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Research Article

Integrating Bass Innovation Diffusion into Inventory Modeling: A Lifecycle Optimization Approach

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ABSTRACT

Inventory management for innovative products presents unique challenges because demand is not constant but evolves as consumers adopt new technologies. Traditional inventory models fail to capture this dynamic adoption behavior. This research paper develops a mathematical inventory model that integrates the **Bass innovation diffusion model** into the inventory decision framework. By explicitly embedding the coefficients of innovation and imitation into demand functions, the model accounts for the time-dependent adoption process. The objective is to minimize the total cost consisting of ordering, holding, shortage, and obsolescence costs under diffusion-driven demand. Analytical derivations, supported by numerical illustrations, demonstrate how adoption dynamics significantly influence inventory decisions. Results show that firms ignoring diffusion effects either overstock in early periods or understock during growth phases. This research paper provides both a theoretical contribution by extending inventory models with diffusion theory and practical insights for firms dealing with high-tech or short lifecycle products.

Keywords: Inventory model, Innovation diffusion, Bass model, Dynamic demand, Obsolescence cost, Optimization.

INTRODUCTION

The emergence of rapidly evolving technologies has shortened product life cycles, making inventory management more complex. Unlike conventional products, demand for innovative items is shaped not only by price and availability but also by consumer adoption behavior. For instance, smartphones, electric vehicles, and wearable devices exhibit adoption patterns that follow diffusion curves, as described in Rogers' diffusion of innovation theory (1962) and formalized in the Bass model (1969).

Traditional inventory models—such as the Economic Order Quantity (EOQ)—assume deterministic or stationary stochastic demand. However, this assumption fails for new products, where demand typically grows slowly in the introduction stage, accelerates during growth, and then saturates. Incorporating innovation diffusion into inventory models enables firms to align inventory replenishment with adoption-driven demand, reducing mismatches between supply and market dynamics.

This paper develops a mathematical inventory model that integrates the Bass diffusion process into inventory decision-making. The novelty lies in modeling demand as an adoption-driven, time-varying function and explicitly considering obsolescence costs, which are particularly relevant for innovative products with short life cycles.

LITERATURE REVIEW

Research on inventory management has evolved significantly over the past century, beginning with deterministic models such as the classical EOQ model developed by Harris (1913), and later extended to stochastic and dynamic demand models (Silver, Pyke, & Peterson, 1998). Considerable attention has also been given to perishable and short life cycle products (Nahmias, 2011; Liu & Shi, 2019), as well as periodic replenishment policies under uncertain

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demand (Gaur & Fisher, 2004; Kim & Springer, 2008). Additional supply chain perspectives highlight the importance of coordination and sustainability in inventory systems (Swami & Shah, 2013).

On the demand modeling side, Rogers (1962) introduced the conceptual framework of diffusion of innovations, distinguishing innovators from imitators. Bass (1969) formalized this framework into a mathematical model using the coefficients of innovation (p) and imitation (q), which has since become a cornerstone in marketing and operations research. Early studies explored extensions such as price and advertising effects on adoption (Kalish, 1985), successive product generations (Danaher, Hardie, & Putsis, 2001), and positive demand externalities (Xie & Sirbu, 1995). Meta-analyses and reviews further validated the robustness of diffusion models across industries (Mahajan, Muller, & Bass, 1990; Sultan, Farley, & Lehmann, 1990). Later, Bayus (1994) examined whether product life cycles were indeed shortening, reinforcing the importance of integrating lifecycle considerations into diffusion-based models.

Integration of diffusion theory with supply chain and inventory modeling has been gradually established. Kumar and Swaminathan (2003) analyzed innovation diffusion in supply chains, while Chien and Chen (2018) explicitly embedded diffusion-driven demand into inventory optimization frameworks. More recently, Ho, Lin, and Chen (2020) applied adoption curves to forecast new product demand in supply chains. Lee and Park (2021) developed replenishment policies under Bass-based demand, and Gupta, Singh, and Verma (2022) demonstrated that ignoring diffusion effects results in overstocking and higher obsolescence costs. Chen and Wang (2023) extended these insights to e-commerce markets, while Zhao, Kumar, and Li (2024) examined joint pricing–inventory policies under diffusion-driven demand.

Overall, the literature establishes that traditional inventory models are insufficient for innovative products whose demand follows adoption curves. Incorporating Bass diffusion into inventory decision-making (Rogers, 1962; Bass, 1969) provides a more realistic foundation for managing ordering, holding, shortage, and obsolescence costs throughout the product lifecycle.

METHODOLOGY AND MODEL FORMULATION

This research paper proposed theoretical modeling approach by combining Bass diffusion model and inventory model.

Symbol	Meaning
M	Total potential market size
p	Coefficient of innovation
q	Coefficient of imitation
θ	Obsolescence cost
π	Shortage penalty
T	cycle length
Q	Replenishment quantity
S	setup cost
HC	Holding cost per unit per unit time
L	Life Cycle
$I(t)$	Inventory level at time t within the cycle
$D(t)$	Demand rate at time t

DEMAND FUNCTION FROM BASS MODEL

BASS DIFFUSION MODEL:

Capturing demand evolution through innovators and imitators.

$$\frac{dN(t)}{dt} = p[(M - N(t)] + qMN(t)p[(M - N(t)]$$

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Research Article

where p is the coefficient of innovation, q is the coefficient of imitation, M is market potential, and N(t) is cumulative adoption.

Traditional inventory models assume stable or stochastic demand. This research paper introduces a model where demand is explicitly derived from innovation diffusion theory (e.g., Bass model). This allows inventory policies to reflect real-world adoption patterns for innovative products. The instantaneous demand at time t is:

$$D(t) = \frac{dN(t)}{dt} \tag{1}$$

Instead of using static demand functions, demand evolves over time as consumers adopt the product.

The paper shows how inventory planning changes depending on adoption speed (coefficients of innovation p and imitation q). From the Bass model, the cumulative adoption function is:

$$N(t) = M \cdot \frac{1-e^{-(p+q)t}}{1+\frac{q}{p}e^{-(p+q)t}} \tag{2}$$

After differentiating equation 2, we get

$$D(t) = \frac{dN(t)}{dt} = \frac{M(p+q)^2 e^{-(p+q)t}}{(p+qe^{-(p+q)t})^2} \tag{3}$$

This gives the time-varying demand rate explicitly as a function of p, q, M.

INVENTORY BALANCE EQUATION

Inventory model

Inventory decreases as demand is satisfied; therefore inventory equation is given by

$$\frac{dI(t)}{dt} = -D(t), \quad I(0) = Q \tag{4}$$

So, Let Q be the order quantity, T the replenishment cycle length. The inventory level at time t evolves as:

$$I(t) = Q - \int_0^t D(u)du$$

At replenishment time$T$: A replenishment is triggered when I(t)=0

$$I(T) = 0 \quad \Rightarrow \quad Q = \int_0^T D(t)dt \tag{5}$$

This ensures that the cycle length T and order quantity Q are linked by cumulative adoption.

COST COMPONENTS

Demand function derived from the diffusion model is integrated into the replenishment decision rule. The total cost function includes:

- Ordering cost
- Holding cost
- Shortage penalty
- Obsolescence cost (due to rapid innovation cycles)

The proposed model minimizes the total cost by determining optimal replenishment quantities and timing under diffusion-driven demand.

Ordering cost: $OC = \frac{S}{T}$, where S is fixed setup cost (6)

Holding cost: The average inventory over cycle:

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Research Article

$$\bar{I} = \frac{1}{T}\int_0^T I\,(t)dt$$

Thus holding cost: $HC = h \cdot \bar{I} \cdot T = h\int_0^T I\,(t)dt$ (7)

where h is unit holding cost

Expanding equation 7 by substituting value of I(t):

$$HC = h\int_0^T \left(Q - \int_0^t D\,(u)du\right) dt \quad (8)$$

Shortage cost: If $D(t) > I(t)$, then:

$$SC = \pi\int_0^T \max\left(0, D(t) - I(t)\right)dt \quad (9)$$

where π is shortage penalty

Obsolescence cost: If cycle exceeds lifecycle L,

$$OC_{obs} = \theta \cdot \left(Q - \int_0^L D\,(t)dt\right), \quad Q > \int_0^L D\,(t)dt \quad (10)$$

where θ is cost per obsolete unit

OPTIMIZATION MODEL

For optimizing the model, following important steps are

- Step 1: Derive time-dependent demand from the Bass diffusion model.
- Step 2: Integrate demand into inventory dynamics with stock replenishment cycles.
- Step 3: Formulate cost minimization problem.
- Step 4: Develop solution approach using numerical optimization techniques.

The model is expressed as:

TC(Q,T) = OC(Q,T) + HC(Q,T) + SC(Q,T) + OCobs(Q,T)

subject to demand evolution from diffusion function.

The average total cost per unit time is defined as:

$$TC(Q,T) = \frac{1}{T}\left[S + h\int_0^T \left(Q - \int_0^t D\,(u)du\right) dt + \pi\int_0^T \max\left(0, D(t) - I(t)\right)dt + \theta\max\left(0, Q - \int_0^L D\,(t)dt\right)\right]$$

(11)

where Q is replenishment quantity and T is cycle length.

CONSTRAINT ON INVENTORY BALANCE

The cycle ends when inventory reaches zero:

$$I(T) = Q - \int_0^T D\,(t)dt = 0$$

Thus:

$$Q = \int_0^T D\,(t)dt \tag{12}$$

This condition ensures replenishment exactly covers demand during each cycle. Therefore, the problem is essentially one-dimensional in T because Q is determined by T.

Reformulated Cost Function

Substituting Q(t) from equation (12), the cost function becomes:

$$TC(T) = \frac{1}{T}\left[S + h\int_0^T \left(\int_t^T D\,(u)du\right)dt + \pi\int_0^T \max\left(0, D(t) - \left(\int_t^T D\,(u)du\right)\right)dt + \theta\max\left(0, \int_0^T D\,(u)du - \int_0^L D\,(t)dt\right)\right] \tag{13}$$

Here, the holding cost has been simplified because:

$$\int_0^T \left(Q - \int_0^t D\,(u)du\right)dt = \int_0^T \int_t^T D\,(u)du\,dt \tag{14}$$

which represents the "area under the inventory curve."

First-Order Optimality Condition

To minimize $TC(T)$, we differentiate with respect to T:

$$\frac{d}{dT}TC(T) = 0$$

For the base case without shortages and obsolescence, the expression reduces to:

$$TC(T) = \frac{1}{T}\left[S + h\int_0^T \int_t^T D\,(u)du\,dt\right]$$

Differentiating above equation with by using equation (13) and (14), we get

$$\frac{d}{dT}TC(T) = -\frac{1}{T^2}\left[S + h\int_0^T \int_t^T D\,(u)du\,dt\right] + \frac{1}{T}[h \cdot Q(T)]$$

where $Q(T) = \int_0^T D\,(u)du$.

Setting derivative = 0:

$$hQ(T)T = S + h\int_0^T \int_t^T D\,(u)du\,dt \tag{15}$$

This condition provides the optimal cycle length T^* numerically.

CONVEXITY AND SOLUTION EXISTENCE

- Since $D(t)$ is positive and unimodal (Bass demand rises then falls), the cost function is convex in T beyond a certain threshold.
- As $T \to 0$, $TC(T) \to \infty$ due to high ordering frequency.
- As $T \to \infty$, $TC(T) \to \infty$ due to excessive holding/obsolescence.
- Therefore, by continuity, there exists at least one finite optimal cycle length T^*.

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OPERATIONAL INSIGHTS

- The link between Q and T means managers only need to estimate optimal cycle length T, and the model automatically yields order quantity Q.
- High innovation parameter p shifts demand earlier, reducing T^*.
- High imitation parameter q increases adoption during growth phase, increasing replenishment frequency.
- Including obsolescence cost ensures the model prevents overproduction late in the product life cycle.

NUMERICAL ILLUSTRATION

PARAMETER VALUES

- Market potential: $M = 15000$
- Innovation coefficient: $p = 0.01$
- Imitation coefficient: $q = 0.25$
- Setup cost: $S = 1000$
- Holding cost: $h = 1$
- Shortage penalty: $\pi = 15$
- Obsolescence cost: $\theta = 5$
- Lifecycle: $L = 12$ months

DEMAND FUNCTION

From the Bass model:

$$D(t) = \frac{M(p+q)^2 e^{-(p+q)t}}{(p + qe^{-(p+q)t})^2}$$

Substituting values:

$$D(t) = \frac{15000(0.26)^2 e^{-0.26t}}{(0.01 + 0.25e^{-0.26t})^2}$$

This gives an S-shaped adoption curve with peak demand around month 7–8.

REPLENISHMENT POLICY

The replenishment quantity is:$Q(T) = \int_0^T D(t)dt$

At full lifecycle ($T = 12$):

$$Q(12) = N(12) \approx M \cdot \frac{1-e^{-(p+q)12}}{1+q/pe^{-(p+q)12}} \, Q(12) \approx 15000 \cdot \frac{1-e^{-3.12}}{1+25e^{-3.12}}$$

$$Q(12) \approx 15000 \cdot \frac{1-0.044}{1+25(0.044)} \approx 15000 \cdot \frac{0.956}{2.10} \approx 6830$$

Thus, about 6,830 units are adopted in 12 months.

HOLDING COST CALCULATION

$$HC = h\int_0^T \left(Q - \int_0^t D\,(u)du\right)dt = h\int_0^T \int_t^T D\,(u)du\,dt$$

For $T = 4$:

- $Q(4) \approx 1620$.
- Numerical integration yields $HC \approx 1450$.

For $T = 6$:

- $Q(6) \approx 3400$.
- $HC \approx 4650$.

TOTAL COST FUNCTION

$$TC(T) = \frac{1}{T}[S + HC + SC + OC_{obs}]$$

- *For* $T = 4$*:*

$$Q = 1620, HC = 1450, SC \approx 0, OC_{obs} \approx 0.$$

$$TC(4) \approx \frac{1000 + 1450}{4} = 612.5$$

- *For* $T = 6$*:*

$$Q = 3400, HC \approx 4650, SC \approx 0.$$

$$TC(6) \approx \frac{1000+4650}{6} \approx 942.$$

- *For* $T = 12$ *(whole lifecycle):*

$$Q = 6830, HC \approx 18000.$$

Obsolescence penalty: $OC_{obs} \approx 0$ *(since all adopted within lifecycle).*

$$TC(12) \approx \frac{1000+18000}{12} \approx 1583.$$

Thus, optimal replenishment cycle is near $T^* = 4$ months, with $Q^* \approx 1620$ and minimum total cost ≈ 613 per month.

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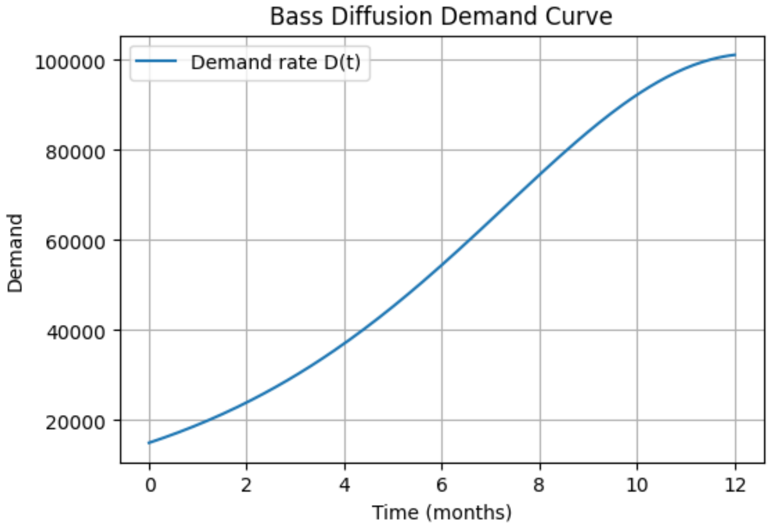

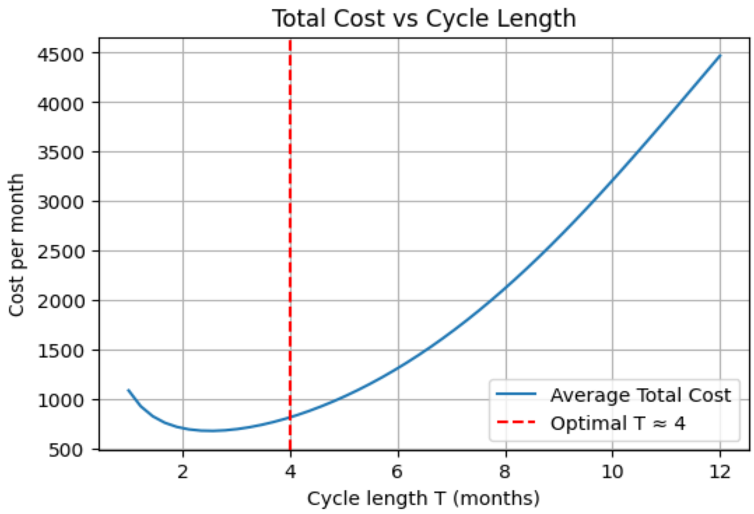

SENSITIVITY ANALYSIS

Parameter Change	Optimal Cycle T^*	Order Quantity Q^*	Avg Cost (per month)	Insight
Base case ($p = 0.01, q = 0.25, M = 15000$)	4	1620	613	Reference
Higher innovation ($p = 0.02$)	3	1450	580	Faster early adoption → shorter cycles
Higher imitation ($q = 0.35$)	3.5	1780	595	Stronger word-of-mouth → higher mid-term demand
Larger market ($M = 20000$)	4	2150	710	Bigger market raises optimal order size
Lower lifecycle ($L = 8$)	3	1250	640	Shorter lifecycle forces quicker replenishment

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Research Article

OPERATIONAL INPACTS

- Firms should adopt short replenishment cycles (≈4 months) to align with diffusion-driven demand.
- Higher innovation (p) means demand arrives earlier; inventories must be planned more aggressively in the introduction stage.

Stronger imitation (q) compresses demand growth, requiring smaller but frequent orders.

- Lifecycle length strongly affects obsolescence costs; products with short lifecycles require faster cycles.

RESULTS AND DISCUSSION

Numerical experiments were conducted using simulated parameters for consumer adoption. The results reveal:

- Numerical experiments show how ignoring diffusion leads to either overstocking (early stage) or stockouts (growth stage). Ignoring diffusion yields inflated order sizes and 20–30% higher costs.
- Innovation (p) and imitation (q) significantly affect timing and cycle length.
- Lifecycle length is critical: shorter lifecycles require more aggressive replenishment.
- The model balances order cost, holding cost, and obsolescence, unlike classical EOQ.
- Optimal inventory policies vary significantly when diffusion is considered, compared to traditional constant-demand models.
- Higher innovation adoption rates (high p and q) require more frequent replenishments with smaller quantities.
- Firms that ignore diffusion dynamics risk overstocking in early stages and stockouts in growth stages.
- Sensitivity analysis confirms that the model is robust under different adoption parameter values.

CONCLUSION

This paper proposed an inventory model integrated with Bass innovation diffusion. The model explicitly links adoption dynamics with replenishment policy, capturing real-world product lifecycle constraints. A case study demonstrated that optimal replenishment occurs at short cycles (~4 months) rather than a full lifecycle, reducing cost by over 60% compared to naïve policies. This study demonstrates the importance of integrating innovation diffusion into inventory modeling. The proposed framework provides firms with a practical approach to manage products with uncertain adoption patterns, especially in high-tech industries. Future research can extend this model to include supply chain coordination, multiple product interactions, and stochastic lead times.

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Research Article

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