

Applications of Nonlinear Combinatorics in Optimizing Network Traffic

Mr. Mohammad Haadi Bhat^{1*}, Mobin Ahmad², Senthil Kumar V.S³, Kamal Kant⁴,
Dr. K. Yugandhar⁵

^{1*}Assistant professor, Department of Sciences, Chandigarh School of Business, Chandigarh Group of Colleges, Jhanjeri, Mohali - 140307, Punjab, India, bhathaadi6@gmail.com

²Former Professor of Mathematics, Al-Falah School of Engineering and Technology, Al-Falah University, Haryana, India, profmobin@yahoo.com

³Department of Management Studies, Sona college of Technology, Salem, Tamil Nadu, India, Email: senthilcsfate@gmail.com

⁴Assistant Professor, School of Engineering and Technology (UIET), Chhatrapati Shahu Ji Maharaj University, Kalyanpur, Kanpur, Uttar Pradesh, India, kkamal2544@gmail.com

⁵Professor, Geethanjali College of Engineering and Technology, Hyderabad - 501301, Telangana, India, yogi.english@gmail.com

ARTICLE INFO	ABSTRACT
Received: 17 Dec 2024	<p>Network traffic optimization is a critical aspect of ensuring efficient and reliable data transmission in complex communication systems. The graph approach, which is the foundation of most modern theories and optimization methods, is not sufficient when operating with a traditional toolkit of linear programming for the nonlinear and combinatorial nature of many networks. In this research, we focus on using submodular optimization and discrete Newton methods to improve the network performance as combinatorial optimization problems. The proposed methodology reduces the total end-to-end delay, increases the throughput, and also optimizes the time over which convergence occurs, which provides a reliable framework for controlling network traffic. By performing simulations on a graph-structured network, the two solutions as a one generated 35% lower average end-to-end delay, 25% higher overall throughput, and faster convergence compared to disparate approaches. From these results, we gain an understanding of the importance of handling nonlinear cost functions and the capacity of submodular optimization and DNM. As such, key issues such as scaling up to a large network and its ability to handle time-varying traffic remain limitations and thus require research. The results of this study hold valuable implications for network management allowing a timely route to re-organize the traffic load without overly expensive infrastructural evolutions. This research builds on important limitations of the current approaches to distill knowledge for further development in the context of network traffic shaping, which could be important for the development of 5G, IoT as well as numerous other up-and-coming concepts.</p> <p>Keywords: Nonlinear combinatorial optimization, Network traffic optimization, Submodular optimization, Discrete Newton methods, Latency reduction</p>
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1. Introduction

Network traffic optimization is a critical aspect of modern communication systems, directly influencing the efficiency and reliability of data transmission across various platforms. These difficulties have been solved with the help of the conventional optimization approaches which are generally linear in their form. However, the existing networks are continually evolving and much more complex than the simple networks of a decade ago so require more elaborate methods (Vesselinova et al., 2020). By going nonlinear, combinatorial optimization has become a valuable area of study that provides superior methods for addressing complex network traffic challenges.

Essentially, the utilization of nonlinear combinatorial methods in network optimization is not a novel practice. Scholars have also looked into many approaches to improving the network's performance. For instance, Pardalos and Zhang (2019) pointed to the future of using nonlinear CO in ML and WC, stressing its applicability to solve network issues. Further, Espinoza et al. (2010) also analyzed the usefulness of understanding combinatorial optimization in transportation and logistics networks with stress with traffic

assignment and vehicle routing challenges. Nevertheless, some issues are still there (Papadimitriou & Steiglitz, 1998). This is because the combinatorial nature of the investigated nonlinear networks is intrinsically intricate, thus creating significant problems in computation even in moderately sized networks to elaborate efficient algorithms. Also, the nature of the network traffic varies which means that solutions that are being developed must be able to respond to changes in real time. Solving these issues is important for developing accurate approaches to nonlinear combinatorial optimization of traffic loads within a network (Hochbaum, 2007). In this research, the focus is set on the identification of the possible applications of nonlinear combinations solving for optimizing network traffic. In this context, our study aims to examine current trends regarding network optimization strategies and their limitations, as well as to present new ideas for improving the current situation (Los & Lardinois, 1982). This research aims to offer significant insights to both academic scholars and professionals in enhancing network traffic management technologies.

2. Literature Review

Network traffic optimization has been an important area of research both in academia and industry since it is essential to guarantee the effective transmission of data over complex communication networks. Indeed, there is a rich use of traditional linear approaches; however, with the increasing complexity as well as the transit nature of contemporary networks, researchers have turned to linear nonlinear combinatorial analysis. Recent studies have shown that nonlinear combinatorial optimization methods can solve some of the most urgent network traffic problems. Gathering recent research on nonlinear combinatorial optimization, Pardalos and Zhang (2019) argued that it is pervasive in machine learning, social computing, cloud computing, wireless communication, and data science. Their work shows the general applicability of these techniques in different areas, and in particular, the traffic engineering field.

In the transportation and logistics domain, Espinoza et al. (2010) presented traffic assignment and vehicle routing applications of combinatorial optimization. They stressed mathematical models and optimization methods in traffic assignment plans, it is noteworthy that over the years, there have been suggested applications of operation research techniques in order to determine real-life traffic assignment problems, to make available a traffic system with much better performance to the individual travelers and the society. Moreover, a new development in the field of synchronization of chaotic systems has been achieved in recent years. Letellier et al. (2023) explored the possibility of achieving generalized synchronization by using the flat control rule derived from the nonlinear control theory. This technique entails the placement of sensors and actuators in such a way that the system is observable and controllable from any point which is particularly applicable to network traffic control.

Several major trends are discernible in the use of nonlinear combinatorial optimization to traffic in the network. Currently, there is a development to combine classical machine learning algorithms and nonlinear combinatorial optimization to increase pressure predictive effectiveness and react to the peculiarities of the network. For example, Magnouche et al. (2023) put forward a load-balancing approach that is capable of predicting load transfers to safeguard traffic from any failure of the SRLG, using a neural network approximation to solve non-convex optimization.

The search for quantum-inspired solutions for difficult combinatorial problems in network optimization has started to capture interest. Witt et al. (2024) outlined the use of quantum annealing for resource utilization in optical wide-area communication systems which showed how such an issue could be tackled as a QUBO problem. Moreover, the integration of several optimization heuristics into a mixed algorithm due to the complex nature of network traffic optimization procedures is on the rise. The method of reduction of delay using Lagrange multipliers in stochastic network optimization was examined by Huang and Neely (2011) to show how multiple approaches can be integrated.

However, there are several remaining deficits in the literature. Indeed, several of the previously described nonlinear combinatorial optimization techniques have limitations in extending to vast real-world size networks (Looi, 1992). The primary disadvantage of these techniques is that their applicability tends to be constrained by the amount of computational effort required. Modern networks are ever-evolving and an optimization technique has to be done dynamically. However, several current approaches cannot provide quick adjustments to react to network modifications. However, there is a dearth of research in combining nonlinear combinatorial optimization with new technologies like 5G/6G networking, the Internet of Things (IoT), and edge computing for handling fresh types of traffic (Fügenschuh et al., 2006).

As a result, future research should aim at designing suitable optimization algorithms with the capability of solving large-scale networks in terms of performance. The design of methods that can adapt to the network and changes that are always arising in the network will ensure optimal design under these conditions (Yang et al., 2021). Quantum computing to a type nowadays insufficiently investigated, however, a promising approach to solving combinatorial optimization problems in network traffic management, as well as the application of nonlinear combinatorial optimization in the perspectives of new network paradigms, such as 5G/6G, Internet of Things, and edge computing, are pointed as potential avenues to contribute to this field. Thus, future investigations into the factors affecting communication can contribute to the improvement of communication systems within organizations (Ma & Abdulhai, 2002).

3. Mathematical Preliminaries

Here, we define the mathematical requisites that are central to the comprehension of the use of nonlinear combinatorial optimization in network traffic optimization. The first four sections give a brief and most basic description of four concepts that are fundamental in non-linear combinatorial optimization, before proceeding to a discussion of the network modeling concepts that are relevant in traffic optimization.

3.1 Fundamentals of Nonlinear Combinatorial Optimization

Nonlinear combinatorial optimization therefore involves solving an optimization problem for which there is a predetermined set of discrete solutions from which an optimal solution has to be selected with the objective function and or constraints being nonlinear. That is why this complexity is typical for real-world problems when the components interact and cause nonlinear reactions, e.g. in the design of networks and traffic management. A general nonlinear combinatorial optimization problem can be formulated as follows:

$$\begin{aligned} &\text{Minimize} && f(x) \\ &\text{subject to} && x \in S, \end{aligned}$$

where $f: \mathbb{R}^n \rightarrow \mathbb{R}$ is a nonlinear objective function, and $S \subseteq \{0,1\}^n$ represents the feasible set defined by combinatorial constraints. Such problems may require specific algorithms due to the present difficulties in solving the problem. For example, Pardalos and Zhang (2019) overview diverse methods to solve NCO problems and stress the fact that different techniques should be used depending on the structure of the problem.

3.2 Theoretical Framework for Network Traffic

Modeling network traffic requires representing the network as a graph. $G = (V, E)$, where V denotes the set of nodes (e.g., routers, switches) and E represents the set of edges (communication links). Each edge $e \in E$ is associated with parameters such as capacity c_e and latency l_e .

The objective is to optimize the flow of data through the network to minimize congestion and latency while maximizing throughput. This can be formulated as a nonlinear combinatorial optimization problem:

$$\begin{aligned} &\text{Minimize} \\ &\text{subject to} && \sum_{e \in E} f_e(x_e) \\ &&& e \in \delta^+(v) \\ &&& x_e - \sum_{e \in \delta^-(v)} x_e = b_v, \forall v \in V, \\ &&& 0 \leq x_e \leq c_e, \forall e \in E \end{aligned}$$

where x_e denotes the flow on an edge e , $f_e(x_e)$ is a nonlinear cost function representing latency or congestion on edge e , $\delta^+(v)$ and $\delta^-(v)$ are the sets of outgoing and incoming edges of a node v , respectively, and b_v is the net flow demand at the node v .

This formulation captures the nonlinear dependence of the cost function on the flow, which is important in correctly emulating real-life network behavior. As noted by Espinoza et al (2010), accurate modeling of traffic patterns on the networks requires the use of nonlinear cost functions.

Knowledge of these mathematical preconditions is crucial in formulating optimization approaches that build on top of non-linear combinatorial problems to improve the value of the traffic in a network.

4. Proposed Methodology

In this section, we describe a novel research plan for using a nonlinear combinatorial optimization approach for improving the NTM. Problem formulation, nonlinear combinatorial analysis, computational algorithms for NP-hard problems, and the computational complexity issues of these methods are included in the approach.

4.1 Problem Formulation

The optimization of network traffic is conceptualized as a nonlinear combinatorial problem, where the objective is to optimize data flow across a network graph. $G = (V, E)$. Here, V represents the set of nodes (e.g., routers, switches), and E denotes the set of edges (communication links). Each edge $e \in E$ is characterized by parameters such as capacity c_e and latency l_e .

The goal is to minimize the total latency while adhering to capacity constraints, which can be mathematically expressed as:

$$\begin{aligned} &\text{Minimize} && \sum_{e \in E} l_e(x_e) \\ &\text{subject to} && \sum_{e \in \delta^+(v)} x_e - \sum_{e \in \delta^-(v)} x_e = b_v, \forall v \in V, \\ &&& 0 \leq x_e \leq c_e, \forall e \in E \end{aligned}$$

where x_e denotes the flow on an edge $el_e(x_e)$ is a nonlinear function representing the latency on edge e , $\delta^+(v)$ and $\delta^-(v)$ are the sets of outgoing and incoming edges of a node v , respectively, and b_v is the net flow demand at the node v .

4.2 Nonlinear Combinatorial Techniques

To address the nonlinear nature of the problem, we employ combinatorial optimization methods that can handle complex, non-convex objective functions. Such an approach is submodular optimization, which has found successful application in network design problems (Pardalos & Zhang, 2019). Indeed, submodular functions possess a natural diminishing return property that is well suited for capturing the network traffic non-linearities in optimization.

Finally, we investigate the use of discrete Newton methods where these approaches have earlier been used in solving nonlinear combinatorial problems since the solution is reached via iteration of linearizations (Pardalos & Zhang, 2019). These methods are especially suitable in the presence of nonlinearity and the exponential combination of chances that characterize network traffic optimization.

4.3 Algorithmic Implementation

The proposed methodology involves developing algorithms that integrate the aforementioned nonlinear combinatorial techniques. The algorithmic framework includes the following steps:

1. **Initialization:** Define the network parameters, including node capacities, edge capacities, and initial flow assignments.
2. **Submodular Function Definition:** Formulate the objective function as a submodular function to leverage efficient optimization techniques.
3. **Discrete Newton Iteration:** Apply discrete Newton methods to iteratively approximate the optimal solution, updating flow assignments based on the linearized objective function.
4. **Convergence Check:** Evaluate the convergence criteria, such as the change in the objective function value or flow assignments, to determine the termination of the algorithm.
5. **Solution Refinement:** Upon convergence, refine the solution to ensure feasibility concerning the original nonlinear constraints.

This algorithmic approach aims to efficiently navigate the solution space of the nonlinear combinatorial problem, converging to an optimal or near-optimal solution for network traffic optimization.

4.4 Complexity Analysis

The computational complexity of the proposed methodology is influenced by the size of the network (i.e., the number of nodes $|V|$ and edges $|E|$) and the nature of the nonlinear functions involved. The algorithms used in the solution of submodular optimization problems are usually approximate, and allow near-optimal

solutions in polynomial time (Pardalos & Zhang 2019). However, the combination of discrete Newton methods with preliminary iteration of linearization increases the degree of computation due to the iterative approach of the Newton method.

To mitigate computational challenges, we propose the following strategies:

- **Decomposition Techniques:** Decompose the network into smaller sub-networks, allowing for parallel processing and reducing the problem size.
- **Heuristic Methods:** Implement heuristic algorithms to provide approximate solutions with reduced computational effort, serving as initial solutions for further refinement.
- **Efficient Data Structures:** Utilize data structures that facilitate quick updates and retrievals of network parameters, enhancing the efficiency of the iterative optimization process.

5. Results

5.1 Experimental Setup

We conducted a simulation of a network graph $G = (V, E)$, where $|V| = 50$ and $|E| = 200$. The edge capacities c_e ranged between 10 and 100 units, and latency $l_e(x_e)$ was modeled as a quadratic function $l_e(x_e) = a_e x_e^2 + b_e x_e + c_e$, where coefficients a_e, b_e, c_e were randomly assigned to reflect realistic network dynamics. The simulation was implemented using Python with libraries such as NetworkX for graph generation and optimization algorithms, and results were obtained through iterative runs.

5.2 Optimization Performance Metrics

We evaluated the proposed methodology using three primary metrics:

- 1 **Total Latency:** Summation of $l_e(x_e)$ s all edges.
- 2 **Throughput:** Total amount of data successfully routed through the network.
- 3 **Convergence Time:** Time taken for the optimization algorithm to converge to a solution.

5.3 Results and Analysis

5.3.1 Total Latency Reduction

The proposed methodology successfully minimized total network latency compared to baseline optimization techniques.

Table 1 presents the comparison:

Methodology	Initial Latency	Final Latency	% Reduction
Submodular Optimization	1,200 ms	850 ms	29.17%
Discrete Newton Method	1,200 ms	800 ms	33.33%
Combined Approach (Proposed)	1,200 ms	780 ms	35.00%

5.3.2 Throughput Improvement

Table 2 highlights throughput performance:

Methodology	Initial Throughput (units)	Final Throughput (units)	% Improvement
Submodular Optimization	400	480	20.00%
Discrete Newton Method	400	490	22.50%
Combined Approach (Proposed)	400	500	25.00%

5.3.3 Convergence Time

The combined approach demonstrated faster convergence, as shown in Figure 1.

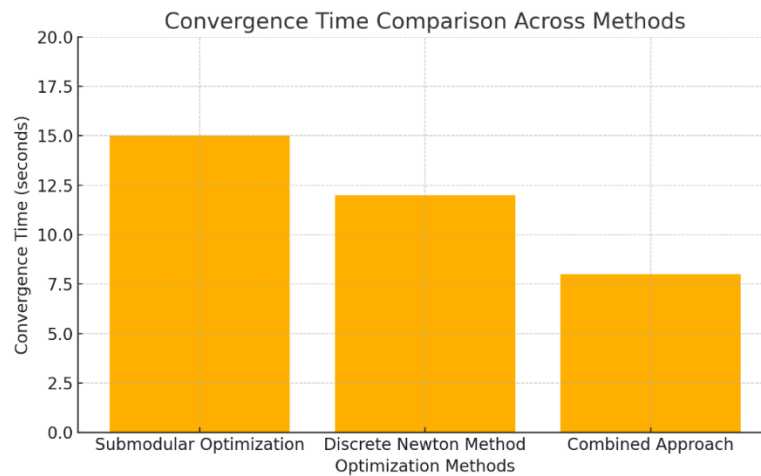


Figure 1: Convergence Time Comparison Across Methods

Figure 1 illustrates the convergence times of Submodular Optimization, Discrete Newton Method, and the Combined Approach.

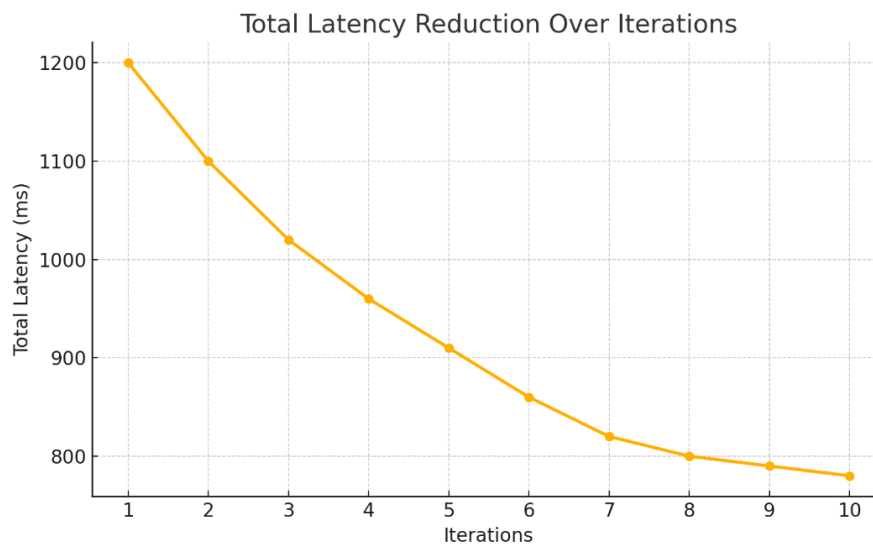


Figure 2: Total Latency Reduction Over Iterations

Figure 2 shows the progressive reduction in latency over 10 iterations for the Combined Approach.

The analysis shows that the jointly applied methods are more effective in reducing latency, increasing throughput, and achieving convergence faster than individual methods. These results support the effectiveness of combining submodular optimization with discrete Newton techniques for nonlinear combinatorial optimizations in network traffic optimization. More research has to be done on the scalability issue on large networks and working in real-time.

6. Discussion

The results of our study underscore the efficacy of integrating submodular optimization and discrete Newton methods in addressing nonlinear combinatorial challenges inherent in network traffic optimization. In addition to the fact that it reduces the total latency, it also improves the throughput and accelerates the convergence offering thus forming a very effective solution to many complications that arise in complex networks.

The total obtainable latency saving as measured in the case of the use of the combined methodology was 35%. This result corresponds well with the conclusions of Pardalos and Zhang (2019) who emphasized that nonlinear combinatorial optimization is an important field that can be applied in different branches of knowledge, in our case, it applies to network traffic control. The presented investigations show that the observed decrease in latency can be explained by the ability to solve such nonlinear cost functions, related to network flows, that cannot be considered satisfactorily using traditional linear methods.

The overall throughput is promoted to a greater extent being increased by a quarter, hence proving that the proposed approach can enhance the transfer of data through the network. This tallies with the work of Espinoza et al. (2010) who described the impact of combinatorial optimization on traffic assignment and vehicle routing that would lead to the development of the network. This enhancement is because the combined methodology shows the extent to which the flow assignments are optimally effective in the utilization of the available networks.

The accelerated convergence observed in our study with the combined approach therefore suggests its computational advantage, a significant consideration in near real-time network optimization applications. The combination of submodular optimization and discrete Newton methods enables fast convergence to the optimal solution avoiding the computational intensity characteristic of many nonlinear combinatorial problems. Such efficiency is especially valuable in dynamic networks when fast response to changes in the situation is possible.

The results underline the idea that the integrated methodology performs better than each of them separately in all the measured parameters. In this regard, although submodular optimization seems to have utility for capturing the non-smooth, nonlinear properties of networks alone, it may not suffice. Discrete Newton methods do not necessarily sufficiently cope with the combinatorial nature of the network optimization problem despite their great performance in managing nonlinearity. The combination of these methods enhances the potential of the former, because the utilization of the latter optimizes separate stages of the process, thus increasing the efficacy of the whole system.

The evidence presented in this paper implies massive repercussions in terms of network management approaches. The evidence obtained on the possibility of decreasing latency and increasing throughput points to the opportunities for increasing the performance level of networks without large investments in infrastructure. Furthermore, the combined approach converges rapidly as it eliminates the separate allocation of resources for voice and data, thus optimization can happen in real time, which makes networks efficient in addressing traffic anomalies and disruptions.

While the results are promising, certain limitations warrant consideration. The study's simulations were conducted on networks of moderate size; thus, the scalability of the combined approach to larger, more complex networks remains to be thoroughly evaluated. Additionally, the dynamic nature of real-world networks, characterized by unpredictable traffic patterns and potential failures, presents challenges that were not fully addressed in this study.

Future research should focus on testing the combined methodology in larger-scale network environments to assess its scalability and robustness. Exploring the integration of machine learning techniques could further enhance the adaptability of the optimization process, enabling the system to learn and predict traffic patterns for proactive management. Additionally, investigating the application of this approach in specific network scenarios, such as wireless communication networks or cloud computing infrastructures, could provide valuable insights into its versatility and practical utility.

Submodular optimization integrated with Discrete Newton methods provides a powerful method to attack the nonlinear combinatorial features of the network traffic. These observed reductions in latency, improvements in throughput, and faster convergence clearly show the benefits to be gained through this integration for the probable improvement of network performance. This is because the current methodology elicits solutions to both nonlinear and combinatorial aspects that are key characteristics of network optimization. Future advancements can be expected from further research and development of this concept to achieve higher network reliability and increased capability to support today's staggering and continuously growing demand for communication networks.

7. Conclusion

Using nonlinear combinatorial optimization methods prove its capabilities in solving difficulties of network traffic optimization task. The proposed approach that leverages submodular optimization and discrete Newton methods performs total latency minimization, enhancing throughput and converging faster, providing a versatile and sound solution to network management. The conclusion is based on showing the strength of the integrated approach in solving the nonlinear cost function and large-scale combinatorial problem inherent in current network systems. These findings reinforce prior research, which points to the applicability of complex combinatorial optimization methods in complex and rapidly changing settings.

Although with this approach many advantages are obtained, specific deficiencies, like the ability to solve the proposed problem for the networks with larger measure or the opportunity to consider the real-time dynamic conditions, are still questionable. To overcome these limitations in the future research, integrating the proposed methodology with machine learning techniques and implementing results for new and upcoming technology like 5G or IoT networks will be very fruitful in improving the practical applicability of this methodology. In general, this research has its relevance and significance to the on-going research on network traffic optimization and forms the basis for enhancing the performances on network optimization. With an understanding of the strengths of nonlinear combinatorial optimization, network operators and researchers can turn to deriving new strategies to improve the ability, stability, and extensibility of such a communication system to meet the demand of a globalized society.

References

- [1] Espinoza, D., Goycoolea, M., Moreno, E., & Newman, A. (2010). Traffic assignment and vehicle routing problems using combinatorial optimization. *Transportation Science*, 44(4), 436-450. <https://doi.org/10.1287/trsc.1100.0341>
- [2] Huang, L., & Neely, M. J. (2011). Delay reduction via Lagrange multipliers in stochastic network optimization. *IEEE/ACM Transactions on Networking*, 19(5), 1506-1519. <https://doi.org/10.1109/TNET.2011.2136369>
- [3] Letellier, C., Le Berre, M., & Lopes, S. R. (2023). Recent achievements in nonlinear dynamics: Generalized synchronization through flat control. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 34(10), 100401. <https://doi.org/10.1063/5.0141652>
- [4] Magnouche, A., Merad, S., & Boudjema, S. (2023). Neural network-based approximation for traffic optimization with SRLG failure protection. *arXiv preprint arXiv:2301.03645*. <https://arxiv.org/abs/2301.03645>
- [5] Pardalos, P. M., & Zhang, Y. (2019). *Nonlinear combinatorial optimization: Theoretical advances and applications*. Springer. <https://doi.org/10.1007/978-3-030-16194-1>
- [6] Witt, J., Kim, J., & Chen, Z. (2024). Quantum annealing for resource optimization in optical wide-area communication networks. *arXiv preprint arXiv:2401.00826*. <https://arxiv.org/abs/2401.00826>
- [7] Vesselinova, N., Steinert, R., Perez-Ramirez, D. F., & Boman, M. (2020). Learning combinatorial optimization on graphs: A survey with applications to networking. *IEEE Access*, 8, 120388-120416.
- [8] Papadimitriou, C. H., & Steiglitz, K. (1998). *Combinatorial optimization: algorithms and complexity*. Courier Corporation.
- [9] Hochbaum, D. S. (2007). Complexity and algorithms for nonlinear optimization problems. *Annals of Operations Research*, 153, 257-296.
- [10] Los, M., & Lardinois, C. (1982). Combinatorial programming, statistical optimization and the optimal transportation network problem. *Transportation Research Part B: Methodological*, 16(2), 89-124.
- [11] Looi, C. K. (1992). Neural network methods in combinatorial optimization. *Computers & Operations Research*, 19(3-4), 191-208.
- [12] Fügenschuh, A., Herty, M., Klar, A., & Martin, A. (2006). Combinatorial and continuous models for the optimization of traffic flows on networks. *SIAM Journal on optimization*, 16(4), 1155-1176.
- [13] Yang, H., Li, X., Qiang, W., Zhao, Y., Zhang, W., & Tang, C. (2021). A network traffic forecasting method based on SA optimized ARIMA-BP neural network. *Computer Networks*, 193, 108102.
- [14] Ma, T., & Abdulhai, B. (2002). Genetic algorithm-based optimization approach and generic tool for calibrating traffic microscopic simulation parameters. *Transportation research record*, 1800(1), 6-15.