated measurements are artificially obtained by the solution of the direct problem in the points of the sensors by the use of the predefined values of these parameters then the algorithm of the method applied to estimate these parameters as unknown parameters. In addition to estimating thermophysical properties, the proposed inverse approach provides a valuable contribution to building energy management by enabling the selection of optimal wall parameters that minimise heat gains from the external environment. By accurately identifying thermal conductivity and volumetric heat capacity, the method makes it possible to adjust air conditioning systems more effectively in response to variations in ambient temperature. This contributes to reducing cooling loads, enhancing thermal comfort, and improving overall energy efficiency in buildings, particularly in hot climate conditions.

PROBLEM STATEMENT AND SOLUTION PROCEDURE

The direct problem

Th direct problem in this communication consists of one-dimensional heat conduction in a slab of thickness L and initial temperature Ti heated on one side by the heat flux q(t) as presented in the figure (1), the thermal properties are considered known, the solution of the direct problem allows us to obtain the transient temperature fields [1,2].

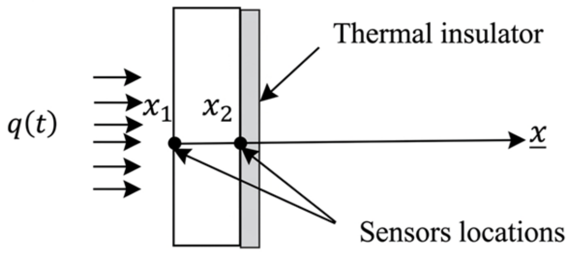

Fig. 1:The plate scheme

The governing equations of the direct problem

$$C\frac{\partial T}{\partial t} = \lambda \frac{\partial^2 T}{\partial x^2} \quad x \in [0, L], t \geq 0 \quad (1-a)$$

$$-\lambda \frac{\partial T}{\partial x} = q(t), \quad x = 0 \text{ et } t \geq 0 \quad (1-b)$$

$$\frac{\partial T}{\partial x} = 0, \quad x = L \text{ et } t \geq 0 \quad (1-c)$$

$$T(0, x) = T_\infty(x), \quad x \in [0, L] \quad (1-d)$$

the inverse problem:

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In the inverse problem the parameters to be estimated are considered unknowns, and the problem become an ill- posed problem, and to overcome this problem the temperature fields simulated in the direct problem are used as an additional information.

The governing equation of the inverse problem are :

$$C(?)\frac{\partial T}{\partial t} = \lambda(?)\frac{\partial^2 T}{\partial x^2} \quad x \in [0, L], t \geq 0 \; (2-a)$$

$$-\lambda(?)\frac{\partial T}{\partial x} = q(t), \quad x = 0 \text{ et } t \geq 0 \; (2-b)$$

$$\frac{\partial T}{\partial x} = 0, \quad x = L \text{ et } t \geq 0 \; (2-c)$$

$$T(0, x) = T_\infty(x), \quad x \in [0, L] \; (2-d)$$

And the temperature measurements inside the solid plate in the positions x_1 and x_2 are:

$T(t_i, x_S) = Y_{is} \; ... (2-e)$ for $x = x_s$ (s = 1 et 2) et $t = t_i$ (i = 1,2, … , I)

The solution of this inverse problem is based on the minimisation of the least squares criterion presented below [1,4]:

$$J(P) = \sum_{s=1}^{I} \sum_{i=1}^{I} [Y_{iS} - T_{iS}(P)]^2 \; ... (3)$$

J : Square sum of errors or objective function.

$\mathbf{P^T} \equiv [\mathbf{C}, \boldsymbol{\lambda}]$: unknown parameters vector.

$\boldsymbol{T_{is}(P)} \equiv \mathbf{T}(\mathbf{P}, \mathbf{t_i}, \mathbf{x_s})$: calculated temperatures at time t_i and location x_s.

$\boldsymbol{Y_{iS}} \equiv \mathbf{Y}(\mathbf{t_i}, \mathbf{x_s})$: measured temperatures at time t_i and location x_s.

s: sensor number, $S = 2$.

i: total number of measures.

Sensitivity study of the problem

The sensitivity matrix is a key concept that relates changes in temperatures to changes in parameters to be estimated. The sensitivity matrix X is the Jacobian of Ti the parameters Pj , it is defined as :

$$\mathbf{X(P)} = \left[\frac{\partial \mathbf{T^T(P)}}{\partial \mathbf{P}}\right]^{\mathbf{T}} \; .... (\mathbf{4})$$

In fact, when sensitivity coefficient are small means that a large changes in Pj yield small changes in the temperatures Ti and consequently $|X^T X| \approx 0$, this means the inverse problem is ill-conditioned and the estimation of parameters is extremely difficult.

Also, when $|X^T X| = 0$, it can be shown that one column of the sensitivity matrix has a linear dependency with another column which means that the corresponding model of parameters estimation are not independently influencing the changes in temperatures, leading to the redundancy in the sensitivity matrix and consequently the solution of the inverse problem is impossible.

In general, before the solution of the inverse problem, the sensitivity analysis must be examined, and the maximization of $|X^T X|$ must be achieved by the optimum design of the experiences.

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THE ALGORITHM OF THE LEVENBERG-MARQUARDT METHODE:

To find the vector of the unknown parameters P we use the iterative method of Levenberg-Marquarts equation presented below [1,4,10]:

$$\mathbf{P^{k+1}} = \mathbf{P^{k+1}} + [(\mathbf{X^K})^\mathbf{T}\mathbf{X^K} + \boldsymbol{\mu}^\mathbf{K}\boldsymbol{\Omega}^\mathbf{K}]^{-\mathbf{1}}(\mathbf{X^K})^\mathbf{T}\left[\mathbf{Y} - \mathbf{T}(\mathbf{P^k})\right] \qquad (\mathbf{5})$$

where:

$\mathbf{T}\left(\mathbf{P^k}\right)$: Estimated temperature at iteration k, $\mathbf{P^k}$: Parameters vector at iteration k

$\mathbf{X^K}$: Sensibility matrix evaluated at iteration k, with $\mathbf{X}(\mathbf{P}) = \left[\frac{\partial \mathbf{T}^\mathbf{T}(\mathbf{P})}{\partial \mathbf{P}}\right]^\mathbf{T}$

$\boldsymbol{\mu}^\mathbf{K}$: damping parameter

$\boldsymbol{\Omega}^\mathbf{K} = diag\ ((\mathbf{X^K})^\mathbf{T}\mathbf{X^K})$.

Stopping criteria:

The criteria suggested to stop the iterative procedure is given by the equation:

$$\mathbf{J}(\mathbf{P^{K+1}}) < \boldsymbol{\varepsilon} \qquad (\mathbf{6})$$

Where $\boldsymbol{\varepsilon}$ a user prescribed tolerance.

The computational algorithm

The computational algorithm of the Levenberg Marquardt method used in this paper for the estimation of parameter is described in the figure (2) below [1,4]:

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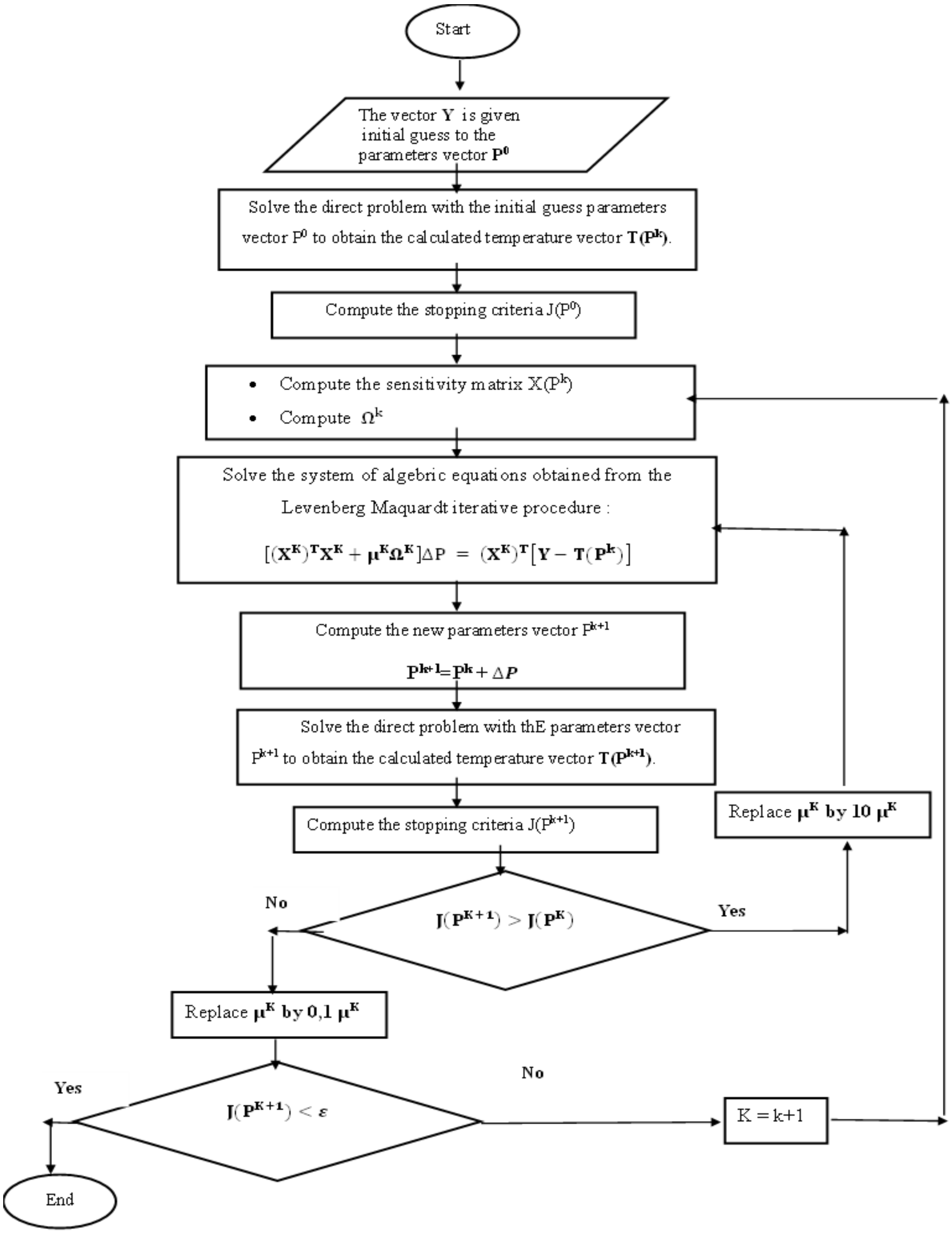

Fig. 2: The Algorithm of the method of Levenberg-Marquardt for parameters estimation

RESULTS

Sensitivity analysis

The sensitivity coefficients that measure the variations of the temperatures with respect to the change of the parameters to be estimated (volumetric heat capacity and thermal conductivity) in the positions of the sensors (at x=0 and x=1). In this operation the direct problem is solved with the parameters: C=ρc_p=1 and λ=1, the heat flow function is represented in figure (3) and the sensitivity coefficients are calculated by formula (4). Figures (3-3) and (3-4) represent the variations of the sensitivity coefficients, these figures show that the absolute values of the coefficients are greater than 0.01 [1,5,7], which gives

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a good sign on the feasibility and ease of the estimation. Also, from figure (3-3) we note that the sensitivity coefficients of the volumetric capacity and the sensitivity coefficients of the thermal conductivity are linearly dependent in the time interval between 0 and 0.35s; which means that the estimation of these parameters simultaneously is impossible. On the other hand, in the rest of the time interval it is clear that the sensitivity coefficients are linearly independent, which means that the estimation is possible and feasible.

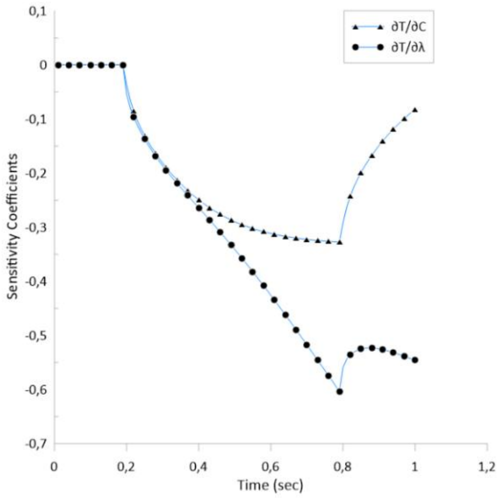

Fig. 4:Variation of sensitivity coefficients with respect tie at X= 0

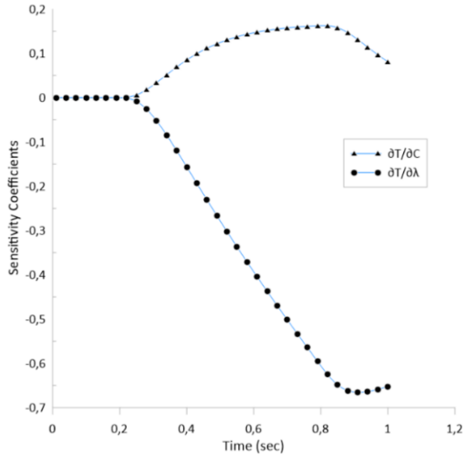

Fig. 3:Variation of sensitivity coefficients with respect tie at X= 0

For the solution of the direct problem the finite difference method is used, and the fields of temperature obtained are presented in the figure (2), the heat flux function used in the direct problem is presented in figure (3).

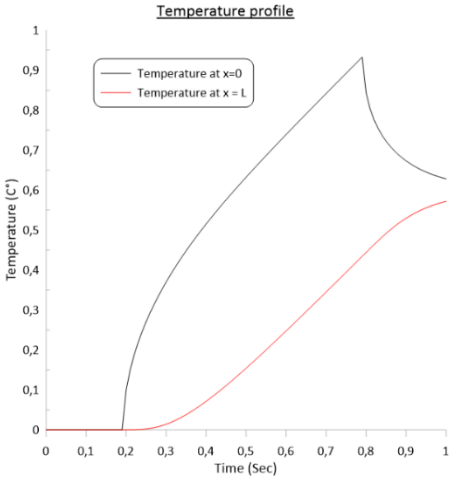

Fig. 6:Temperatures profile simulated without noise

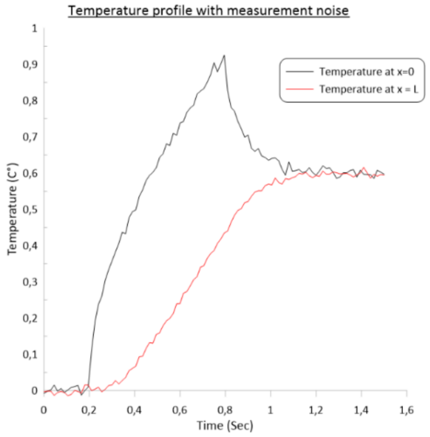

Fig. 5:Temperatures profile simulated with measurement noise

The temperature fields presented in figure (4) are considered as exact solutions of the direct problem in eqauations (1). To achieve the experimental conditions of tese temprature fields, a random noise of type $\sigma\omega$ is added to the exact solution and the measered tempratures become[1,7] :

$$\mathbf{Y_{mes} = T_{ex} + \sigma\omega} \quad (7)$$

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Y_{mes}: simulated measures containing random errors. T_{ex}: exacts measures simulated σ: standard deviation of measurement errors. ω: random variables ($-2.576 < \boldsymbol{\omega} < 2.576$).

Simulated measures containing random errors obtaained are presented in the figure (5).

Table(1): Values of the identified thermophysical properties parameters using L-M

Parameters	**Exact value**	**Initial guess**	***σ***	**Estimated value**	**Error(%)**
λ (W/m/°c)	1	0.01	0	1	0
			0.01	0.9952	0.48
ρc(**J/k m3**)	1	0.01	0	1	0
			0.01	0.9990	0.1
Steel					
λ (W/m/°c)	64	0.01	0	64	0
			0.01	63.527	0.73
ρc(**J/k m3**)	3398220	0.01	0	3398220	0
			0.01	3394298.655	-0.11
Copper					
λ (W/m/°c)	398	0.01	0	398	0
			0.01	383.016	3.76
ρc(**J/k m3**)	3438336	0.01	0	3438336	0
			0.01	3435251.860	0.89

Where **λ** : is thermal coductivity and $\boldsymbol{\rho c}$: is calorific capacity

DISCUSSION

The results obtained from solving the inverse problem using the Levenberg–Marquardt method are presented in Table (1). A detailed analysis of these results demonstrates the high efficiency, robustness, and accuracy of the proposed estimation procedure. In the case where the exact temperature fields are employed as input data, the estimated thermophysical properties perfectly coincide with their exact theoretical values, indicating that the algorithm is capable of accurately reconstructing the unknown parameters without introducing numerical inaccuracies. This excellent agreement confirms the reliability of the Levenberg–Marquardt optimization technique and highlights its strong convergence characteristics when accurate thermal measurements are available. Furthermore, the obtained solutions validate the mathematical formulation of the inverse problem and demonstrate the stability of the computational implementation adopted in this study.

On the other hand, when the measured temperature data contain random errors with a magnitude of 0.01 Tmax, slight deviations between the estimated and exact parameters are observed. Nevertheless, these deviations remain relatively small, ranging from only 0.09% to 3.76%, which indicates that the estimation procedure preserves a high level of accuracy even in the presence of measurement noise and experimental uncertainties. Such performance clearly demonstrates the robustness and practical applicability of the Levenberg-Marquardt method for inverse heat transfer problems involving thermophysical parameter estimation. The results therefore confirm that the method not only provides highly accurate solutions under ideal conditions, but also maintains excellent predictive capability and numerical stability when realistic noisy experimental data are considered.

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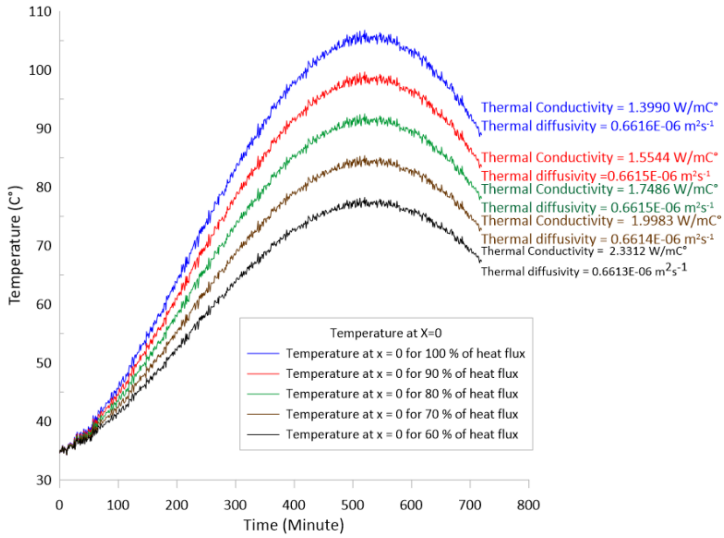

Fig. 7: temperatures at x=0

The results demonstrate that, by simultaneously analysing system performance and the influence of ambient temperature, we were able to identify the optimal features that reduced the cooling load by approximately 15% and thereby improved overall energy efficiency.

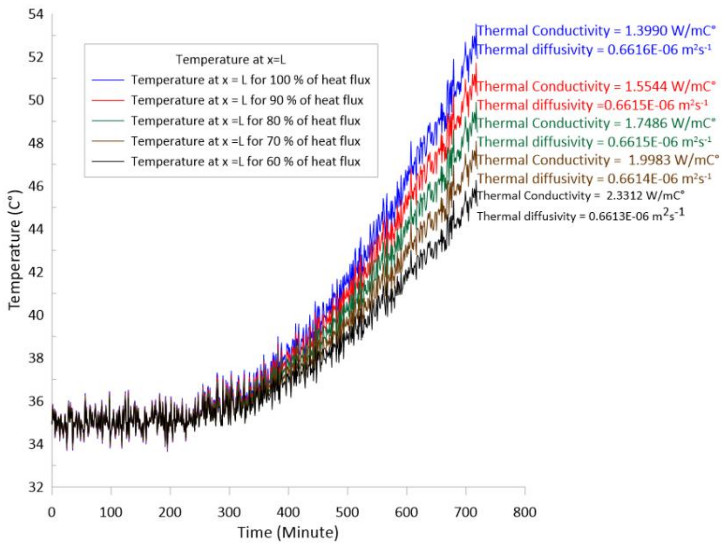

Fig. 8:temperature at x=L

The results indicate that, by concurrently evaluating system performance and the effect of ambient temperature, we were able to identify the most effective features, which simultaneously reduced the cooling load and enhanced overall energy efficiency.

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CONCLUSION

In this communication the method of Levenberg Marquardt is presented as a technique for the estimation of thermophysical properties in inverse heat conduction problems, the steps of these technique resumed in, first the sensitivity study which carried out to ensure the feasibility of the method indeed the optimization of the experience, like the location of sensors, then the temperature measurements are simulated by the solution of the direct problem and finally these measurement are used in the algorithm of the inverse method, the results obtained show that the estimated thermophysical properties are exacts if the exact temperature are used, however when the simulated temperatures are with random noise the estimated properties deviate from the exact solution, but still in a good precision.

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