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Research Article

Effects of Deterioration Rate and Transportation Costs on Supply Chain Strategies Considering Partial Backlog and Linear Demand with Permissible Delay in payment

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ABSTRACT

In this study, we consider an inventory model for the effect of the deterioration rate and transportation cost on the supply chain strategies in which shortages are allowed with partial backlog and salvage value is incorporated to the deterioration items. The holding cost is a linear function of time. In this model, the demand rate and the deterioration rate are both linear time-dependent. The objective of this paper is to develop and analyze an inventory model that captures these complexities and provides insights into optimal decision-making strategies under the given conditions. The model is solved analytically by minimizing the total inventory cost. Numerical analysis is provided to illustrate the solution and application of the model.

Keywords: Supply chain management, Linear demand, Partial backlog, Salvage value, Deterioration rate, Transportation cost, Permissible delay in payment.

INTRODUCTION

The global market is currently experiencing intense competition. The emergence of products with shorter life cycles, alongside increasing customer expectations, has compelled businesses to invest in and concentrate on their supply chains. In addition, advances in communication and transportation technologies—such as mobile communication, the internet, and overnight delivery—have driven the continual evolution of supply chain management techniques. Moreover, the competitive pressures combined with advancements in information technologies have significantly impacted production system structures, necessitating a reduction of time to market, Increased flexibility of systems, Substantial cost reductions, and a broader concept of quality.

A supply chain is a system of organizations, people, technology, activities, information, and resources involved in moving a product or service from supplier to customer. A supply chain is a network that includes retailers, distributors, transporters, storage facilities, and suppliers, all of whom collaborate in the production, delivery, and sale of a product to the consumer. These activities involve the movement and transformation of goods from the raw materials stage to the end user, along with the corresponding flows of information and funds. Supply chain activities convert natural resources, raw materials, and components into a finished product that is then delivered to the end customer. In simple terms, a supply chain is the connection between a business and the suppliers that provide it with materials, as well as the customers who buy its products. The supply chain, often called the logistics network, includes suppliers, manufacturing plants, warehouses, distribution centers, and retail stores. It also involves the movement of raw materials, work-in-progress inventory, and finished products between these facilities.

Figure 1.1: A conceptual model of a basic supply chain

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The supply chain, commonly known as the logistics network, comprises suppliers, manufacturing centers, warehouses, distribution centers, and retail outlets. It includes the movement of raw materials, work-in-progress inventory, and finished products that circulate between these different facilities.

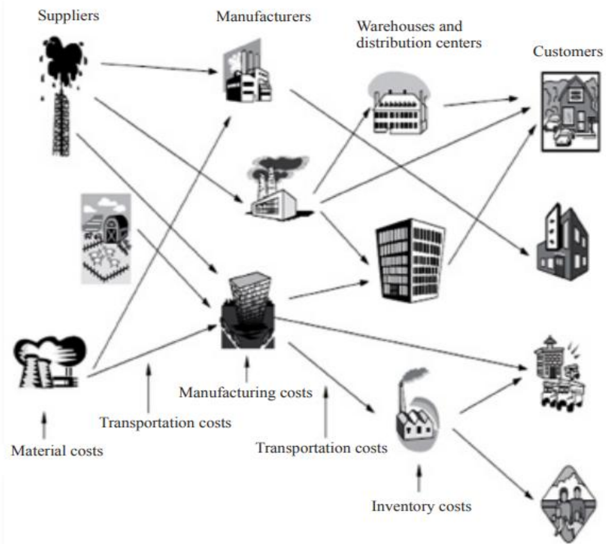

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Figure: 1.2 A supply chain network

In today's highly competitive market, effective supply chain management (SCM) has become a crucial aspect for businesses aiming to optimize their operations and maximize profitability. Managing inventory efficiently is a core element of SCM, especially when dealing with products that have a linear demand pattern and are subject to deterioration over time. This paper addresses a comprehensive inventory model that integrates several key factors: linear demand, partial backlogging, holding costs, permissible delay in payment, linear deterioration rate, and transportation cost. Supply chains are complex networks involving multiple stakeholders, from suppliers to end customers. The need for effective coordination across these networks is amplified when products are perishable or have a defined shelf life. Deterioration of goods not only affects inventory levels but also imposes additional costs, necessitating strategic decisions regarding inventory replenishment and backlogging.

In a traditional inventory model, it is generally assumed that the demand rate is independent of factors such as stock availability, price of items, time, etc. But, in actual practice, it is seen that demand for certain items is greatly influenced by the factor of time. For example, the demand for seasonal foods and clothing is strongly time-dependent. Therefore, it can be concluded that demand for such items fluctuates over time, either increasing or decreasing at different periods. Many authors have studied other types of demand. Dave and Patel (1981) were the first to study a deteriorating inventory model where the demand rate is a linear increasing function of time and shortages are not allowed. Pervin et. al (2018 analysed an inventor model with shortage under time-dependent demand and time-varying holding cost including stochastic deterioration.

Transportation costs are a significant component of total logistics costs and can have a major impact on a company's inventory strategy. These costs can include the expenses associated with moving goods from suppliers to warehouses, between distribution centers, and to end customers. In inventory management, transportation costs must be carefully managed to ensure that products are delivered to the right place at the right time while minimizing costs. Transportation costs can be particularly challenging in industries where demand is highly variable, and products are subject to deterioration. In such cases, companies must carefully plan their transportation strategies to ensure that products are delivered in a timely manner and in optimal condition. This may involve optimizing transportation routes, selecting the most cost-effective transportation modes, and coordinating with suppliers and logistics providers to ensure timely delivery.

Partial backlogging refers to a situation where a portion of the unmet demand during a stockout period is backordered and fulfilled at a later date. This strategy is particularly useful in industries where customers are willing to wait for their orders to be fulfilled rather than canceling them entirely. However, partial backlogging also introduces additional costs and risks into the inventory management process. The decision to implement partial backlogging must be carefully considered, as it requires companies to balance the cost of backordering (including potential lost sales and customer dissatisfaction) against the cost of carrying excess inventory. Additionally, partial backlogging can complicate the process of forecasting demand and managing inventory levels, as companies must account for the impact of backorders on future demand and inventory requirements.

Holding costs, also known as carrying costs, represent the expenses associated with storing and maintaining inventory over time. These costs can include warehousing expenses, insurance, taxes, depreciation, and opportunity costs associated with the capital tied up in inventory. In the context of linear demand and partial backlogging, holding costs play a critical role in determining the optimal inventory levels. High holding costs can significantly impact a company's profitability, particularly in industries where inventory turnover is low, or products have a limited shelf life. As such, companies must carefully manage their inventory levels to minimize holding costs while still meeting customer demand. This requires a delicate balance between maintaining sufficient stock to avoid stockouts and minimizing excess inventory that can lead to high holding costs. Mishra (2012) developed the inventory model for time-dependent holding cost and deterioration with salvage value and shortages.

Many products, particularly perishable goods, are subject to deterioration over time. The rate of deterioration can vary depending on the nature of the product and the conditions under which it is stored. In some cases, the deterioration rate is linear, meaning that the quality of the product decreases at a constant rate over time. The linear deterioration rate presents unique challenges for inventory management. Companies must account for the impact of deterioration on their inventory levels and adjust their inventory strategies accordingly. This may involve implementing more frequent replenishment cycles, optimizing storage conditions, or offering discounts on products that are nearing the end of their shelf life. Additionally, the linear deterioration rate can impact the cost of carrying inventory, as products that deteriorate more quickly may require more frequent replenishment and higher levels of safety stock to avoid stockouts.

Permissible delay in payment refers to the period during which a buyer can delay payment to the supplier without incurring any penalties or interest charges. This practice is common in many industries and can provide significant financial flexibility for companies, allowing them to manage their cash flow more effectively and invest in other areas of the business. In the context of inventory management, a permissible delay in payment can have a significant impact on a company's inventory strategy. By delaying payment, companies can reduce their working capital requirements and improve their cash flow, which can, in turn, allow them to invest in additional inventory or other areas of the business. However, this strategy also introduces additional risks, as companies must carefully manage their payment schedules to avoid penalties or damage to supplier relationships. Aggarwal and Jaggi (1995) developed an inventory model for Ordering policies of deteriorating items under permissible delay in payments. Chu et. al (1998) created the Economic order quantity model for deteriorating items under permissible delay in payments.

In the process of developing a mathematical model for inventory control, it is typically assumed that payments to suppliers are made immediately upon receipt of the goods. However, in practical scenarios, suppliers often grant a specific period for settling accounts. During this grace period, no interest is charged; yet, any payments made beyond this timeframe incur interest as per the terms agreed upon. Since inventories are generally financed through debt or equity, it is important to note that debt financing is frequently short-term. Consequently, the interest paid in such cases represents the cost of capital or opportunity cost.

Short-term loans can be conceptualized as financing sourced from suppliers once the credit period expires. Importantly, before the account settlement is due, customers have the opportunity to sell the goods they have received, allowing them to accumulate revenue and potentially earn interest. This situation may prevent the need for an overdraft that would be required if the supplier demands immediate payment after the replenishment. The interest earned during this period can be viewed as a return on investment, as the generated revenue can be reinvested back into the business.

We have extended the work of Khare and Sharma (2024) by introducing an inventory model that incorporates a linear demand function over time, we investigate the Effects of the deterioration rate and transportation Costs on Supply Chain Strategies. Our model allows for shortages with partial backlogging and Permissible Delay in payment. The

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main objective of this model minimize the total inventory cost. We present a numerical example to validate the model and demonstrate parameter sensitivity graphically.

Notations

- C_p is the purchase cost per unit time.
- h_1 is the holding cost per unit time.
- h_2 is the holding cost per unit time.
- $I_1(t)$ is inventory level at a time t_1.
- $I_2(t)$ is inventory level at a time T.
- A is the inventory order cost per order.
- C_D is the deterioration cost per unit time.
- C_s is the shortage cost per unit time.
- t_1 is stock exhausts time.
- T is the length of a process duration.
- I_b is the highest level of delay purchase.
- I_m is the maximum level of stock during the period [0, T]
- TC is the total inventory cost.
- Q is the total quantity of an order.
- SC is the shortage cost per cycle.
- PC is the purchase cost per cycle.
- HC is the holding cost.
- $R(t)$ is the deterioration rate.
- OC is the ordering cost.
- δ is the salvage value.
- f is the transportation cost.
- M is the permissible delay in settling the accounts.
- IP_1 is the interest payable when $0 \leq M \leq t_1$.
- IP_2 is the interest payable when $t_1 \leq M \leq T$.
- IE_1 is the interest earned when $0 \leq M \leq t_1$.
- IE_2 is the interest earned when $t_1 \leq M \leq T$.
- I_e is the interest that can be earned per rupee in a year.
- I_p is the interest paid per rupee investment in stocks per year.

Assumptions

- Replenishment rate is infinite, i.e. Replenishment rate is instantaneous.
- Lead time is negligible.
- The demand rate of the item is considered a linear and continuous function of time. $D(t)$ is the time-dependent demand function, which is defined by
  $D(t) = (a + bt)$. Where a, b are constant $(a, b, > 0)$. Here a is the initial rate of demand, b is the rate at which the demand rate increases.
- The deterioration rate is variable at time t, and its parameter is R(t)= θt.
- Here, holding cost is time sensitive i.e., where $h_1 > 0, h_2 > 0$.
- Units that have deteriorated over the period of a cycle are related with the salvage value δ, where $0 \leq \delta < 1$.
- Shortage is allowed, unmet demand fulfilled has been described as partially backlogged. As consumer's waiting period, $(T - t)$ reduces, the percentage of backorders increases. The partial reserve rate is equal to $e^{-\mu(T-t)}$ where μ is the parameter for positive backlogging.

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Mathematical Model

Figure 1, This study assumes that a depreciating product will need to be replenished with variable holding and ordering costs. We provide the appropriate order quantity Q and the ideal total cost of inventory. The inventory level $I(t)$ falls to zero at $t = t_1$ due to both demand and deterioration throughout the period $[0, t_1]$, whilst shortages occur during the period $[t_1, T\ ]$due to demand, a portion of the requirements are backlogged.

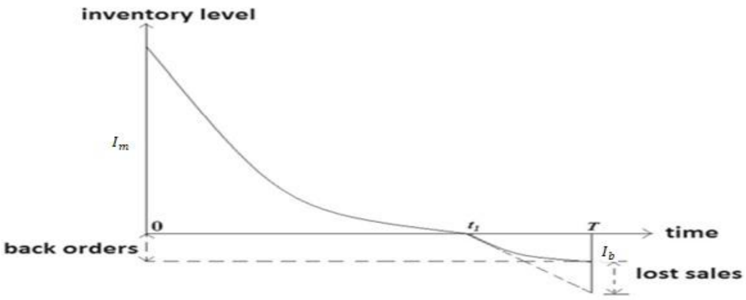

Figure 1: Inventory Level Over Time T

$$\frac{dI_1(t)}{dt} + R(t)I_1(t) = -D(t); \quad 0 \le t \le t_1 \qquad \dots (1)$$

$$\frac{dI_2(t)}{dt} = -D(t)e^{-\mu(T-t)}; \quad t_1 \le t \le T \qquad \dots (2)$$

Using the Deterioration rate $R(t) = (a + bt)$ and Demand rate $D(t) = \theta t$ with the condition $t = t_1$, and $I_1(t_1) = 0$ in equation (1)

By equation (1) we get,

$$\frac{dI_1(t)}{dt} + \theta t\, I_1(t) = -(a + bt); \quad 0 \le t \le t_1$$

$$I_1(t) = \left\{a(t_1 - t) - \frac{a\theta}{2}(t_1t^2 - t^3) + \frac{b}{2}(t_1^2 - t^2) - \frac{b\theta}{4}(t_1^2t^2 - t^4) + \frac{a\theta}{6}(t_1^3 - t^3) + \frac{b\theta}{8}(t_1^4 - t^4)\right\} \qquad \dots (3)$$

Now, by equation (2) we get,

Using the boundary condition $t = t_1$, and $I_2(t_1) = 0$ in equation (2), we get

$$\frac{dI_2(t)}{dt} = -D(t)e^{-\mu(T-t)}; \quad t_1 \le t \le T$$

$$I_2(t) = \left\{e^{-\mu(T-t_1)}\left[\frac{a}{\mu} + \frac{b}{\mu^2}(t_1\mu - 1)\right] - e^{-\mu(T-t)}\left[\frac{a}{\mu} + \frac{b}{\mu^2}(t\mu - 1)\right]\right\} \qquad \dots (4)$$

Now, at $t = 0$ the maximum storage amount for every period is given by $I_m = I_1(0)$

$$I_m = \left\{at_1 + \frac{bt_1{}^2}{2} + \frac{a\theta t_1{}^3}{6} + \frac{b\theta t_1{}^4}{8}\right\} \qquad \dots (5)$$

And at $t = T$ the maximum negative inventory (back-ordered unit) is $I_b = -I_2(T)$

$$I_b = \left\{\left[\frac{a}{\mu} + \frac{b}{\mu^2}(T\mu - 1)\right] - e^{-\mu(T-t_1)}\left[\frac{a}{\mu} + \frac{b}{\mu^2}(t_1\mu - 1)\right]\right\} \qquad \dots (6)$$

Thus, the total quantity in the inventory [0, T] is

$$Q = I_m + I_b$$

$$Q = \left\{at_1 + \frac{bt_1{}^2}{2} + \frac{a\theta t_1{}^3}{6} + \frac{b\theta t_1{}^4}{8}\right\} + \left\{\left[\frac{a}{\mu} + \frac{b}{\mu^2}(T\mu - 1)\right] - e^{-\mu(T-t_1)}\left[\frac{a}{\mu} + \frac{b}{\mu^2}(t_1\mu - 1)\right]\right\} \qquad \dots (7)$$

Now,

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Holding cost during the interval $[0, t_1]$ is given by

$$HC = \int_0^{t_1} h(t)\, I_1(t)dt$$

$$HC = \int_0^{t_1}(h_1 + h_2 t)\, I_1(t)dt$$

$$HC = h_1\left\{\frac{at_1^2}{2} + \frac{bt_1^3}{3} + + \frac{a\theta t_1^4}{12} + \frac{b\theta t_1^5}{15}\right\} + h_2\left\{\frac{at_1^3}{2} + \frac{bt_1^4}{8} + \frac{a\theta t_1^5}{40} + \frac{b\theta t_1^6}{48}\right\} \quad \ldots (8)$$

Now, the deterioration cost is given by

$$DC = C_D\left\{I_1(0) - \int_0^{t_1} D(t)\, dt\right\}$$

$$DC = C_D\left\{\frac{a\theta t_1^3}{6} + \frac{b\theta t_1^4}{8}\right\} \quad \ldots (9)$$

The shortage cost during $[t_1, \text{T}]$ is evaluated as follows.

$$SC = -C_S \int_{t_1}^{T} I_2(t)\, dt$$

$$SC = -C_S\left\{e^{-\mu(T-t_1)}(T - t_1)\left[\frac{a}{\mu} + \frac{b}{\mu^2}(t_1\mu - 1)\right] - \left[\frac{a}{\mu^2} + \frac{b}{\mu^3}(T\mu - 1)\right] + e^{-\mu(T-t_1)}\left[\frac{a}{\mu^2} + \frac{b}{\mu^3}(t_1\mu - 1)\right] + \frac{b}{\mu^3}\left[1 - e^{-\mu(T-t_1)}\right]\right\} \quad \ldots (10)$$

Now, the Purchasing cost per cycle is given by: -

$$PC = C_P\, .Q$$

$$PC = C_P\left[\left\{at_1 + \frac{bt_1^2}{2} + \frac{a\theta t_1^3}{6} + \frac{b\theta t_1^4}{8}\right\} + \left\{\left[\frac{a}{\mu} + \frac{b}{\mu^2}(T\mu - 1)\right] - e^{-\mu(T-t_1)}\left[\frac{a}{\mu} + \frac{b}{\mu^2}(t_1\mu - 1)\right]\right\}\right] \quad \ldots (11)$$

Now, the ordering cost is $OC = A$.

Now, the salvage value is given by: -

$$S_V = \delta . DC$$

$$S_V = \delta . C_D\left\{\frac{a\theta t_1^3}{6} + \frac{b\theta t_1^4}{8}\right\} \quad \ldots (12)$$

Now, transportation cost is given by: -

$$MF = f . Q$$

$$MF = f\left[\left\{at_1 + \frac{bt_1^2}{2} + \frac{a\theta t_1^3}{6} + \frac{b\theta t_1^4}{8}\right\} + \left\{\left[\frac{a}{\mu} + \frac{b}{\mu^2}(T\mu - 1)\right] - e^{-\mu(T-t_1)}\left[\frac{a}{\mu} + \frac{b}{\mu^2}(t_1\mu - 1)\right]\right\}\right] \quad \ldots (13)$$

Now, there will be two cases, namely

(i) $0 \le M \le t_1$ (ii) $t_1 \le M \le T$

Case 1: $0 \le M \le t_1$

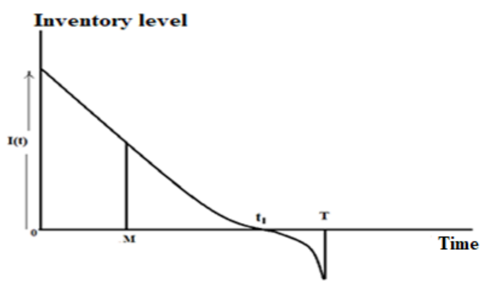

Figure 2: Inventory System for Case 1 ($0 \le M \le t_1$) with Partial Backlogging

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There, the length of the period with positive inventory stock of the items is larger than the credit period the retailer can continue to accumulate revenue and earn interest with an annual rate I_e on it for the rest period in the cycle.

Hence, the interest earned per cycle is: -

Interest Earned

$$IE_1 = C_P.I_e \int_0^M t.D(t)\, dt$$

$$IE_1 = C_P.I_e \left\{\frac{aM^2}{2} + \frac{bM^3}{3}\right\} \quad \ldots (14)$$

However, beyond the fixed credit period, the products still in stock need to be financed with an annual rate I_p.

The interest payable per cycle is: -

Interest Payable

$$IP_1 = C_P.I_p \int_M^{t_1} I_1(t)\, dt$$

$$IP_1 = C_P.I_p \Big\{\Big[\frac{at_1^2}{2} + \frac{bt_1^3}{3} + \frac{a\theta t_1^4}{12} + \frac{b\theta t_1^5}{15}\Big] - \Big[\frac{aM}{2}(2t_1 - M) - \frac{a\theta M^3}{24}(4t_1 - 3M) + \frac{bM}{6}(3t_1^2 - M^2) - \frac{b\theta M^3}{60}(5t_1^2 - 3M^2) +$$
$$\frac{a\theta M}{24}(4t_1^3 - M^3) + \frac{b\theta M}{40}(5t_1^4 - M^4)\Big]\Big\} \quad \ldots (15)$$

The total cost is given by: -

$$TC = \frac{1}{T}\{ \mathrm{HC} + \mathrm{SC} + \mathrm{PC} + \mathrm{DC} + \mathrm{MF} + \mathrm{OC} + IP_1 - S_V - IE_1\}$$

$$TC = \frac{1}{T}\Big\{h_1 \Big\{\frac{at_1^2}{2} + \frac{bt_1^3}{3} + + \frac{a\theta t_1^4}{12} + \frac{b\theta t_1^5}{15}\Big\} + h_2 \Big\{\frac{at_1^3}{2} + \frac{bt_1^4}{8} + \frac{a\theta t_1^5}{40} + \frac{b\theta t_1^6}{48}\Big\} - C_S \Big\{e^{-\mu(T-t_1)}(T - t_1)\Big[\frac{a}{\mu} + \frac{b(t_1\mu - 1)}{\mu^2}\Big] - \Big[\frac{a}{\mu^2} +$$
$$\frac{b(T\mu - 1)}{\mu^3}\Big] + e^{-\mu(T-t_1)}\Big[\frac{a}{\mu^2} + \frac{b(t_1\mu - 1)}{\mu^3}\Big] + \frac{b}{\mu^3}\big[1 - e^{-\mu(T-t_1)}\big]\Big\} + C_P \Big\{\Big[at_1 + \frac{bt_1^2}{2} + \frac{a\theta t_1^3}{6} + \frac{b\theta t_1^4}{8}\Big] + \Big[\frac{a}{\mu} + \frac{b(T\mu - 1)}{\mu^2}\Big] - e^{-\mu(T-t_1)}\Big[\frac{a}{\mu} +$$
$$\frac{b(t_1\mu - 1)}{\mu^2}\Big]\Big\} + C_D \Big\{\frac{a\theta t_1^3}{6} + \frac{b\theta t_1^4}{8}\Big\} + f.\Big\{\Big[at_1 + \frac{bt_1^2}{2} + \frac{a\theta t_1^3}{6} + \frac{b\theta t_1^4}{8}\Big] + \Big[\frac{a}{\mu} + \frac{b(T\mu - 1)}{\mu^2}\Big] - e^{-\mu(T-t_1)}\Big[\frac{a}{\mu} + \frac{b(t_1\mu - 1)}{\mu^2}\Big]\Big\} + \mathrm{A} +$$
$$C_P.I_p \Big\{\Big[\frac{at_1^2}{2} + \frac{bt_1^3}{3} + \frac{a\theta t_1^4}{12} + \frac{b\theta t_1^5}{15}\Big] - \Big[\frac{aM}{2}(2t_1 - M) - \frac{a\theta M^3}{24}(4t_1 - 3M) + \frac{bM}{6}(3t_1^2 - M^2) - \frac{b\theta M^3}{60}(5t_1^2 - 3M^2) +$$
$$\frac{a\theta M}{24}(4t_1^3 - M^3) + \frac{b\theta M}{40}(5t_1^4 - M^4)\Big]\Big\} - \delta.C_D \Big\{\frac{a\theta t_1^3}{6} + \frac{b\theta t_1^4}{8}\Big\} - C_P.I_e \Big\{\frac{aM^2}{2} + \frac{bM^3}{3}\Big\}\Big\}$$
...(16)

Case 2: $t_1 \leq M \leq T$

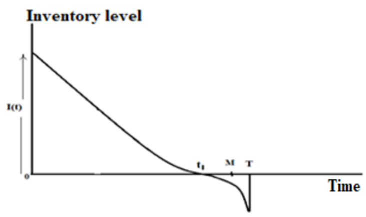

Figure 3: Inventory System for Case 2 ($t_1 \leq M \leq T$) with Partial Backlogging

$IP_2 = 0$ Where I_p is the interest purchased per cycle. Then, in this policy, it is assumed that the account is to be settled during the shortage period, and the retailer pays no interest per cycle.

The interest earned per cycle is

$$IE_2 = C_P.I_e \Big\{\int_0^M t.D(t)\, dt + (M - t_1)\int_0^{t_1} D(t)\, dt\Big\}$$

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$$IE_2 = C_P.I_e\left\{\frac{at_1^2}{2} + \frac{bt_1^3}{3} + (M - t_1)\left[at_1 + \frac{bt_1^2}{2}\right]\right\} \quad \ldots (17)$$

The total cost is given by: -

$$TC = \frac{1}{T}\{\mathrm{HC} + \mathrm{SC} + \mathrm{PC} + \mathrm{DC} + \mathrm{MF} + \mathrm{OC} + IP_2 - S_V - IE_2\}$$

$$TC = \frac{1}{T}\Bigg\{h_1\left\{\frac{at_1^2}{2} + \frac{bt_1^3}{3} + + \frac{a\theta t_1^4}{12} + \frac{b\theta t_1^5}{15}\right\} + h_2\left\{\frac{at_1^3}{2} + \frac{bt_1^4}{8} + \frac{a\theta t_1^5}{40} + \frac{b\theta t_1^6}{48}\right\} - C_S\left\{e^{-\mu(T-t_1)}(T-t_1)\left[\frac{a}{\mu} + \frac{b(t_1\mu-1)}{\mu^2}\right] - \left[\frac{a}{\mu^2} + \frac{b(T\mu-1)}{\mu^3}\right] + e^{-\mu(T-t_1)}\left[\frac{a}{\mu^2} + \frac{b(t_1\mu-1)}{\mu^3}\right] + \frac{b}{\mu^3}\left[1 - e^{-\mu(T-t_1)}\right]\right\} + C_P\left\{\left[at_1 + \frac{bt_1^2}{2} + \frac{a\theta t_1^3}{6} + \frac{b\theta t_1^4}{8}\right] + \left[\frac{a}{\mu} + \frac{b(T\mu-1)}{\mu^2}\right] - e^{-\mu(T-t_1)}\left[\frac{a}{\mu} + \frac{b(t_1\mu-1)}{\mu^2}\right]\right\} + C_D\left\{\frac{a\theta t_1^3}{6} + \frac{b\theta t_1^4}{8}\right\} + f.\left\{\left[at_1 + \frac{bt_1^2}{2} + \frac{a\theta t_1^3}{6} + \frac{b\theta t_1^4}{8}\right] + \left[\frac{a}{\mu} + \frac{b(T\mu-1)}{\mu^2}\right] - e^{-\mu(T-t_1)}\left[\frac{a}{\mu} + \frac{b(t_1\mu-1)}{\mu^2}\right]\right\} + \mathrm{A} - \delta.C_D\left\{\frac{a\theta t_1^3}{6} + \frac{b\theta t_1^4}{8}\right\} - C_P.I_e\left\{\frac{at_1^2}{2} + \frac{bt_1^3}{3} + (M - t_1)\left[at_1 + \frac{bt_1^2}{2}\right]\right\}\Bigg\}$$

... (18)

The necessary condition to be reduced is

$\frac{\partial TC}{\partial t_1} = 0, \frac{\partial TC}{\partial T} = 0$ i.e., And $\frac{\partial^2 TC}{\partial t_1^2} > 0$, $\frac{\partial^2 TC}{\partial T^2} > 0$

And $\left(\frac{\partial^2 TC}{\partial t_1^2}\right)\left(\frac{\partial^2 TC}{\partial T^2}\right) - \left(\frac{\partial^2 TC}{\partial t_1 \partial T}\right) > 0$

Numerical Examples

CASE 1:

Values of parameters used in our Inventory Model are as follows:

A=100, $h_1 = \$10$, $h_2 = \$25$, C_P=\$50, $\delta = \$25$ C_s=\$10, C_D=\$50, θ=0.01, a=100, b=8, $\mu = 0.1$, $f = 2.5$, $I_{E1} = \$\,0.15$, $I_{P1} = \$\,0.18$, and $M = 20/365$ year in appropriate units. We obtained the optimal value $\boldsymbol{t_1}$**=0.1019 year**, $\boldsymbol{T} =$ **0.4988 year**, $\boldsymbol{TC}$**=5641.0274**, and $\boldsymbol{Q}$**= 50.0917 units.**

Case 1: Table 1

Effect of Permissible Delay in Payment and Deterioration Rate on The Total Inventory Cost

M / θ	Total Cost	15	20	25	30	35
0.008	TC	5643.3804	5641.1086	5638.7884	5636.4203	5634.0044
0.009	TC	5643.3446	5641.0681	5638.7429	5636.3693	5633.9476
0.01	TC	5643.3087	5641.0274	5638.6971	5636.3180	5633.8905
0.011	TC	5643.2725	5640.9865	5638.6509	5636.2663	5633.8329
0.012	TC	5643.2361	5640.9452	5638.6045	5636.2143	5633.7750

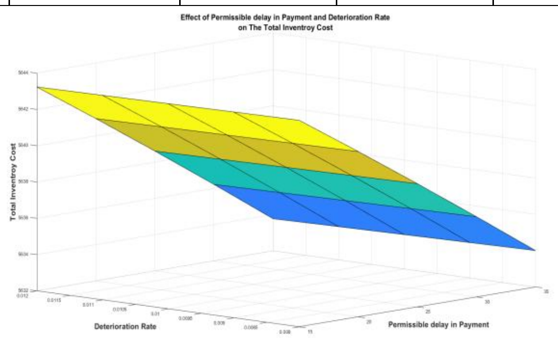

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Figure 4: Effect of Permissible Delay in Payment and Deterioration Rate on The Total Inventory Cost

Case 1: Table 2

Effect of Permissible Delay in Payment and Backlogging Parameter on The Total Inventory Cost

M / μ	Total Cost	15	20	25	30	35
0.08	TC	5659.9145	5657.2587	5654.5576	5651.8115	5649.0205
0.09	TC	5651.8677	5649.3947	5646.8744	5644.3073	5641.6936
0.1	TC	5643.3087	5641.0274	5638.6971	5636.3180	5633.8905
0.11	TC	5634.1750	5632.0956	5629.9651	5627.7839	5625.5524
0.12	TC	5624.3913	5622.5250	5620.6055	5618.6333	5616.6089

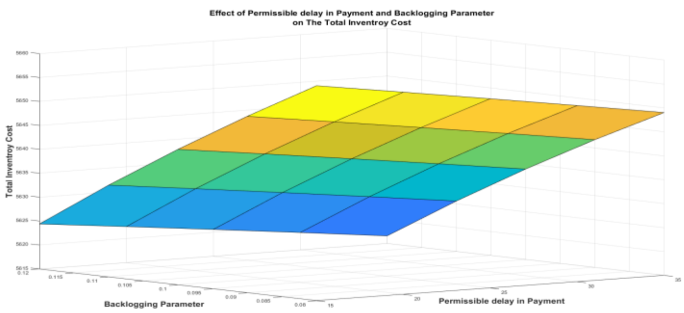

Figure 5: Effect of Permissible Delay in Payment and Backlogging Parameter on The Total Inventory Cost

Case 1: Table 3

Effect of Deterioration Rate and Backlogging Parameter on The Total Inventory Cost

θ / μ	Total Cost	0.008	0.009	0.01	0.011	0.012
0.08	TC	5657.3713	5657.3152	5657.2587	5657.2019	5657.1447
0.09	TC	5649.4913	5649.4431	5649.3947	5649.3459	5649.2968
0.1	TC	5641.1086	5641.0681	5641.0274	5640.9865	5640.9452
0.11	TC	5632.1620	5632.1289	5632.0956	5632.0621	5632.0284
0.12	TC	5622.5774	5622.5513	5622.5250	5622.4985	5622.4719

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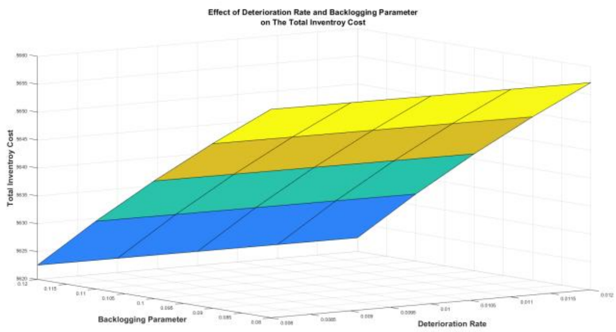

Figure 6: Effect of Deterioration Rate and Backlogging Parameter on The Total Inventory Cost

Sensitivity analysis

By changing the values of parameters used in our model and reading out the effects on t_1 and TC. The rate of changes in values of parameters are taken -20%, -10%, +10%, and 20%.

Case 1: Table 4 Sensitivity Analysis

Parameter	%	-20%	-10%	0%	+10%	+20%
h_1	t_1	0.1094	0.1056	0.1019	0.0985	0.0953
	T	0.4999	0.4994	0.4988	0.4983	0.4979
	TC	5638.7806	5639.9433	5641.0274	5642.0401	5642.9877
h_2	t_1	0.1048	0.1033	0.1019	0.1006	0.0994
	T	0.4996	0.4992	0.4988	0.4985	0.4982
	TC	5640.4734	5640.7561	5641.0274	5641.2883	5641.5395
A	t_1	0.0943	0.0983	0.1019	0.1054	0.1087
	T	0.4457	0.4730	0.4988	0.5234	0.5469
	TC	5598.6828	5620.4499	5641.0274	5660.5904	5679.2747
C_D	t_1	0.1015	0.1017	0.1019	0.1022	0.1024
	T	0.4987	0.4988	0.4988	0.4989	0.4990
	TC	5641.1124	5641.0700	5641.0274	5640.9846	5640.9414
C_S	t_1	0.0785	0.0913	0.1019	0.1109	0.1186
	T	0.5551	0.5239	0.4988	0.4783	0.4610
	TC	5605.4968	5624.3891	5641.0274	5655.8498	5669.1763
C_P	t_1	0.1190	0.1103	0.1019	0.0938	0.0858
	T	0.5035	0.5009	0.4988	0.4974	0.4964
	TC	4636.1951	5138.8138	5641.0274	6142.8516	6644.3004
δ	t_1	0.1015	0.1017	0.1019	0.1022	0.1024
	T	0.4987	0.4988	0.4988	0.4989	0.4990
	TC	5641.1159	5641.0718	5641.0274	5640.9828	5640.9378
I_e	t_1	0.1020	0.1020	0.1019	0.1019	0.1019
	T	0.4994	0.4991	0.4988	0.4986	0.4983
	TC	5641.4799	5641.2537	5641.0274	5640.8010	5640.5745
I_P	t_1	0.1051	0.1035	0.1019	0.1005	0.0991
	T	0.4999	0.4994	0.4988	0.4983	0.4979
	TC	5640.5966	5640.8190	5641.0274	5641.2232	5641.4073
M	t_1	0.0987	0.1003	0.1019	0.1036	0.1052
	T	0.4994	0.4991	0.4988	0.4986	0.4983

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	TC	5642.8564	5641.9458	5641.0274	5640.1011	5639.1670
a	t_1	0.1055	0.1037	0.1019	0.1003	0.0988
	T	0.5238	0.5109	0.4988	0.4876	0.4770
	TC	4573.6955	5107.4775	5641.0274	6174.3603	6707.4898
b	t_1	0.1061	0.1039	0.1019	0.1001	0.0984
	T	0.5284	0.5129	0.4988	0.4859	0.4739
	TC	5619.1714	5630.2585	5641.0274	5651.5044	5661.7125
θ	t_1	0.1015	0.1017	0.1019	0.1022	0.1024
	T	0.4987	0.4988	0.4988	0.4989	0.4989
	TC	5641.1086	5641.0681	5641.0274	5640.9865	5640.9452
μ	t_1	0.1119	0.1071	0.1019	0.0962	0.0899
	T	0.4755	0.4865	0.4988	0.5126	0.5283
	TC	5657.2587	5649.3947	5641.0274	5632.0956	5622.5250
f	t_1	0.1026	0.1023	0.1019	0.1016	0.1013
	T	0.4989	0.4989	0.4988	0.4988	0.4987
	TC	5590.8220	5615.9251	5641.0274	5666.1291	5691.2302

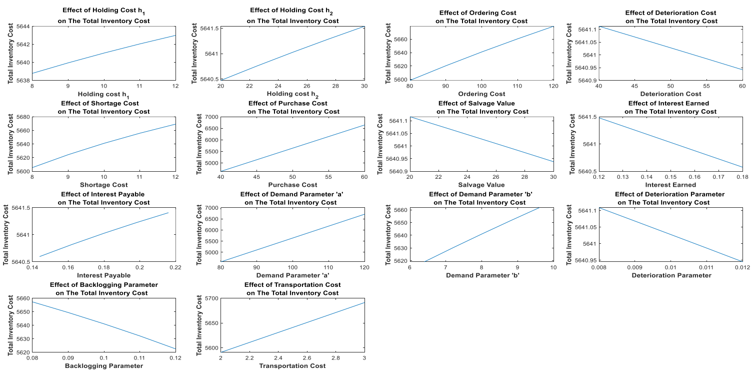

Figure 7: Graphical Representation of Sensitivity Analysis

CASE 2:

Values of parameters used in our Inventory Model are as follows:

A=100, $h_1 = \$10$, $h_2 = \$25$, C_P=\$50, $\delta = \$25$ C_s=\$10, C_D=\$50, θ=0.01, a=100, b=8, $\mu = 0.1$, $f = \$2.5$, $I_{E2} = \$\ 0.15$, and $M = 90/365$ year in appropriate units. We obtained the optimal value t_1**=0.1019 year**, $T = 0.4988$, **year** TC**=5641.0274**, and Q**= 50.0917 units.**

Case 2: Table 5

Effect of Permissible Delay in Payment and Deterioration Rate on The Total Inventory Cost

M / θ	Total Cost	80	85	90	95	100
0.008	TC	5622.4035	5621.9374	5621.4694	5620.9995	5620.5276
0.009	TC	5622.2077	5621.7393	5621.2690	5620.7967	5620.3225
0.01	TC	5622.0087	5621.5380	5621.0654	5620.5908	5620.1142
0.011	TC	5621.8066	5621.3336	5620.8586	5620.3816	5619.9027
0.012	TC	5621.6013	5621.1258	5620.6484	5620.1690	5619.6876

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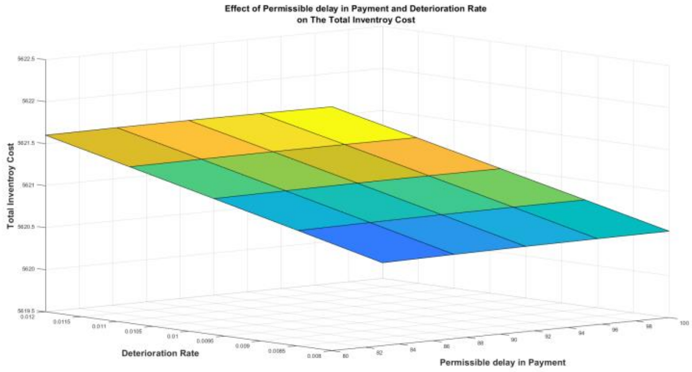

Figure 8: Effect of Permissible Delay in Payment and Deterioration Rate on The Total Inventory Cost

Case 2: Table 6

Effect of Permissible Delay in Payment and Backlogging Parameter on The Total Inventory Cost

M / μ	Total Cost	80	85	90	95	100
0.08	TC	5633.2897	5632.7656	5632.2396	5631.7117	5631.1820
0.09	TC	5627.8720	5627.3736	5626.8733	5626.3711	5625.8670
0.1	TC	5622.0087	5621.5380	5621.0654	5620.5908	5620.1142
0.11	TC	5615.6330	5615.1924	5614.7497	5614.3051	5613.8584
0.12	TC	5608.6616	5608.2538	5607.8438	5607.4318	5607.0177

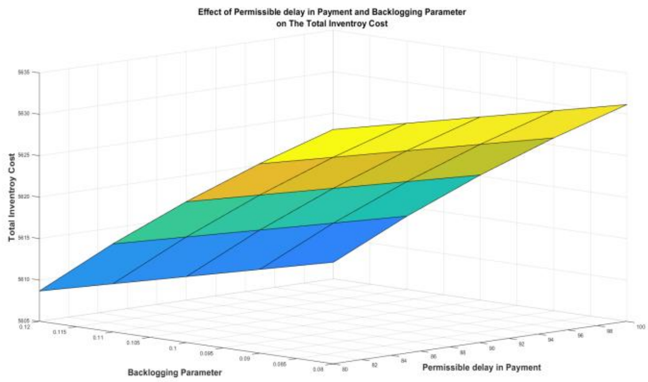

Figure 9: Effect of Permissible Delay in Payment and Backlogging Parameter on The Total Inventory Cost

Case 2: Table 7

Effect of Deterioration Rate and Backlogging Parameter on The Total Inventory Cost

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θ / μ	Total Cost	0.008	0.009	0.01	0.011	0.012
0.08	TC	5632.7530	5632.4983	5632.2396	5631.9768	5631.7098
0.09	TC	5627.3326	5627.1047	5626.8733	5626.6382	5626.3994
0.1	TC	5621.4694	5621.2690	5621.0654	5620.8586	5620.6484
0.11	TC	5615.0976	5614.9250	5614.7497	5614.5716	5614.3906
0.12	TC	5608.1350	5607.9906	5607.8438	5607.6947	5607.5432

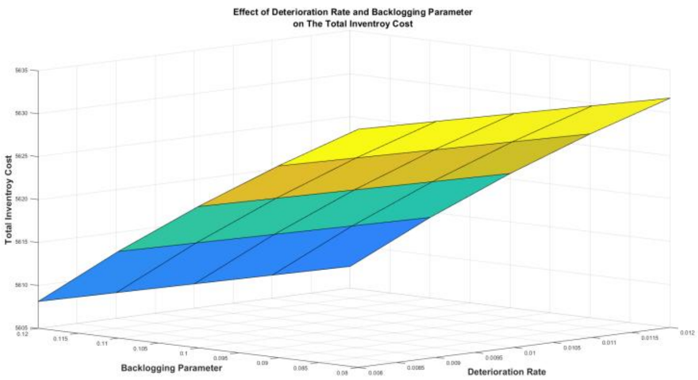

Figure 10: Effect of Deterioration Rate and Backlogging Parameter on The Total Inventory Cost

Case 2: Table 8 Sensitivity Analysis

Parameter	%	-20%	-10%	0%	+10%	+20%
h_1	t_1	0.1956	0.1854	0.1759	0.1671	0.1589
	T	0.5144	0.5134	0.5124	0.5113	0.5103
	TC	5614.2901	5617.8544	5621.0654	5623.9642	5626.5869
h_2	t_1	0.1900	0.1825	0.1759	0.1701	0.1649
	T	0.5160	0.5140	0.5124	0.5109	0.5097
	TC	5618.0884	5619.6599	5621.0654	5622.3342	5623.4888
A	t_1	0.1641	0.1702	0.1759	0.1812	0.1861
	T	0.4589	0.4864	0.5124	0.5371	0.5607
	TC	5579.8884	5601.0428	5621.0654	5640.1215	5658.3395
C_D	t_1	0.1740	0.1749	0.1759	0.1769	0.1780
	T	0.5119	0.5121	0.5124	0.5126	0.5129
	TC	5621.4880	5621.2784	5621.0654	5620.8488	5620.6286
C_S	t_1	0.1443	0.1622	0.1759	0.1869	0.1960
	T	0.5622	0.5344	0.5124	0.4944	0.4795
	TC	5594.9815	5609.1118	5621.0654	5631.3567	5640.3375
C_P	t_1	0.1802	0.1781	0.1759	0.1736	0.1710
	T	0.5174	0.5148	0.5124	0.5100	0.5077
	TC	4616.2612	5118.6730	5621.0654	6123.4372	6625.7868
	t_1	0.1739	0.1749	0.1759	0.1770	0.1780

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δ	T	0.5119	0.5121	0.5124	0.5126	0.5129
	TC	5621.5053	5621.2872	5621.0654	5620.8397	5620.6101
I_e	t_1	0.1629	0.1693	0.1759	0.1830	0.1904
	T	0.5108	0.5116	0.5124	0.5131	0.5139
	TC	5625.3078	5623.2666	5621.0654	5618.6887	5616.1195
M	t_1	0.1736	0.1748	0.1759	0.1771	0.1782
	T	0.5132	0.5128	0.5124	0.5120	0.5115
	TC	5622.7577	5621.9147	5621.0654	5620.2097	5619.3477
a	t_1	0.1811	0.1784	0.1759	0.1736	0.1713
	T	0.5359	0.5237	0.5124	0.5017	0.4917
	TC	4557.2736	5089.2669	5621.0654	6152.6816	6684.1269
b	t_1	0.1825	0.1791	0.1759	0.1730	0.1703
	T	0.5437	0.5273	0.5124	0.4987	0.4860
	TC	5598.6401	5610.0210	5621.0654	5631.8016	5642.2541
θ	t_1	0.1741	0.1750	0.1759	0.1769	0.1779
	T	0.5119	0.5121	0.5124	0.5126	0.5129
	TC	5621.4694	5621.2690	5621.0654	5620.8586	5620.6484
μ	t_1	0.1880	0.1823	0.1759	0.1687	0.1604
	T	0.4923	0.5018	0.5124	0.5243	0.5379
	TC	5632.2396	5626.8733	5621.0654	5614.7497	5607.8438
f	t_1	0.1768	0.1764	0.1759	0.1755	0.1750
	T	0.5127	0.5125	0.5124	0.5122	0.5121
	TC	5570.5971	5595.8316	5621.0654	5646.2985	5671.5308

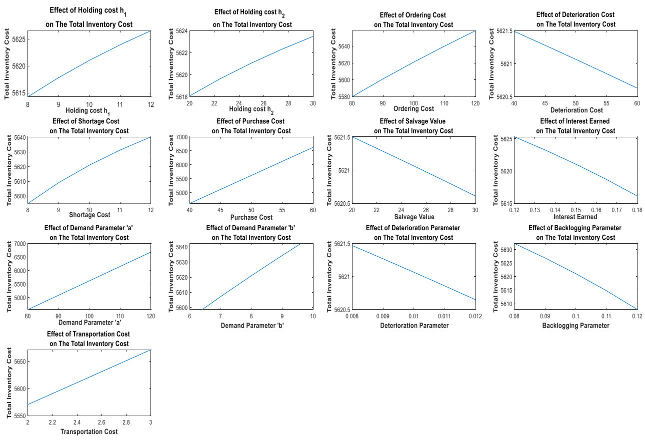

Figure 11: Graphical Representation of Sensitivity Analysis

CONCLUSIONS

We created an inventory model that incorporates a linear demand function over time, we investigate the effects of the deterioration rate and transportation costs on supply chain strategies. Our model allows for shortages with partial backlogging and Permissible Delay in payment. The main objective of this model minimize the total inventory cost. This given model is supported by a numerical example along with sensitivity analysis is carried out to measure the effect of parameters on the total average inventory cost. Table 1 illustrates how changes in permissible delay in payment and deterioration rate on the total inventory cost, as depicted in Figure 4. Table 2 illustrates how changes in permissible delay in payment and backlogging parameter on the total inventory cost, as depicted in Figure 5. Table 3 illustrates how changes in deterioration rate and backlogging parameter on the total inventory cost, as depicted in Figure 6. A sensitivity analysis of case (1) is conducted to examine the effects of boundary values on the optimal arrangement. Based on the model analysis, it has been concluded that in Table 4, as depicted in Figure 7, these effects are illustrated. Table 5 illustrates how changes in permissible delay in payment and deterioration rate on the total inventory cost, as depicted in Figure 8. Table 6 illustrates how changes in permissible delay in payment and backlogging parameter on the total inventory cost, as depicted in Figure 9. Table 7 illustrates how changes in deterioration rate and backlogging parameter on the total inventory cost, as depicted in Figure 10. A sensitivity analysis of case (2) is conducted to examine the effects of boundary values on the optimal arrangement. Based on the model analysis, it has been concluded that in Table 8, as depicted in Figure 11, these effects are illustrated.

In real-world applications, this integrated inventory management system has the potential to transform how businesses operate across various industries. Here are some specific examples of how this invention could be applied in practice: Retail Industry, Perishable Goods, Manufacturing Sector, E-commerce and Distribution, Healthcare Industry, Automotive Industry. Ultimately, this invention provides businesses with the tools they need to thrive in today's complex, fast-paced, and highly competitive market.

From the sensitivity analysis of the model, it has concluded that:

Case 1:

- As the holding cost, purchase cost, demand parameters, transportation cost, backlogging parameter, interest earned, and interest payable increases, the stock exhaust time (t_1) decreases marginally.
- As the ordering cost, permissible delay in payment, deterioration cost, shortage cost, deterioration rate, and salvage value increases, the stock exhaust time (t_1) is increased marginally.
- As holding cost, shortage cost, demand parameters, transportation cost, interest earned, interest payable, permissible delay in payment, salvage value, and purchase cost increase, the length of a process duration (T) decreases marginally.
- As the deterioration rate, salvage value, ordering cost, and deterioration cost increases, the length of a process duration (T) increased marginally.
- As the deterioration rate, demand parameter (b), permissible delay in payment, deterioration cost, interest earned, backlogging parameter, and salvage value increase, the total inventory cost decreases marginally.
- As the holding cost, shortage cost, transportation cost, and interest payable increase, the total inventory cost increases marginally.
- As purchase cost and demand parameter (a) increase, the total inventory cost increased significantly.

Case 2:

- As the holding cost, purchase cost, demand parameters, transportation cost, and backlogging parameter increase, the stock exhaust time (t_1) decreases marginally.
- As the ordering cost, deterioration cost, shortage cost, deterioration rate, salvage value, permissible delay in payment, and interest earned increase, the stock exhaust time (t_1) is increased marginally.
- As the holding cost, purchase cost, shortage cost, demand parameters, transportation cost, and permissible delay in payment increase, the length of a process duration (T) decreases marginally.
- As the deterioration rate, interest earned, backlogging parameter, salvage value, ordering cost, and deterioration cost increase, the length of a process duration (T) increased marginally.
- As the deterioration rate, permissible delay in payment, deterioration cost, interest earned, backlogging parameter, and salvage value increase, the total inventory cost decreases marginally.
- As the holding cost, ordering cost, shortage cost, demand parameter (b), and transportation cost increases, the total inventory cost increases marginally.
- As the purchase cost and demand parameter (a) increase, the total inventory cost increased significantly.

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