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Modelling Innovation Diffusion Using Optimized Marketing Mix Strategies

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ABSTRACT

In this paper, we studied the primary limitation of the original Bass Diffusion Model by developing an extension of the Generalized Bass Model (GBM) that introduces key marketing mix variables into the model. Since innovation and imitation are not the only drivers of product adoption given the competitive and technologically advancing nature of markets, the study presents a time-varying marketing influence function as a linear combination of advertising expenditure, customer service quality, product performance, innovation capability, and extension of distribution. Built multiplicatively into the equation of diffusion, this function is designed to reflect the accelerating influence of marketing activities on adoption rates. The study also utilizes Linear Programming (LPP) to maximize marketing impact function under practical constraints, allowing for the computation of optimal resource allocation between marketing variables. This upgraded model is a marketing planning support tool, performance measurement tool, and resource allocation tool, especially in sectors such as smartphones, electric vehicles, and consumer electronics, enabling firms to determine high-impact areas for investments and develop effective marketing campaigns for maximizing product diffusion effect.

Keywords*:* Innovation diffusion, Technology adopting, Marketing Mix optimization, LPP.

1. INTRODUCTION:

The Bass Diffusion Model [2] has long served as a widely accepted framework for understanding how innovative products are adopted in the market. However, a key limitation of this model is its failure to incorporate essential marketing mix elements such as advertising, customer service, and distribution. Existing Bass Diffusion Models primarily focus on innovation and imitation effects, often neglecting crucial determinants like advertising intensity, customer service, product quality, technological advancements, and distribution coverage**.**

Over the past few years, several diffusion models have been proposed that include both social contagion and marketing mix variables. Such models are used not only to describe and predict how these two forces jointly drive new product sales but also to provide normative insights into how to manage advertising to maximize profits while considering social contagion. Dodson and Muller [4] extended the Bass model by introducing the Repeat Purchase Diffusion Model, which incorporated repeat purchases made by consumers. According to their model, the total market is divided into three segments:

- Individuals unaware of the product,
- Individuals aware but yet to purchase, and
- Adopters who have already purchased the product.

They identified advertising and word-of-mouth as two independent driving forces of adoption. Horsky and Simon [7] proposed a new approach by modelling advertising's impact as a conditional probability of adoption. Dockner, Feichtinger, and Sorger [3] extend the model of Horsky and Simon with price and again find that optimal advertising levels decrease over time. Using different model structures incorporating both price and advertising, Mahajan, Muller and Srivastava [13], Kalish [9], Jedidi, Eliashberg, and DeSarbo [8].

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The Generalized Bass Model (GBM)**,** proposed by Bass, Krishnan, and Jain [1], has been particularly popular in both descriptive and normative applications (Gupta [5], Krishnan & Jain [10], Vakratsas and Kolsarici [17]).

Mahajan and Peterson [11,12] proposed the mixed influence diffusion model and assume potential market that varies with certain variables. They proposed the following model

$$\frac{d\,n(t)}{dt} = \left(\int_{t_0}^{t} G(Z(y))\,dy - n(t) \right)[\,p + q\,\frac{n(t)}{N}]$$

Where n(t) is the cumulative number of adopters at time t, $\int_{t_0}^{t} G(Z(y))\,dy$ is a function of relevant variables affecting the potential market at time t and p, q are the parameters.

Hahn, Park, Krishnamurthi and Zeltner's [6] extend the model of mixed influence diffusion by allowing for non-durable products and incorporating promotional efforts into the model and consider that promotional efforts by the firm and its competitions affect external influence. They propose the following two sets of diffusion models for external influence:

$$\frac{d\,n(t)}{dt} = (N - n(t))\{[p + k\ln(\frac{A_E}{A_{EC} + A_E})] + q\,\frac{n(t)}{N}\}$$

$$\frac{d\,n(t)}{dt} = (N - n(t))[\,p + k\ln(\,A_E\,) + q\,\frac{n(t)}{N}]$$

Where A_E is the advertising efforts associated with the product in time t,$A_{EC}$is the advertising efforts associated with competitor products in time t and p and k are parameters to be estimated.

Parker and Gatignon [14] address the impact of marketing variables (price and advertising) on the diffusion process of competing brands. They propose the following response of functions for marketing variables :

External influence (p) = p(t)

$$=[\,P(t)^{a+bC_B(t)}\,A(t)^{c+d\,C_B\,(t)}\,]$$

Internal influence (q) = q(t)

$$=P_M\,(t)^{a+bC_B(t)}\,A_M\,(t)^{c+dC_B(t)}$$

Thus, they proposed the following diffusion model

$$\frac{d\,n(t)}{dt} = (N - n(t))\{[\,P(t)^{a+bC_B(t)}\,A(t)^{c+d\,C_B\,(t)}\,\} + \{P_M\,(t)^{a+bC_B(t)}\,A_M\,(t)^{c+dC_B(t)}\}\,\frac{n(t)}{N}$$

where $P_M\,(t) = \frac{P(t)}{\frac{\sum_{i=1}^{B(t)} P_i\,(t)}{B(t)}}$, $A_M\,(t) = \frac{A(t)}{\sum_{i=1}^{B(t)} A_i\,(t)}$,

P(t) is the price at time t ,A(t) is advertising at time t ,$C_B\,(t)$ is the number of competing brands in the product category at time t ,and a,b,c,d are constants The results show that brand is characterized by a different diffusion model . Although marketing variables are critical in the diffusion of brands , their impact is not identical across brands .Their results also show that the sensitivity can be insignificant ,increase or decrease over time ,depending on the order of entry

Robinson and Lakhani [16] incorporated the price effect into the model, showing that a rise in price negatively affects product sales. To capture this, they introduced an exponential decay term to represent price sensitivity:

$$\frac{d\,n(t)}{dt} = (N - n(t))[\,p + q\,\frac{n(t)}{N}]\,e^{-\lambda P(t)}$$

Where $P(t)$ is the price at time t and λ is the price coefficient. This simplistic model does not explain why or when purchase decision is made. This formulation captures the consumer's sensitivity to price over time.

The classical diffusion model to explain the diffusion of new products, but it did not include crucial marketing mix variables such as pricing strategies, advertising, distribution and promotion. This deficiency limits the model's utility in real world scenarios, where these variables significantly impact product success. Due to

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technology advancements and market dynamic product life cycle are rapidly shorting. Bass diffusion model (assume a stable diffusion pattern over time) fails to accommodate these changes. Thus there is a need for models that can incorporate control variables and reflect the evolving market landscape.

Bass, Krishnan, and Jain [1] identified the limitations of the traditional Bass Model and proposed the Generalized Bass Model (GBM) by introducing marketing effort as a multiplicative factor. The enhanced model assumes that marketing accelerates the rate of adoption. The equation is modified as:

$$\frac{d\,n(t)}{dt} = \left(N - n(t)\right)\left[\, p + q\frac{n(t)}{N}\right] x(t) \qquad \dots (1)$$

where $x(t)$ is a function of the marketing mix variable (advertising and price) in time period t

$$x(t) = 1 + \beta_1 \frac{[P(t) - P(t-1)]}{P(t-1)} + \beta_2 \max\{0, \frac{[A(t) - A(t-1)]}{A(t-1)}\}$$

β_1 = coefficient capturing the percentage increases in diffusion speed resulting from a 1% in decreases in price.

β_2 = coefficient capturing the percentage increases in diffusion speed resulting from 1% increases in advertising.

$P(t)$ = Price in period t.

$A(t)$ = advertising in period t.

In equation (1), we show that by increasing marketing effort, a firm can increase the likelihood of adoption of the innovation i.e. marketing effort speeds up the rate of diffusion of the innovation in the population. For implementing the model, we can measure marketing effort relative to a base level indexed to 1.0. Then if advertising at time t is double the base level, $x(t)$ will be equal to 2.0. Over the past decade, there has been extensive research on modified Bass models to better understand innovation diffusion and consumer adoption behaviour. Given the limitations of the traditional Bass Model, many scholars have proposed enhanced versions by incorporating various external and behavioural factors. In today's highly competitive and technologically advanced markets—especially in sectors like smartphones, electric vehicles, and consumer electronics—companies often maintain stable prices while relying on innovation and service improvements to attract customers. Present paper is an extension of the work of Bass, Krishnan, and Jain [1] and Peres,Muller and Mahajan[15] that combines several marketing mix variables to better represent innovation diffusion in competitive and technology-intensive markets.

In the present paper , a time-varying marketing influence function is proposed as a linear function of five key marketing factors: advertising spend, customer service quality, product performance, innovation capacity, and distribution extension. This function is added multiplicatively to the diffusion equation to accelerate adoption based on actual marketing actions. The goal is to optimize this marketing impact function under feasible constraints such as overall marketing expenditure, service quality thresholds, distribution capacity, and operational balance across activities. The problem is optimized using Linear Programming (LPP) methods, which allow us to calculate optimal allocations for each marketing variable without violating cost or operational boundaries. This model assists companies in identifying high-impact areas for strategic investment and crafting effective marketing campaigns that encourage product adoption. The model's applications include marketing planning, performance measurement, and resource allocation across industries like smartphones, electric cars, and consumer electronics. Ultimately, this model can serve as a decision-support tool for managers aiming to balance marketing investments and achieve maximum diffusion impact

2. MODEL CONSTRUCTION:

The Generalized Bass Model and its extension have been used widely to explain the diffusion of new products over time. Conventionally, these models are based on the premise that the marketing influence function is mainly a function of advertising and price, meaning that changes in price directly affect the consumer choice of adopting a product. Nevertheless, in the technologically advanced as well as highly competitive markets of the present day, this is no longer tenable. In product categories like smartphones, electric vehicles, smartwatches, and other consumer

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electronics, firms tend to use a fixed price policy across the life of a product. Rather than change prices, they tend to launch new and better technological variants of the product at regular intervals. The aim here is to maintain consumers' interest, establish brand positioning through stable pricing, and achieve competitive advantage. For instance, Apple and Tesla usually have a consistent price for one generation of product and depend on innovation-led upgrades to drive adoption. In such a market scenario, price ceases to be the overarching consideration determining customer choice—particularly when customers have already embraced the price level. Therefore, until a much better technological version is introduced, for the time t, consumer adoption is affected mainly by non-price characteristics, above all, advertising intensity and quality of customer care and post-sales support. Advertising is essentially responsible for creating brand awareness and interest among prospective consumers, while robust service infrastructure generates consumer confidence and satisfaction resulting in positive word-of-mouth and imitation-based adoption. Perceiving this change in behavior, the current research formulates a revised Generalized Bass Model (GBM), specifically designed for such contemporary market conditions.

In this paper we consider for the fixed price until the company launches a newer and more technologically advanced product. The marketing impact function x(t) depends advertising expenditure and market exposure $X_1(t)$ also customer service quality and after-sales support $X_2(t)$ product quality and performance $X_3(t)$, Innovation capability $X_4(t)$ and distribution reach and availability $X_5(t)$ at time t. A general marketing impact function at each time t is given by:

$$x(t) = f(X_1(t), X_2(t), X_3(t), X_4(t), X_5(t))$$

We proposed a model based on the generalized Bass model (GBM) and above marketing impact function is given by:

$$\frac{d\,n(t)}{dt} = \big(N - n(t)\big)\left[p + q\frac{n(t)}{N}\right] x(t) \text{ .... } (2)$$

Where

$$x(t) = 1 + \sum_{i=1}^{5} \gamma_i \max\{0, \frac{[X_i(t) - X_i(t-1)]}{X_i(t-1)}\} \qquad \text{...... . } (3)$$

where γ_i $\forall i = 1,2,3,4,5$ are coefficient capturing the percentage increases in diffusion speed resulting from a percentage in increasing in advertising, customer service, product quality, innovation capability and distribution reach availability respectively. When advertising ,customer service, product quality ,innovation capability and distribution reach availability change at more or less a constant rate over time ,these five factors would be a constant ,yielding x(t) to be some constant .In such a case , equation(3)would become observationally equal to the Bass model .

Let n(t) = f(t) N

$$\frac{d\,n(t)}{dt} = N\frac{d\,f(t)}{dt}$$

$$N\frac{d\,f(t)}{dt} = (N - f(t)N)\left[p + q\frac{f(t)\,N}{N}\right] x(t)$$

$$\frac{d\,f(t)}{dt} = (1 - f(t))[p + q\,f(t)]\,x(t)$$

$$\frac{d\,f(t)}{(1 - f(t))[p + q\,f(t)]} = x(t)\,dt$$

$$q\int \frac{df(t)}{p + q\,f(t)} + \int \frac{df(t)}{1 - f(t)} = (p + q)\int x(t)dt$$

$$q\int_0^t \frac{df(t)}{p + q\,f(t)} + \int_0^t \frac{df(t)}{1 - f(t)} = (p + q)\int_0^t x(t)dt$$

$$\{\log(p + q\,f(t)) - \log(1 - f(t))\}_{t=0}^{t=t} = (p + q)\int_0^t x(t)dt$$

$$\log\frac{(p + q\,f(t))}{(1 - f(t))} - \log\frac{(p + q\,f(0))}{(1 - f(0))} = (p + q)\int_0^t x(t)dt$$

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$$\frac{(p+q\,f(t))}{(1-f(t))} = \frac{(1-f(0))}{(p+q\,f(0))}\; e^{(p+q)\int_{t=0}^{t=t} x(t)dt}$$

$$f(t) = \frac{\frac{(p+qf(0)}{(1-f(0))}e^{(p+q)\int_{t=0}^{t=t} x(t)dt} - p}{q+\frac{p+qf(0)}{(1-f(0))}e^{(p+q)\int_{t=0}^{t=t} x(t)dt}}$$

$$n(t) = N\,\frac{\frac{(p+qf(0)}{(1-f(0))}e^{(p+q)\int_{t=0}^{t=t} x(t)dt} - p}{q+\frac{p+qf(0)}{(1-f(0))}e^{(p+q)\int_{t=0}^{t=t} x(t)dt}}$$

Initially if n(0) =0 , f(0) =0

$$f(t) = \frac{p\,e^{(p+q)\int_{t=0}^{t=t} x(t)dt} - p}{q+\;p\,e^{(p+q)\int_{t=0}^{t=t} x(t)dt}}$$

$$f(t) = \frac{e^{(p+q)\int_{t=0}^{t=t} x(t)dt} - 1}{\frac{q}{p}+\;e^{(p+q)\int_{t=0}^{t=t} x(t)dt}}$$

$$n(t) = N\,\frac{e^{(p+q)\int_{t=0}^{t=t} x(t)dt} - 1}{\frac{q}{p}+\;e^{(p+q)\int_{t=0}^{t=t} x(t)dt}}$$

$$x(t) = \begin{cases} = 1 & \text{, standard Bass model} \\ > 1 & ,\textit{for accelerated adoption} \\ < 1 & ,\textit{for slower adoption} \end{cases}$$

For an industry their goal is to maximize $x(t) > 1$ under a cost constraint for the optimum trajectories to achive the higest adoption influence without exceeding the total marketing budget.

Assume $\frac{[X_i(t)-X_i(t-1)]}{X_i(t-1)} > 0$

$$\Rightarrow \max\{0, \frac{[X_i(t)-X_i(t-1)]}{X_i(t-1)}\} = \frac{[X_i(t)-X_i(t-1)]}{X_i(t-1)}$$

$$x(t) = 1 + \textstyle\sum_{i=1}^{5} \gamma_i\left(\frac{X_i(t)}{X_i(t-1)} - 1\right)$$

$$= 1 - \sum_{i=1}^{5}\gamma_i + \sum_{i=1}^{5}\left(\frac{\gamma_i}{X_i(t-1)}X_i(t)\right\}$$

Let $a_i = \frac{\gamma_i}{X_i(t-1)}$, $c = (1 - \sum_{i=1}^{5}\gamma_i)$

$$x(t) = c + \sum_{i=1}^{5} a_i X_i(t)$$

It is the linear objective function with variable $X_i(t)$.

Now we formulate a LPP,

Find the values of $X_1(t)$,$X_2(t)$,$X_3(t)$,$X_4(t)$, $X_5(t)$ which maximize

$x(t) = c + \sum_{i=1}^{5} a_i X_i(t)$ (objective function)

Subject to constraints

$X_1(t) \geq a$.......(i)

$X_3(t) \geq \alpha X_2(t)\ \ldots\ldots\ldots.(ii)$

$X_4(t) \leq b\ \ldots\ldots.(iii)$

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$$X_4(t)+X_5(t) \leq c \dots \dots \dots \dots (iv)$$

$$X_1(t) + X_2(t)+X_3(t)+X_4(t) + X_5(t) \leq B \dots \dots \dots \dots . . (v)$$

And non- negative restriction

$$X_1(t), X_2(t), X_3(t), X_4(t) X_5(t) \geq 0$$

Where $a > 0$ is service mini, b> 0 is max distribution capacity, c> 0 is marketing cap, B is total budget.

Constraint (a) refers the customer service must meet a baseline standard to maintain customer loyalty and satisfaction. Constraint (ii) refers the product quality and innovation must move together. This ensures innovation doesn't fall behind product improvements. Constraint (iii) refers the distribution reach is limited due to logistic constraints, From Constraint (iv) we say that a maximum amount is allowed for total distribution and marketing efforts. From Constraint (v) we can say that he overall resource allocation across all five efforts is limited by a total budget B.

3.APPLICATION OF THE MODEL:

a) Strategic Investment Planning:
The model helps firms identify the operational areas that offer the highest returns on marginal investments so that resources can be better allocated. For example, investments could be directed toward customer service or innovation.

b) Marketing Optimization:
After measuring the impact of advertising and reach of distribution, the model helps firms design campaigns that are cost effective from the perspective of market penetration.

c) Development and Prioritization of Product Planning:
The model measures how changes in product quality and innovations affect performance and help firms prioritize R&D investments.

d) After sales and servicing efficiency:
Customer retention and satisfaction are represented via $X\_1(t)$, which could be optimized for increasing companies' goodwill and repeating purchase.

e) Scenario Analysis and Risk Management:
The firm can analyze certain market or internal scenarios, like cuts in the budget or an expansion plan, in order to weigh the possible outcome and hedge against the risk.

f) Policy Formation for Managers:
It is a decision-support application aiding operations and marketing managers in justification of budget allocation and performance KPIs. It is a decision-support application aiding operations and marketing managers in justification of budget allocation and performance KPIs.

4.CONCLUSION:

In this paper, A mathematical model of organizational performance optimization is proposed with the impact of key strategic variables—after-sales support, distribution reach, in terms of rate-of-change formulations and applying Linear Programming (LP) the research shows a framework for firms to achieve optimal functional efficacy given real-world constraints. Such a methodology equips decision-makers with the ability to measure the marginal value of individual functions over time, allowing for efficient resource allocation. The model enhances data-based decision-making, highlights the need for ongoing improvement, and is capable of incorporating real-world constraints like budget or minimum service requirements. Essentially, the present study stresses the ability of coupling mathematical modelling and managerial knowledge to optimize operations strategy and competitiveness. Future extensions might include dynamic constraints, nonlinear objectives, or uncertainty in parameter estimation, thus allowing the model to become more flexible with respect to fast-changing industrial environments. Based on the general organizational problem of having limited resources (e.g., budget, manpower, time), the model helps managers allocate these resources between different business functions (e.g., innovation, service, distribution) optimally to yield maximum performance results. Given that the real world has boundaries—budgets, minimum level of performance, operating limits—the research allows the definition of linear constraints representing such limits, while it still determines the optimal level of performance. The diversity of this model provides for its use in many industries, to maximize quality control, maintenance planning, and logistics networks. To balance wisely investment in innovation, network excellence, and customer service. To optimally allocate budgets between facilities, staff, and technological innovation. To invest wisely in delivery systems, customer care programs, and product lines. In addition, the model gives useful insights into the returns on investment across various departments, which enable companies to know potential gains or losses from the levels of investment adjustment. Going beyond optimization, the model can be further expanded for forecasting, which allows organizations to forecast the impact different investment strategies are likely to have on future performance. Prescribe action plans based on future trends and scenarios.

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