

Integrating Acceptance Prediction, Assignment, and Pricing Optimization to Maximize the Expected Revenue of Ride-Hailing Transactions

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ABSTRACT

The ride-hailing platform mediates transactions between driver and passenger through an assignment algorithm. However, more than 40% of assignments are rejected either by drivers or passengers, leading to inefficient system performance. Predicting which assignment would be accepted becomes necessary to avoid such inefficiency. Big data technology and the availability of ride-hailing daily transactional data can enable platforms to dive into driver's and passenger's behavior and determine factors influencing their acceptance. This paper aims to incorporate prediction ability into an assignment optimization model. We perform numerical experiments and conclude that the proposed model could provide behavior-custom assignment, thus ensuring a high acceptance rate by drivers and passengers. Unfortunately, We found that the global optimal solution for the model can only be found for small-sized problems. When dealing with large-sized problems, standardized optimization software can only provide local optimal or no feasible solution. We designed an assignment algorithm based on particle swarm optimization to improve the solution-finding process. The algorithm can achieve an average of 1.78% error for small-sized problems, up to 6.47% better solution compared to local optimal solutions found by standard optimization software, and it can give feasible solutions when standard optimization software fails to provide them.

Keywords: Big Data, Tailored Assignment, Predict Then Optimize, Logistic Regression, Particle Swarm Optimization.

1. INTRODUCTION

The rapid development of online ride-hailing (ORH) platforms has globally revolutionized our transportation systems. Over the past decade, ride-hailing platforms like Uber in the USA, Didi Chuxing in China, and Grab and Gojek in Southeast Asia have successfully transformed how people travel [1] [2]. Despite regulatory challenges [3] [4] and incumbent resistance [5], ORH strives to provide easy commuting for passengers and job opportunities for drivers. Researchers found that the adoption would continue to grow [6] [7], and the business is projected to reach a market size worth \$285bn by 2030 [1]. A recent study from Indonesia, a country with more than 270 million people, revealed that 1 out of 4 citizens depend on ride-hailing for their daily transportation and business [8].

ORH mediates passengers who need transportation services and drivers who provide the service. The mediation is done through the ORH digital application, wherein lies an optimization engine capable of assigning drivers to passengers and determining the price for each trip. Assignment and pricing algorithms ensure effective and efficient ORH operational management. For these reasons, literature regarding assignment and pricing has expanded rapidly in recent years. Some specifically addressed the assignment problem [9] [10] [11] [12] [13], pricing problem [14] [15] [16] [17] or integration of assignment and pricing [18] [19]. A more comprehensive comparison will be given in the next section. Most research concerning ORH assignment and pricing assumes drivers and passengers always accept the assignment result. This assumption is not necessarily valid in the ORH business context, where drivers and

passengers can independently accept or reject an assignment.

Empirical data shows that driver's rejection rate reached 41% in New York in 2014, while Didi Chuxing reported that more than 40% of ride-hailing requests in Shanghai received no response from drivers in 2017 [20]. Assignment rejection by drivers causes higher passenger waiting time and dissatisfaction, thus leading to system inefficiency [21]. Conversely, it is common to find passengers rejecting assignments due to high waiting times or concerns about a driver's low rating score. This rejection lowers the availability of drivers in other areas and causes drivers to ride a wild goose chase [22]. Rejection from either driver or passenger leads to a negative impact; thus, it must be avoided and considered when deciding on an assignment. The appearance of big data and machine learning technology, along with the availability of ORH's daily historical data, can help ORH gain visibility into the behavior of drivers and passengers during transactions, including revealing factors affecting their decisions. Knowing these factors would help predict whether drivers and passengers would accept or reject a particular assignment [23] [24]. This capability allows researchers to consider acceptance prediction, assignment, and pricing in an integrated manner.

Research [25] proposed a mathematical model for assigning drivers to passengers and determining the optimal payment for drivers to enforce assignment acceptance. Even though their research is one of the first to integrate acceptance prediction, assignment, and payment, they only focused on the drivers and assumed perfect passenger acceptance, which is still unrealistic. Moreover, they applied a sequential approach when trying to find the solution, meaning that the assignment is decided first and payment second, leading to a suboptimal solution. Through this paper, we extend the work of [25] by incorporating the passenger's acceptance into the assignment and pricing decision, thus bringing the model into a more realistic situation. Furthermore, we propose a simultaneous approach for finding the solution [25].

The contribution of this paper is as follows: we develop a mathematical model for simultaneous assignment and pricing, integrate a prediction and optimization model for tailored offerings for drivers and passengers, provide numerical experiments, and analyze the model applicability regarding solution-finding efficiency; finally, we propose a solution searching algorithm based on particle swarm optimization to increase the efficiency of solution-finding process. This paper is divided into several sections. They are introduction, literature review, model development, numerical experiment and analysis, conclusions, and further research exploration.

2. LITERATURE REVIEW

In recent years, literature concerning ORH services has been numerous. Although there are many research topics surrounding ORH services, we limit the discussion in this section to the literature regarding assignment and pricing mechanisms. Research concerning the ORH assignment algorithm can be divided into three research groups, which differ in their assignment mechanisms. The first group proposed an instant assignment, where any incoming passenger's request is instantly matched to a nearby driver [9] [10]. All platforms used this assignment mechanism early in ORH appearance. Instant assignment provides the timeliest response, yet a greedy approach that only considers immediate benefit.

The second research group proposed batched assignment or delayed matching, where passenger's requests are accumulated for a specific interval (usually seconds) and then get matched together to drivers at the end of the interval, which allows more extensive exploration for better matching quality [11] [12] [13] [26]. When dealing with the batched assignment, the main challenge is determining the optimal length of the batching interval [27]. Implementing batched assignments has been reported to bring higher efficiency to ORH operational management than instant assignments. The third research group aims for further benefit by considering the future while making matching decisions in the current batched interval through forward-looking batched assignment [28] [29] [30] [31] [32] [33] [34] [35] [36] [37] [38].

In each research concerning the assignment mentioned earlier, there was an underlying assumption that drivers and passengers always accept the assignment result, which is not necessarily valid. In the ORH business context, the driver and passenger can accept or reject the assignment. Determining assignment decisions without considering the driver's and passenger's acceptance would lead to system inefficiency.

Research by [19] modeled how pricing can affect the driver's and passenger's decisions on accepting or rejecting an

assignment, specifically how trip price affects their pick-up distance tolerance limit. For the driver, the higher trip price allows a higher pick-up tolerance limit, while the contrary applies to the passenger. Their proposed pricing strategy is called the per-service pricing strategy, marking the integration of assignment and pricing decisions. Following the work of [25], [39] suggested simultaneous assignment and pricing, which considers the behavior heterogeneity among drivers and passengers and the platform's multi-objective target, allowing personalized assignment and pricing. However, the research failed to consider that the driver's and passenger's acceptance decision might not be perfectly influenceable and did not measure how likely drivers and passengers would accept the assignment given. Other researchers consider pricing to be a separate decision from assignment. Most of them deal with the problem of using price to balance the number of drivers and passengers at a particular time [14] [15] [16] [17] [40] [41] [42] [43] or area [44] [45] [46] [47] and do not explicitly consider pricing in the context of assignment acceptance decision-making.

Seeing the importance of ensuring assignment acceptance, researchers attempt to uncover more comprehensive factors affecting the decision. To do this, [21] and [48] conducted surveys involving drivers, while [20], [49], and [50] analyzed historical data gathered from ride-hailing platforms to build explanatory models for driver's decisions. On the passenger side, [22] built behavior modeling of order cancellation using a two-month hourly average dataset provided by Didi Chuxing. Combining the knowledge regarding driver's and passenger's decisions, [23] proposed a matching success rate (MSR) prediction model using a multi-viewed model, while [24] built a deep learning model to predict the probability of order cancellation for Didi Chengdu. The research opens further potential exploration to integrate acceptance prediction while doing assignments, which was then pursued by [26], which proposed per-service payment for drivers to ensure assignment acceptance. However, they only focus on the drivers and assume perfect acceptance by passengers, which leads to unrealistic simplification. Furthermore, they took a sequential approach to find the solution, meaning that the assignment was determined first, and payment was second. This sequential approach would lead to a sub-optimal solution. Table 1 summarizes the classification of research explained in this section. Our paper joins the per-service pricing research category introduced by [19], which integrates assignment and pricing decisions and extends the work of [25] and [39]. We consider batched assignment, per service pricing, and modeled the situation where driver's and passenger acceptance is partially influenceable.

Table 1. Classification of assignment and pricing research

Aspect Considered		Reference Number
Assignment	<i>Instant</i>	[9][10][19]
	<i>Single Batched</i>	[11] [12] [13] [25] [26]
	<i>Forward-Looking</i>	[28][29] [30] [31] [32] [33] [34] [35] [36] [37] [38]
Pricing	<i>Temporal</i>	[14] [15] [16] [17] [40] [41] [42] [43]
	<i>Spatial</i>	[44] [45] [46] [47]
	<i>Per service</i>	[19] [25] [39]
Driver's acceptance modeling	<i>Always accept</i>	[9][10] [11] [12] [13] [26] [27] [28][29] [30] [31] [32] [33] [34] [35] [36] [37] [38]
	<i>Completely influenceable</i>	[14] [15] [16] [17] [19] [40] [41] [42] [43] [44] [45] [46] [47]
	<i>Partially influenceable</i>	[25]
Passenger's acceptance modeling	<i>Always accept</i>	[9][10] [11] [12] [13] [16] [25] [26] [27] [28] [30] [31] [32] [33] [34] [35] [36] [37] [38] [40] [47]
	<i>Constant probabilistic acceptance</i>	[29]
	<i>Completely influenceable</i>	[14] [15] [19] [41] [42] [43] [44] [45] [46]
	<i>Partially influenceable</i>	
Solution finding technique	<i>Exact</i>	[9] [14] [15] [16] [19] [25] [28] [29] [31] [32] [40] [41] [42] [43] [44] [45] [46] [47]
	<i>Heuristic</i>	[11] [12] [13] [26]

Aspect Considered	Reference Number
Metaheuristic	[30]
Machine Learning	[27] [30] [33] [34] [36] [38]
Simulation	[11]

3. METHOD

3.1 Problem Description

We consider the situation where the ORH platform works to mediate transactions between drivers and passengers by accumulating and assigning drivers to passengers and determining the trip price for each assignment. The ORH implemented a batched assignment. This mechanism divides their operational time into smaller intervals with equivalent lengths. Fig.1 illustrates the batched assignment mechanism for a single interval. Fig.1 shows a situation where ORH operates for a particular operational time (H). The operational time is then divided into smaller intervals (h), where $h = 1, 2, \dots, H$; each interval starts at b_h^s and ends at b_h^e . Suppose that at the end of a particular batching interval (b_h^e) there are I idle drivers, and J passenger's requests accumulated. Drivers are denoted by the d_i ($i = 1, 2, \dots, I$).

Their original location is visible through the ride-hailing app and denoted by $(x_{d_i}^o, y_{d_i}^o)$. On the other side, passengers are denoted by the p_j ($j = 1, 2, \dots, J$). As they open the ride-hailing app, they would input their pick-up $(x_{p_j}^o, y_{p_j}^o)$ and destination point $(x_{p_j}^d, y_{p_j}^d)$ and the platform would log their request's submission time, which is t_{p_j} . The task of the platform is to determine the assignment of driver- i to passenger- j , which is denoted by X_{d_i, p_j} , as well as the trip fare associated with the assignment (F_{d_i, p_j}).

The basic assumption proposed in this paper is that both driver and passenger do not always accept the assignment results. Suppose that driver- i acceptance probability when matched to passenger- j is denoted by P'_{d_i, p_j} , while passenger- j acceptance probability when matched to driver- i is denoted by P''_{d_i, p_j} . Given that the driver and passenger independently choose their decision, the probability of both driver and passenger accepting the assignment (P_{d_i, p_j}) is the product between P'_{d_i, p_j} and P''_{d_i, p_j} . Throughout its operational activities, the ORH platform will have significant historical data that can be used to predict P'_{d_i, p_j} and P''_{d_i, p_j} . In this paper, we assume the platform uses a logistic regression-based machine learning model that can predict assignment acceptance probability by the driver and passenger using multiple predictors. The logistic regression model also appeared in [23] and [24].

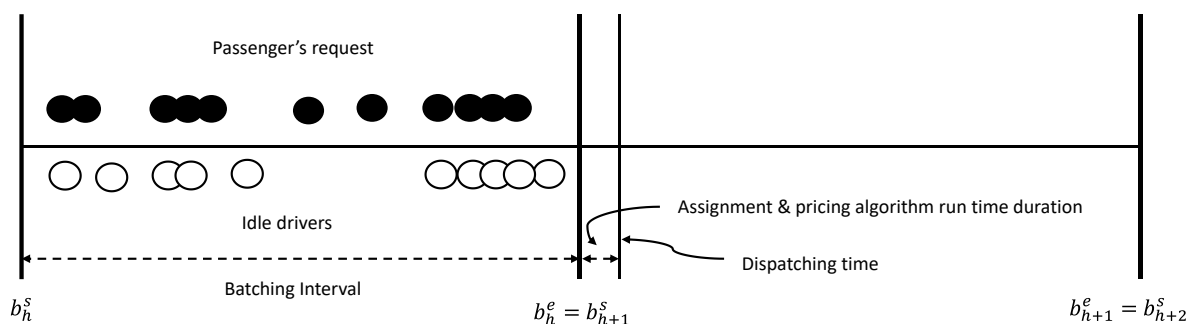


Figure 1. Illustration of Batch Assignment

The factors and coefficient used to predict are uncovered through historical data. In an operational context, ORH could customize the number of prediction factors by choosing the most significant among available factors. This paper illustrates a situation where three factors are used in the logistic regression model to predict the acceptance probability of both drivers and passengers. For drivers, the factors are driver's profit percentage when assigned to passenger- j (A_{d_i, p_j}), the traffic conditions at the pick-up point of passenger- j (B_{p_j}), and the traffic to the destination

point of passenger- j (Γ_{pj}). Meanwhile, for passengers, the factors used are the waiting time that passenger- j would have to bear when assigned to driver- i (Π_{d_i,p_j}), the trip fare per kilometer they must pay (f_{d_i,p_j}), and the rating of driver- i (Λ_{d_i}).

These factors are selected based on their frequent appearance among many research on the field [20] [21] [48] [49] [50]. The ORH must determine two decision variables: the assignment pair (which driver serves a particular passenger) and the trip fare per kilometer. These decisions are determined to maximize the expected revenue at a single batching interval.

3.2 Mathematical Model

We proposed an optimization model to determine driver-to-passenger assignment and trip price for each assignment. Logistic regression prediction models are embedded as optimization constraints inside the optimization model. Therefore, simultaneous combinatorial optimization would be performed. The quality of all possible driver and passenger pairs and their pricing decision would be evaluated, and finally, the optimal assignment and pricing would be selected. In this setting, our approach differs from the one that sequentially optimizes the assignment decision and proceeds to payment determination. The complete formulation of the assignment and pricing optimization model is given below.

Objective Function

$$\text{Max } Z = \sum_{i=1}^I \sum_{j=1}^J P_{d_i,p_j} X_{d_i,p_j} F_{d_i,p_j} \quad (1)$$

Subject to

$$P_{d_i,p_j} = P'_{d_i,p_j} \cdot P''_{d_i,p_j}, \forall i, j \quad (2)$$

$$P'_{d_i,p_j} = \frac{1}{1+e^{-(\alpha_{d_i} A_{d_i,p_j} + \beta_{d_i} B_{p_j} + \gamma_{d_i} \Gamma_{p_j})}}, \forall i, j \quad (3)$$

$$P''_{d_i,p_j} = \frac{1}{1+e^{-(\pi_{p_j} \Pi_{d_i,p_j} + \epsilon_{p_j} f_{d_i,p_j} + \lambda_{p_j} \Lambda_{d_i})}}, \forall i, j \quad (4)$$

$$A_{d_i,p_j} = \frac{\theta F_{d_i,p_j} - C_{d_i,p_j}}{\theta F_{d_i,p_j}}, \forall i, j \quad (5)$$

$$\theta F_{d_i,p_j} > C_{d_i,p_j} - (1 - X_{d_i,p_j}) M, \forall i, j \quad (6)$$

$$F_{d_i,p_j} = f_{d_i,p_j} (|x_{p_j}^O - x_{p_j}^D| + |y_{p_j}^O - y_{p_j}^D|), \forall i, j \quad (7)$$

$$C_{d_i,p_j} = c \left((|x_{d_i}^O - x_{p_j}^O| + |y_{d_i}^O - y_{p_j}^O|) + (|x_{p_j}^O - x_{p_j}^D| + |y_{p_j}^O - y_{p_j}^D|) \right), \forall i, j \quad (8)$$

$$\Pi_{d_i,p_j} = \frac{(|x_{d_i}^O - x_{p_j}^O| + |y_{d_i}^O - y_{p_j}^O|)}{v_{d_i}}, \forall i, j \quad (9)$$

$$f^{\min} < f_{d_i,p_j} < f^{\max}, \forall i, j \quad (10)$$

$$X_{d_i,p_j} = \begin{cases} X_{d_i,p_j}, & P_{d_i,p_j} > e \\ 0, & P_{d_i,p_j} \leq e \end{cases}, \forall i, j \quad (11)$$

$$\sum_{j=1}^J X_{d_i,p_j} \leq 1, \forall i \quad (12)$$

$$\sum_{i=1}^I X_{d_i,p_j} \leq 1, \forall j \quad (13)$$

$$X_{d_i,p_j} = \{0, 1\}, \forall i, j \quad (14)$$

Equation (1) defines the objective function of the optimization model, which is to maximize the total expected revenue at a single batching interval (Z). The expected revenue is calculated as the total product of the driver's and passenger's joint acceptance probability, the assignment decision of drivers to passengers, and the associated trip fare. Equation (2) defines the driver's and passenger's joint acceptance probability calculation. It is the product of the driver's and the passenger's acceptance probability. The driver's acceptance probability is defined in Eq. (3). It is calculated by taking logistic regression equations of three factors: the driver's profit percentage when assigned to passenger- j (A_{d_i,p_j}), the traffic conditions at the pick-up point of passenger- j (B_{p_j}), and the traffic to the destination point of passenger- j (Γ_{p_j}). The factor's coefficient (α_{d_i} , β_{d_i} , and γ_{d_i}) determines the magnitude of each factor's effect on the acceptance probability.

The higher the profit percentage that driver- i earned when assigned to passenger- j , the lighter the traffic condition both on pick up and destination point would result in higher assignment acceptance probability by the driver. While the driver's profit percentage can be calculated, the traffic condition at the pick-up and destination is pulled through the Google map API readily available in the ride-hailing app. Equation (4) shows the calculation of the passenger's acceptance probability. As for drivers, we also consider three factors to predict passenger ride acceptance probability. The factors are the waiting time that passenger- j would have to bear when assigned to driver- i (Π_{d_i,p_j}), the trip fare per kilometer they must pay (f_{d_i,p_j}), and the rating of driver- i (Λ_{d_i}). The coefficient for each predictor is denoted by π_{p_j} , ε_{p_j} , and λ_{p_j} .

The assignment acceptance probability by the passenger would be high if the waiting time is short, the trip price is low, and the driver's rating is high. Equation (5) defines the driver's profit percentage calculation. Generally, ORH applies fixed revenue sharing with the driver. If the driver's earning share from each assignment is denoted by θ ($0 < \theta < 100\%$), then for each assignment, the driver would get $\theta F_{d_i,p_j}$ as their revenue. The driver's profit is their revenue minus their operating cost when serving passenger- j (C_{d_i,p_j}). The profit should always be greater than zero to maintain economic feasibility (See Eq. (6)). The trip fare (F_{ij}) and the driver's operating cost while serving passenger- j (C_{d_i,p_j}) is calculated through Eq. (7) and Eq. (8). The driver's cost is primarily determined by the ride distance, which covers the pick-up and destination trips. Equation (9) defines the passenger's waiting time, which depends on the pick-up distance and the speed of the driver's vehicle (v_{d_i}).

Equation (10)-(14) constrained the value of the decision variables. Equation (10) states that fares per kilometer should fall between the minimum and maximum value regulated by the government. Equation (11) defines the relation between the value of joint acceptance probability and the assignment decision, where the assignment of driver- i to passenger- j would not be made if the joint acceptance probability is lower or equal to a specific threshold value (e). Equations (12) and (13) limit the assignment solution to a one-to-one assignment, and Eq. (14) defines the binary nature of the assignment decision. Compared to [25], we can conclude that through this assignment and pricing model, we do not necessarily minimize the distance or waiting time of the driver and passenger pair but look into their behavior and determine the optimal distance and price based on that behavior.

4. RESULT AND DISCUSSION

4.1 Model Validation

To validate whether the proposed model has worked as intended, we presented a small case depicting the assignment of 2-drivers and 2-passengers (Case 1). The following parameters were used: $f^{min} = 2$, $f^{maks} = 5$ (both in thousands Indonesian Rupiah), $c = 0.7$; $\theta = 0.8$; $e = 0.5$ and $M = 1000$. Data related to the passengers and drivers are given in Table 1 and Table 2.

Table 2. Parameter values for Drivers on Case 1

i	$x_{d_i}^0$	$y_{d_i}^0$	Λ_{d_i}	α_{d_i}	β_{d_i}	γ_{d_i}
1	1	0	5	10	-1	-1

2	1	0	5	15	-1	-1
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Table 3. Parameter values for Passenger on Case 1

j	$x_{p_j}^O$	$y_{p_j}^O$	B_{p_j}	$x_{p_j}^D$	$y_{p_j}^D$	Γ_{p_j}	π_{p_j}	ε_{p_j}	λ_{p_j}
1	0	0	1	2	5	1	-2	-4	3
2	0	0	1	2	5	1	-2	-6	3

Table 1 shows two drivers with different behavior toward profit percentage, shown by α_{d_1} and α_{d_2} . Driver-2 is more sensitive to the change in profit percentage, which means that the acceptance probability would be higher for driver-2 compared to driver-1 for the same amount of improvement in profit percentage. On the other side, there are two passengers, both with different behavior towards the trip price, shown by the value of ε_{p_1} and ε_{p_2} (see Table 2). With this value, the acceptance probability of Passenger-2 would be lower than Passenger-1, given the same trip price. Except for their behavior, the drivers or passengers can be seen as identical. A list of possible solutions for Case 1 is given in Table 3, which profiled all possible trip fares per kilometer and joint probabilities when the assignment is made for a particular pair of driver and passenger (d_i, p_j)

Table 4. Possible assignment on Case 1

Driver- i / Passenger- j	p_1	p_2
d_1	$f_{d_1,p_1} = 5$	$f_{d_1,p_2} = 3.875$
	$P'_{d_1,p_1} = 0.880$	$P'_{d_1,p_2} = 0.805$
	$P''_{d_1,p_1} = 0.975$	$P''_{d_1,p_2} = 0.603$
	$P'''_{d_1,p_1} = 0.858$	$P'''_{d_1,p_2} = 0.485$
	$Z_{d_1,p_1} = 30.06$	$Z_{d_1,p_2} = 17.23$
d_2	$f_{d_2,p_1} = 5$	$f_{d_2,p_2} = 3.454$
	$P'_{d_2,p_1} = 0.998$	$P'_{d_2,p_2} = 0.991$
	$P''_{d_2,p_1} = 0.975$	$P''_{d_2,p_2} = 0.950$
	$P'''_{d_2,p_1} = 0.973$	$P'''_{d_2,p_2} = 0.941$
	$Z_{d_2,p_1} = 34.03$	$Z_{d_2,p_2} = 22.75$

Given the list in Table 3, we can see that the optimal solution to maximize the total expected revenue is reached when we assign driver-1 to passenger-1 and driver-2 to passenger-2 ($X_{11}=1, X_{12} = 0, X_{21} = 0, X_{22} = 1$) resulting in total expected revenue of 52.810 (thousand Rupiahs). Selecting another feasible pair would lead to a lower objective value. Assigning driver-1 to passenger-1 allows us to set the trip price per kilometer at its highest value and still have a high acceptance probability by both parties; this is not seen when assigning driver-2 to passenger-2 due to the passenger-2 sensitivity towards the trip price. The key decision variable impacting the assignment acceptance by both driver and passenger is the trip fare per kilometer. Fig. 2(a) and 2(b) represent the drivers' and passengers' acceptance probability given various trip fares per kilometer. The driver's acceptance probability increases along with the increment of the trip price, while the passenger's acceptance probability shows a decreasing pattern. Fig. 2(c) represents driver and passenger joint probability when being assigned to each possible pair, and Fig. 2(d) gives the potential revenue gained for each possible assignment pair. The optimal assignment is determined by selecting pairs of drivers and passengers that maximize the expected revenue. Fig. 2(d) shows that the objective function expected revenue for the platform is convex, and there is a single optimal solution for the problem described in Case-1.

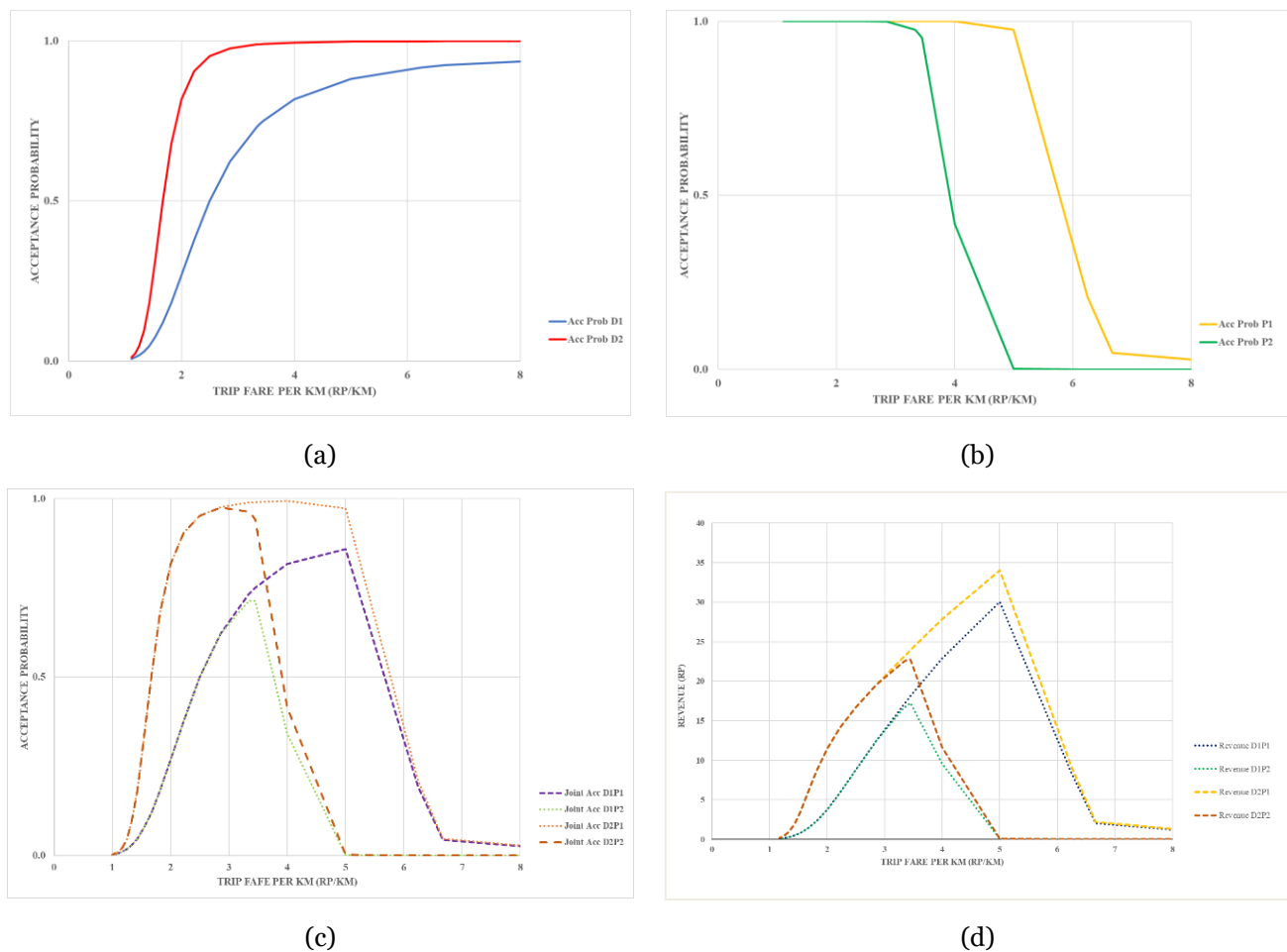


Figure 2. Graphical representation of driver's acceptance probability (a), Passenger's acceptance probability (b), Driver's and Passenger's Joint Probability (c), and Expected revenue for each potential assignment (d), all plotted against trip fare per kilometer as the decision variable.

Research by [25] solved the problem by applying two stages of decisions. In the first stage, they determine the assignment decision based on the target of minimizing the pick-up distance. Later, in the second stage, they determine the payment for the driver given the assignment. Looking at the driver and passenger profile in Case 1, we can see two possible sets of solutions available. The first set is assigning driver-1 to passenger-1, and driver-2 to passenger-2 ($X_{11}=1, X_{12}=0, X_{21}=0, X_{22}=1$) and the second set is assigning driver-1 to passenger-2 and driver-2 to passenger-1 ($X_{11}=0, X_{12}=1, X_{21}=1, X_{22}=0$). When being compared, both sets of solutions lead to similar pick-up distances. Applying the same approach as [25] would lead us to choose one of the sets arbitrarily. Choosing the first set would result in the same expected revenue as we proposed, but choosing the second set would result in lower expected revenue. Applying this approach means that for the situation profiled in Case 1, there is a 50% chance we would arrive at non-optimal expected revenue. Taking a simultaneous approach, as we proposed in this paper, would guarantee that each feasible assignment is evaluated regarding driver's and passenger's acceptance, which would finally lead to higher expected revenue. Using standardized optimization software, we experimented with another 250 cases of two-driver to two-passenger assignment (2D-2P), resulting in similar patterns.

4.2 Finding Solutions for a Large Problem Size

We provide a larger experiment to learn about the characteristics of the solution-finding process when the problem size for the assignment is varied. For each problem size, multiple cases are tested. The data we used here is randomly generated within practical limits. We defined a cutoff time for the solution-searching process, which is 1200 seconds. Each time we find a globally optimal solution within the cutoff time, we log the objective function value and the

computation time to reach that solution. However, if no global solution was found after the cutoff time limit was reached, we stopped the computation and logged whatever solution was found. Several cases lead to local optimal solutions, while other cases lead to only feasible solutions or no feasible solutions at all. As mentioned, all experiments are conducted using a standardized optimization (SOS) and run on Intel(R) Core (TM) i5-8250U CPU @ 1.60GHz 1.80 GHz computer hardware. Table 3 shows the results of the experiment.

Table 5. Solution optimality status and average computation time for various problem sizes using standardized optimization software (SOS)

Problem size	Number of cases experimented	Optimality status of the solution found (percentage)		Average computation time to reach global optimal (second)
		Global optimal	Local optimal	
2D-2P	250	100%	0	0.54
3D-3P	90	75.56%	24.44%	15.67
4D-4P	54	53.57%	46.43%	123.16
5D-5P	36	38.89%	61.11%	1156.00
6D-6P	20	0	100%	N/A
7D-7P	44	0	100%	N/A
8D-8P	16	0	0	N/A
...	N/A
25D-25P	10	0	0	N/A

The first column in Table 3 identifies the size of the assignment problem or, specifically, the number of drivers and passengers involved in the assignment. We arbitrarily select a balanced number of drivers and passengers, but the model can also handle imbalance cases. The second column identifies the number of cases experimented for a particular problem size. The third column identifies the optimality status of the solution found for each case. As we explained earlier, we discern the percentage of cases that could reach global optimal and those that could only reach local optimal. The fourth column identifies the average time it takes to reach a global optimal solution.

The data compiled in Table 3 shows that only in 2D-2P cases can the global optimal solution be 100% found. For cases in the problem size of 3D-3P up to 5D-5P, a globally optimal solution could only be reached for some parts of the cases, while for all cases experimented in the problem size of 6D-6P up to 7D-7P, no feasible solution is found. These are all logged within the 1200-second cutoff time limit for computational time. This information shows that with the increase in problem size, finding the global optimal solution for the model is not easy. Standard optimization software can only efficiently find globally optimal solutions for small-size problems. Larger problem sizes require a very long computational time (more than 1200s, which is not feasible for the ORH operational framework).

4.3 Reaching Efficient Computational Time

As pointed out in the previous sub-section, other means to improve the efficiency of the solution-finding process for the model are needed. Therefore, we designed a solution-finding technique based on the Particle Swarm Optimization (PSO) algorithm. PSO is selected due to its superiority in solving combinatorial problems such as assignment problems [51]. We designed the algorithm in Python and ran it on the same hardware used for standardized optimization software. The pseudocode of the algorithm is given in Table 4.

Particle Swarm Optimization (PSO) is a metaheuristic algorithm based on the behavior of a swarm. In the swarm, each individual (also called a particle) will show their intelligence through a particular behavior; this behavior will affect the behavior of other particles. Thus, if one particle finds the fastest path to a food source, other particles in the same swarm can immediately follow the discovery through information between them.

Table 6. Pseudocode for PSO-based ORH assignment

Start:

PSO Parameter initialization and driver and passenger data input

Set PSO parameter (number of particles, maximum iteration, individual factor, social factor, inertia weight)

Set the number of drivers and passengers to be assigned

Set data related to driver and passenger (driver and passenger initial point, destination point, behavior characteristics)

Iteration = 0; (starting point)

While iteration \leq Maximum Iteration

Do

For i = 0 to Number of Particles

Determine the assignment of driver to passenger

Evaluate the feasibility of one-to-one assignment

Determine the trip price of each assignment

Evaluate whether the joint acceptance probability minimum limit is achieved.

Calculate the objective function

Record P_{best} as the best objective value for individual particle

Find Z_{best} as the best objective value for the entire swarm

Update the position of each particle

Iteration = Iteration + 1

End while

Output Z_{best} for the entire swarm, assignment, and trip price.

In PSO, the swarm is assumed to have a specific size that describes the number of particles in it. Particles have positions and velocities that change over time. In the optimization context, the particle's position represents the value of the decision variable. This position changes over time, describing the exploration of the value of the decision variable in the solution space. The change in position is determined by the particle velocity (vector), which has a

direction of movement and a magnitude of displacement. The particle velocity is iteratively calculated based on the history of the best position of a particular particle (P_{best}) and the best position of the swarm (G_{best}). By changing the position of the particles over time, it is expected that all particles in the swarm will eventually converge to one point in the solution space. This convergence point is then considered the best solution.

Before starting the PSO experiment, we first determine the optimal PSO parameter values to be used. We start by using small-sized cases (2D-2P) and varying PSO parameter values such as the number of particles, maximum iterations, individual and social learning factors, and inertia weight. We then compared the average objective value and computational time achieved by each parameter value set. When a particular set of parameters does not significantly improve the objective value yet lengthens the computational time, the set is denied. The following PSO parameter values are selected through this process: the number of particles is 50, maximum iterations are 1000, individual and social learning factors and inertia weight subsequently are 0.5, 0.5, and 2 [51].

Once the parameter value is set, we ran the PSO algorithm to solve the cases previously listed in Table 3 and summarized the result in Table 5. Moreover, we compared the solution quality found by the PSO algorithm and the output of the standardized optimization software (SOS) in terms of optimality and computational time. The summary presented in Table 5 shows four groups of cases that have different comparison patterns. The first group is 2D-2P cases, which show the SOS's superiority over PSO regarding solution quality (gap of 0.72%) and average computational time (gap of -5.40 seconds). The second group consisted of 3D-3P up to 5D-5P cases. In this group, the SOS maintains its superiority in solution quality (gap of 1.24% to 1.78%) but is worse in average computational time (gap of 12.67 to 514.10 seconds).

The third group consisted of 6D-6P up to 7D-7P cases. In this group, the SOS only gave local optimal solutions within the cutoff time (1200 seconds), while PSO can give better solutions than the local optimal provided by the SOS. In terms of computational time, the PSO can give a relatively short computational time. In fact, by this far, we can see that the computational time for PSO has increased quite a linear pattern compared to SOS (see Fig. 3). The last group comprises cases with large problem sizes (more than 7D-7P). In this group, SOS cannot find any feasible solution within the cutoff point. On the other hand, PSO can still come up with solutions in a relatively short computational time. By projecting this pattern, we can conclude that, given larger cases, the proposed PSO algorithm can solve the ORH assignment problem effectively and efficiently.

Table 7. Comparison of standard optimization software (SOS) and PSO algorithm output.

Problem Size	Comparison of AZ value			Comparison of ACTvalue		
	AZ _{SOS} (in thousand Rp)	AZ _{PSO} (in thousand Rp)	Gap (in percentage)	ACT _{SOS} (in second)	ACT _{PSO} (in second)	Gap (in second)
2D-2P	51.88	51.87	0.72%	0.54	5.94	-5.40
3D-3P	85.34	85.28	1.24%	22.61	7.19	12.67
4D-4P	124.50	122.28	1.78%	129.10	9.94	119.16
5D-5P	153.05	150.65	1.57%	524.56	10.46	514.10
6D-6P	180.98	192.69	-6.47%*	≥ 1200	14.24	≥ 1185.76
7D-7P	210.41	220.33	-4.71%*	≥ 1200	17.72	≥ 1182.28
8D-8P	No FS	250.02	-∞	∞	18.10	∞
9D-9P	No FS	280.88	-∞	∞	19.49	∞
15D-15P	No FS	426.26	-∞	∞	36.05	∞
20D-20P	No FS	559.00	-∞	∞	48.98	∞

25D-25P	No FS	673.80	$-\infty$	∞	59.75	∞
30D-30P	No FS	806.59	$-\infty$	∞	80.95	∞

*The PSO output is compared to the local optimal of standard optimization software (SOS)

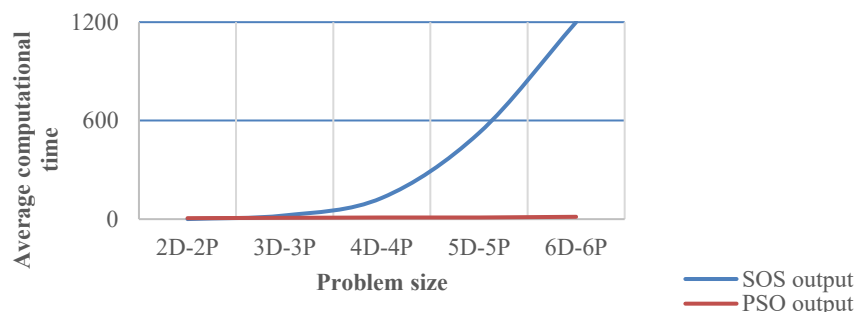


Figure 3. Computational time for standard optimization software (SOS) and PSO plot against the problem size

5. CONCLUSION

This paper proposed a simultaneous model for assigning and determining trip prices used in online ride-hailing businesses. The model has been developed, validated, and experimented on various problem sizes. The result shows that the model can find solutions accommodating different driver and passenger behavior settings. Behavior differences among drivers and passengers lead to different optimal assignments and tailored pricing schemes. This mechanism would reduce the likelihood of the driver and/or passenger rejecting the assignment, thus increasing the system's efficiency. Regarding the solution-finding process, it was found that using standardized optimization software (SOS), the global optimal solution can only be found for small-size problems. Given a larger problem size, the average computational time becomes poor, and the optimality status of the solution deteriorates. We proposed using the particle swarm optimization (PSO) algorithm to overcome this.

The proposed algorithm can achieve an average of 1.78% error for small-sized problems, up to 6.47% better solution compared to local optimal solutions found by standard optimization software, and can give feasible solutions when standard optimization software fails to provide one. A further issue to be tackled is finding data sets to build empirical logistic regression, especially for the Indonesia Ride Hailing market, and integrating them into this research. Another opportunity exists to apply multiple service types provided by ORH in the form of vehicle type. It aims to facilitate the different wants and needs of ORH passengers. Investigating how pricing is determined in such situations can be beneficial for improving ORH's business.

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