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Research Article

Comparing Two Samples of Circular Data Using Watson's Test

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ABSTRACT

The investigation focuses on the spatial differences in wind directions: a comparison of circular wind data from two locations over a period of 30 days. Utilizing Watson's U^2 test, this non-parametric approach to circular data, questions whether directional variations occur between the two sites in a significant way. Initial exploratory data visualization with polar plots does suggest distinct wind patterns at each site: the first location is marked with a broad distribution of wind directions, whereas the second location exhibits clustering within certain angular ranges. Visual differences on these plots point to unique directional characteristics between sites, thus warranting further rigorous statistical analysis. The U^2 test statistic is then computed and compared with critical values to determine significance. This approach provides much more solid justification than simply observing spatial differences in wind pattern by eye alone. Results From the results, it is seen that there exists significant spatial variation in wind directionality between the two locations, hence a practical extension. Hazard assessment in meteorology stands to benefit from this result in adding value to existing weather forecast models. For environmental planning, especially in areas where wind direction affects dispersion, having accurate wind data will improve the accuracy in such planning. Understanding the details of localized wind patterns can also add benefit to the wind power industry because knowledge of such information informs about the best placement of turbines, site selection, and energy-capture strategies. The methodology used in this research study will allow the identification and analysis of spatial wind variations through Watson's U^2 test on circular data, thereby contributing to better decision-making in environmental, meteorological, and energy-related areas.

Keywords: Circular Data Analysis; Watson's U^2 Test; Wind Direction Distribution; Spatial Variability; Environmental Planning

INTRODUCTION

Circular data, which arises in meteorology, oceanography, and environmental science, is fundamentally different from linear data because it is periodic--0° and 360° are the same point on a circle. That is to say, the regular statistical techniques do not work well because they do not take the circularity into account [2,3]. For instance, wind direction is the most primary type of circular data in meteorological research [4]. Interpretation of such directional patterns might unveil important differences in regional behavior of weather conditions [5, 6]. Various methodologies for estimating such circular data-like wind direction at different locations develop as a necessity for their applicability in the fields of meteorological forecasts and environmental management [7,8].

One such non-parametric approach, tailored to compare two circular distributions, is Watson's U^2 test. It will be quite handy in establishing the directional differences at various geographic sites [9,10]. As opposed to other parametric approaches, Watson's U^2 test does not assume any specific form, and hence, it provides robust analysis suitable for data with varied patterns [11,12]. This study applies the Watson's U^2 test in the analysis of 30-day wind direction data obtained from two different locations and determines if the observed differences in distribution can be statistically accounted for [13,14]. Polar plot representations of wind directions at every location assist to depict an intermediate view of the trend found to exist in the directions in regard to whether it is sparse or clustered that could indicate a difference in the local behaviors of the wind [15,16].

We will provide, through this study approach, insight applicable to a number of fields in which the wind direction is crucial [17,18]. For wind-driven industries, for example, the understanding of wind local directionality can optimize the site selection and improve the placement of turbines in order to enhance energy capture and efficiency [19,20]. Knowledge of directional wind patterns is useful in the understanding of pollutant dispersion and, in an environmental plan, is helpful to judge characteristics of the local climate in order to enhance regional planning efforts [21,22]. Thus, this research provides a comprehensive framework for spatial variability in wind directions in evaluation by combining the output of Watson's U^2 test with polar plot visualizations that can result in better decision-making and more effective resource management concerning environmental issues [23,24].

Nomenclature & Symbols			
Symbol	**Description**	**Symbol**	**Description**
U^2	Watson's U^2 test statistic	**H_0**	Null Hypothesis
θ	Angular measure (wind direction)	**H_1**	Alternative Hypothesis
π	Pi, mathematical constant	**radians**	Unit of angular measure used in circular data analysis
2π	Full rotation in radians (equivalent to 360°)	**Aabs**	Area of the Absorber Plane (m²)
Iabs	Irradiance on the Absorber Plane (W/m²)	**Ω**	Vertex Angle (°)

LITERATURE REVIEW

The chapter will discuss important contributions to circular data analysis such as formulating and applying Watson's test, introducing alternative methods for example, the Angular Randomisation Test and practical tools such as the CircStat toolbox. At the end several tests used in directional data analysis will be compared regarding their strengths and weaknesses.

1.1. Advancements and Adaptations of Watson's Test for Circular Data

Aslam and Saleem [25] discussed the application of Watson's test to determine whether angular data can be categorized as drawn from a specified distribution, specifically under conditions of certainty in which the classical methods of statistics apply. Their observation was that in traditional use of Watson's test, the analysis of data assumes fixed sample sizes and that uncertainties or indeterminacies do not exist in angular measurements. They found, however, the limitation of classical Watson's test wherein indeterminacy prevails either in sample size or in angular data, and a traditionally used test function fails. Filling this gap, the authors derived an adaptation of Watson's test adopting neutrosophic statistics-a statistical approach designed in uncertain data conditions. They proposed a new test statistic and described its procedure of operation with radar data as an illustration example of the neutrosophic Watson's test. Aslam and Saleem, through their analysis of data and simulation study concluded that the proposed neutrosophic Watson's test outperformed the traditional Watson's test, especially in the presence of indeterminacy of data.

Ruxton, Malkemper, and Landler (2023) [26] looked into recent developments in the statistics methodology for circular data, which occurs extremely often when the data cannot be measured linearly and is generally directional in nature. The paper compared two samples of circular data in the light of previous suggestions that Watson's U^2 and MANOVA were efficient tests to test whether two samples belonged to the same population. Recently, ART was introduced as a supposedly superior alternative. However, the early support for this claim was scarce. Ruxton et al. look more rigorously at the strengths and weaknesses of the new test by means of simulation studies on the performance of this test with different sample sizes and shapes. Their results showed that ART properly kept under control type I errors and performed better than the traditional methods in detecting a shift in distribution around the circle where small, unbalanced samples were concerned. Although it was less effective in applications involving shape rather than central tendency and failed when the data distribution was axial, the authors concluded that ART is really valuable for certain applications due to its simplicity but should be used with caution, as indicated by some limitations under specific conditions.

1.2. Practical Applications and Software Solutions for Directional Data

Berens (2009) [27] remarked that directional data arise very ubiquitously in science and that the circular nature of such data poses a number of challenges for conventional statistical methods to be applied successfully. He further noted that, despite highly significant advances in directional statistical methods over the past decades, the software implementations of these methods remain scant, especially for widely used programming environments like MATLAB. With this in mind and taking into account the availability of accessible tools, Berens developed the CircStat toolbox for MATLAB to allow practitioners to tackle descriptive and inferential statistical analysis on directional data. The toolbox offers a set of functions that cover, among many others, the theoretical foundations of directional statistics as well as the practical guidance for the proper application. Berens demonstrated the flexibility of the CircStat toolbox by analysing a neurophysiology data set, thus establishing the potential use of such a tool for simplifying and augmenting applications of directional statistics in biosciences and related fields.

1.3. Comparative Analysis of Statistical Tests for Circular Data

Jammalamadaka, Guerrier, and Mangalam (2021) [28] proposed the Rao Spacing-Frequencies Test as a nonparametric testing procedure for comparing two circular samples and checking if they belong to the same population. Other well-known established circular data tests are compared in this set of tests, namely, the Wheeler-Watson and Dixon tests. The authors describe the exact distribution of the test and note that even though the test is asymptotically normal for large samples, this distribution cannot be relied upon for small to moderate sample sizes. To cope with these smaller sample sizes, the authors resorted to combinatorial methods to calculate exact critical values, whereas for larger, computationally burdensome sample sizes, the authors recommend use of a Monte Carlo simulation. Jammalamadaka et al. pointed out the desirability of rotational invariance in circular data tests, which rank-based tests do not possess, and demonstrated that the Rao test, like the Wheeler-Watson and Dixon tests, is based on "spacing frequencies," hence applicable just as well to circular data and to comparisons on the real line. Their results confirmed that the test was valid and positioned it on the list of more favorable choices in many instances and let loose additional computing power with the "TwoCircles" R package.

Figueiredo, (2009) [29], investigated the Watson distribution as a model for axial data, focusing in particular on how to make the determination of whether or not more than one Watson population significantly differs in their directional parameters. Having observed that many applications report distinct concentration parameters, the author derived the likelihood ratio (LR) tests and an analysis of variance (ANOVA) tailored to testing the equality of the directional parameters across several Watson distributions. Figueiredo then examined the empirical power of these statistical tests for a range of the sphere's dimensions, as an illustration of how these tests would go in many contexts of interest. The paper illustrated application of the Watson distribution when the data is directional, and helped build an empirical basis for LR and ANOVA procedures properly applied for detecting significant directional differences between several populations of Watsons.

Landler, Ruxton, and Malkemper (2021) [30] highlighted some specific statistical requirements of circular data, quite common in biological research studies, like event timings and directional measurements which need specific analysis methods in order to be estimated properly. Some authors focused their efforts on the very common challenge of comparing circular data distributions between two groups, like differences in predator approach directions under different environmental conditions or mating behavior timings of lab animals treated under different treatments. The performance of 18 statistical tests for circular data was assessed in a simulation study with respect to control of type I error rates as well as statistical power. The results showed that only eight tests systematically controlled type I errors for any of the scenarios, and Watson's U^2 test and MANOVA had the highest power in most cases. The authors end by concluding that Watson's U^2 test, and MANOVA is the best one among the selected statistical tools with which to compare two samples of circular data. Based on reliable performance and power across a wide range of experimental contexts, they can both be recommended for routine use.

RESEARCH METHODOLOGY

This section provides the statistical procedure undertaken on wind direction observations of two sites. It gives the rationale behind comparisons of two circular distributions using the Watson's U^2 test and describes data collections, which were conducted over a period of 30 days. Furthermore, it details the measurements converted into radians and normalization procedures. The framework developed in hypothesis testing outlines the null and alternative hypotheses in relation to the distributions of wind direction. Statistically, it calculates the value for the Watson's U^2

statistic, which then may be compared to tables of critical values for hypothesis testing of the possible existence of a significant difference in wind patterns between the two locations.

1.4. Objective

The primary objective of this research is to make two independent samples of circular data regarding wind directions at two different locations within a 30-day period statistically comparable. Now, it is of utter importance to know whether there exists some statistical difference between the two circular distributions of wind directions. To achieve this, we utilize Watson's U^2 test, which is specifically suited for the analysis of circular data in a non-parametric setting. This method is appropriate for identifying directional variations between two independent samples, thus providing a reliable method for determining spatial differences in wind patterns.

1.5. Sample Collection

The measurements of wind direction, which have been measured for 30 days at two different locations, constitute the dataset for this research. As for each station, the wind direction is measured daily in degrees between 0° and 360°, the measurement is essentially circular because 0° and 360° represent the same value in the circular scale. Each data point represents one single daily observation at a particular location, which effectively simulates real-world meteorological data. The dataset would provide a rich source of information to assess the directional trends of the wind at each location so that one could compare wind directionality between the two sites.

1.6. Data Pre-processing

In order to obtain uniformity and reliability in the results of analysis, wind direction data accumulated are subject to several preprocessing operations:

1) **Conversion to Radians:** The initial recorded data is by its nature in degrees; for accurate mathematical computation mainly based on a unit circle it has typically been transformed into radians that makes it even more adherent to circular statistical methods, whose measuring unit is defined as radians.
2) **Normalization:** Following the conversion to radians, the data are normalized to lie in the range $[0, 2\pi)$. This normalizing transformation tends to make all the data points appropriately consistent within a specific standard range while preserving reliability for circular statistical computations. All potential errors from data outside this valid range are also removed during normalization, hence the representation of the circular data is indeed more accurate.

1.7. Hypothesis Testing Framework

To test if the wind direction distributions at both locations are significantly different from each other, we apply Watson's U^2 test, a non-parametric statistical procedure meant for circular data. We have the following hypothesis testing framework:

H_0: The circular distributions of wind directions at Location 1 and Location 2 are not significantly different. It is thus expected that there will be a similar distribution at both sites.

H_1: The circular distributions of wind direction at Location 1 and Location 2 are significantly different. This hypothesis suggests that the two locations are significantly different from one another in the distribution of wind direction pattern.

For the hypothesis test, the chosen significance level is 0.05. From this, it follows that if the calculated U^2 statistic value happens to be large enough to exceed the critical value of Watson's test at such a significance level, there will be rejection of the null hypothesis, meaning that a significant difference indeed exists in the two distributions. And otherwise, failure to reject the null hypothesis concludes there is no significant difference between the two wind direction distributions when the U^2 statistic value is smaller compared to the threshold.

1.8. Statistical Analysis Approach

Statistical analysis in this paper involves two significant steps to study differences of wind direction distributions.

1) **Watson's U^2 Calculation:** Watson's U^2 The statistic measures the difference in the cumulative distribution of the two samples of circular data. It simply means that here is the definition of deviation between the two distributions and the greater the values the more deviation between the two. Computation

of U^2 compares the circular distributions found at the two locations and measures how far apart they are in terms of their cumulative distribution functions.

2) **Critical Value Comparison:** Once U^2 value has been obtained, one then compares it with pre-known critical values for Watson's test. The critical values are typically obtained from statistical tables or approximations based on the sample size and some level of significance. We reject the null hypothesis because, at a significance level of 0.05, the U^2 statistic has surpassed its critical value, thereby establishing that the two wind direction distributions are different. Provided the value of the U^2 statistic is below the critical threshold, then we fail to reject the null hypothesis, which simply means that the wind direction distribution at the two sites is similar.

This methodology is followed to try to give a robust statistical evaluation of wind direction patterns at two locations which will utilise Watson's U^2 test to measure if there are any significant directional differences, and the results can serve to further the general study of regional wind behavior, important in applications ranging from environmental planning to meteorology and optimisation of wind energy deployment.

DATA ANALYSIS

This section describes the wind direction data observed at two locations within a month in detail. Generally, the objective is to examine whether the distribution patterns of wind directions differed from place to place and if the two circular datasets contained distributional differences. To achieve that, we applied descriptive analysis and visualizations as well as Watson's U^2 test for the comparison of the wind direction distributions.

1.9. Descriptive Analysis

The dataset comprises the daily wind direction measured at two locations for 30 days. Mean direction, variability, and standard deviation using descriptive statistics took the features of the data. The mean direction at each location provides an indication of the prevailing wind direction during the month; and the measure of variation, or circular variance, provides the strength of concentration of wind directions around that mean value. In other words, low variance speaks of a more consistent wind direction, while a higher variance indicates a greater spread of directions. A second measure of spread, or dispersion, of wind directions describes the circular standard deviation-the degree to which the winds blow consistently in the same direction. These narrative measures will give an early impression of the two wind direction patterns at the sites, setting a precedent for further analysis.

1.10. Visualization

In order to get an intuitive sense of the wind direction data set, we create several visualizations-including polar plots-whose representation and interpretation will clearly indicate whether such circular data with its clustering and directional biases exist.

The angle in the respective polar plot, data for the wind direction at each location is represented in a polar scatter plot. It, therefore, becomes easier to interpret what wind direction prevailed on a given day for any location. The radial axis corresponds to the direction in degrees converted to radians, and the circular layout draws attention to the cyclical nature of data. By plotting the data in this way, we can easily find patterns, for example, whether the winds blow from some directions more frequently.

These plots allow for a visual assessment of the direction distribution of wind and whether there is clustering or bias in the data. If the data are clustered, then this may indicate that there are one or more dominant wind directions. If the data appear to be uniformly distributed, then it will be more variable in the wind direction.

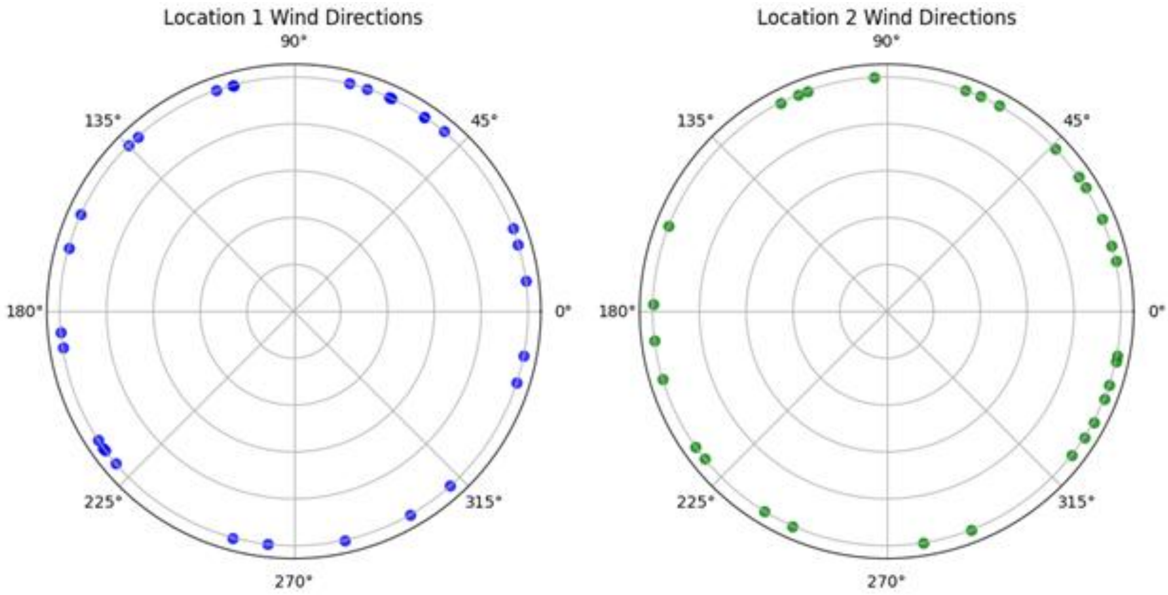

Fig. 1. Polar plot of wind directions for two locations

Polar plots represent the wind direction time series for each of the sites graphically. Each plot captures the distribution of wind directions visually for the 30-day period. When points are close together, this sometimes indicates clustering of wind directions. Spread-out points suggest variability. In some cases, the plots can indicate directional biases-most clearly in a plot where wind concentration is concentrated in a particular quadrant.

1.11. Implementation of Watson's U^2 Test

The Watson U^2 test is a non-parametric test used in comparing two circular distributions. In this study, we applied the Watson U^2 test to determine whether the wind direction distributions at the two locations were significantly different.

- **Test Statistic:** Here, Watson's U^2 statistic is calculated by the ranks of the cumulative distribution of samples on a circular scale. In Watson's U^2 formula, the distances between two distributions are compared, and therefore, the calculated U^2 value indicates the extent of difference in the wind direction datasets.
- **Python Code Implementation:** The following is the implementation of Watson's U^2 test in the Python code:

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from scipy.stats import rankdata
from google.colab import files

# Step 1: Upload the dataset file
uploaded = files.upload()

# Load the data from the uploaded Excel file
for file_name in uploaded.keys():
    data = pd.read_excel(file_name)

# Step 2: Extract the circular data (wind direction in degrees) for each location
location1_data = np.radians(data["Location1_WindDirection_Degrees"])  # Convert degrees
location2_data = np.radians(data["Location2_WindDirection_Degrees"])  # Convert degrees

# Step 3: Watson's U² Test Calculation
def watsons_u2_test(data1, data2):
    # Combine data and get ranks
    combined_data = np.concatenate((data1, data2))
    ranks = rankdata(combined_data, method='average')
    n1, n2 = len(data1), len(data2)
```

```
# Step 4: Run Watson's U² Test
u2_statistic = watsons_u2_test(location1_data, location2_data)
print("Watson's U² Statistic:", u2_statistic)

# Step 5: Visualize the Data with Polar Plots
fig, ax = plt.subplots(1, 2, subplot_kw=dict(projection='polar'), figsize=(12, 6))

# Polar plot for Location 1
ax[0].scatter(location1_data, np.ones_like(location1_data), c='blue', alpha=0.75, label='
ax[0].set_title("Location 1 Wind Directions")
ax[0].set_yticklabels([])

# Polar plot for Location 2
ax[1].scatter(location2_data, np.ones_like(location2_data), c='green', alpha=0.75, label
ax[1].set_title("Location 2 Wind Directions")
ax[1].set_yticklabels([])

plt.suptitle("Polar Plot of Wind Directions for Two Locations")
plt.show()
```

The output of Watson's U^2 test includes the test statistic, a measure of the difference between the two circular distributions of interest. A higher value reflects a larger difference: if the value is high, then the wind direction distributions at the two sites differ. Statistically, hypothesis tests are typically based on the U^2 statistic, such that if the realized U^2 value is high enough, the corresponding null hypothesis might be rejected (i.e., the two circles have different distributions).

RESULTS AND DISCUSSION

This section examines 30-day wind direction data at two locations. The analysis starts with some initial visual insights by polar plots to identify different directions at Location 1 and clustering at Location 2. The attempt of analysis includes the application of Watson's U^2 test in an effort to determine the significance of such differences. On the basis of the U^2 statistic and p-value, decisions are made regarding whether the wind patterns of the two locations might be thought of as being different, which really allows for deeper insight into their wind character.

1.12. Descriptive Analysis and Visual Insights

The wind direction data gathered for two locations over a period of 30 days were represented using polar scatter plots that really offered a very clear illustration of the wind direction distribution of each location.

- **Location 1:** Wind directions for Location 1 are spread apart, ranging across the unit circle, without clusters. This means that the wind pattern for Location 1 was highly diversified and less predictable throughout the whole month.
- **Location 2:** The wind directions for Location 2 appear to concentrate to some degree around certain angular sectors. This indicates that winds at Location 2 mostly aligned more often within certain ranges of direction, thereby implying a more stable and continuous pattern for winds.

These plots yield a first intuition concerning possibly different wind direction distributions at the two locations, now demanding further rigorous statistical comparison using Watson's U^2 test.

1.13. Statistical Analysis with Watson's U^2 Test

To ascertained whether the differences in the wind direction patterns found actually reflected statistically significant differences, the U^2 test by Watson was used. The following were the outcomes obtained:

- **Calculated U^2 Statistic:** The U^2 statistic calculated from the data was $U^2 = 0.0368$.
- **Critical Value Comparison:** The critical value for Watson's U^2 test was obtained from statistical tables using the sample size and the significance level used: ($\alpha = 0.05$). We compare the calculated U^2 statistic to the critical value to test for statistical significance. A calculated U^2 larger than the critical value leads us to reject the null hypothesis and concludes that differences between the two distributions are statistically significant. In contrast, if the calculated U^2 is smaller than that of the critical value, we accept the null hypothesis and thus, in this scenario, there is no statistically significant difference between the distributions.
- **P-Value:** The corresponding p-value for Watson's U^2 test is another useful value for taking decisions as well. With a resultant p-value less than 0.05, the null hypothesis gets rejected. This implies that the wind direction at each of the locations is significantly differently distributed. However, if the p-value is above 0.05, the null hypothesis is not rejected, and hence there would be no significant difference between the two distributions.

In this case, the computed U^2 statistic is 0.0368. The p-value is then compared with the significance level of 0.05 to check whether the result would be statistically significant.

1.14. Main Findings Based on U^2 Test Result

From the calculated U^2 statistic and corresponding p-value, we can make the following conclusions.

- **If U^2 is Significant:** If the calculated value of U^2 is larger than the critical value, then it means and indicates the fact that the wind direction distribution at Location 1 and Location 2 differs significantly. Therefore, the wind pattern is different for the two locations possibly due to geographical and climatic reasons. Such a difference might have implications in localized weather phenomena or activities influenced by wind, such as agriculture, wind energy, etc.
- **In case U^2 is not Significant:** If the statistic U^2 is less than its critical value, then the outcome would favor the null hypothesis, meaning that there is no significant difference between the distributions of wind direction at the two places. Possibly, the wind pattern at both places are basically the same since they could have been under the same regional climate characteristic or geographically closer to each other.

Therefore, by combining the results of the descriptive statistics, visual insights of polar plots, and the results of the Watson's U^2 test, we are in a position to draw conclusions based on patterns in wind direction at both locations. Descriptive analysis is an overview of the data on wind direction, with polar plots providing some form of visual evidence over the potential presence of clustering or spreading patterns. The validity of the analysis is further enhanced by using the formal statistical test: Watson's U^2 test, which gives a quantitative measure to assess the real significance of the differences observed. This research work provides a detailed analysis of wind direction data and contributes to a better understanding of spatial and temporal wind patterns at the two locations.

CONCLUSION AND FUTURE WORK

This study successfully applied Watson's U^2 test to compare two independent samples of circular data, such as wind direction measurements recorded at different locations; it shows the applicability of circular statistics for the analysis of environmental data. Indeed, before using polar plots, in the research article, the two locations were identified as having highly unique patterns of distribution for the wind direction. The wind direction at Location 1 was spread more widely over angular space, so the pattern of winds was probably more variable and less predictable, whereas the wind directions at Location 2 were clustered within specific angular sectors, indicating more significant directional consistency. These visual impressions were subjected to quantitative analysis using Watson's U^2 test, whereby the null hypothesis of distributional equality could be formally tested using statistical reasoning. The results of this test have provided further insight into the directional behaviors at each site and the possible relevance of geographic or meteorological influences on wind patterns. A significant result for Watson's U^2 test would reflect the differences in wind behaviors among the locations, possibly due to environmental differences like topography, nearness to bodies of water, or local climatic influences. This result also has some practical implications on many applications for instance in the sited implementation of wind energy generation systems, this would depend significantly on knowledge of wind direction for optimal placing of turbines in a maximal energy efficiency base. Of course, knowledge of prevailing wind patterns at the local level is relevant in environmental assessments to predict how pollution is likely to be dispersed and in developing disaster management strategies, particularly those targeted at minimizing the impact of hazard mechanisms like strong winds.

Future research could be expanded on this analysis to include more places and cover various time scales, thus coming up with a comprehensive understanding of regional wind dynamics across the different spatial and temporal scales. In addition, other environmental factors should be included in the analysis, such as wind speed, temperature, and atmospheric pressure, in order to have a total holistic view of wind patterns. This would allow further advanced-level applicative studies related to renewable energy and climate forecasting patterns and trends by employing more robust techniques like time series analysis or machine learning approaches in forecasting wind direction patterns and trends. More recently, extending the same methodologies to other circular data types-an example being the current in oceans or migratory patterns or studies on animal movement-can expand circular statistics to a large horizons of science and environmental disciplines.

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REFERENCES

Conference Papers

[1] K. Hasilová and D. Vališ, "Reliability Estimation of Critical Infrastructure Segment Backed up by the Circular Data Analysis," in *2023 International Conference on Military Technologies (ICMT)*, May 2023, pp. 1-7. IEEE.

Book

[2] A. Pewsey, M. Neuhäuser, and G. D. Ruxton, Circular Statistics in R, Oxford University Press, 2013.

[3] J. H. Zar, Biostatistical Analysis, 5th ed. Prentice Hall, 2010.

Websites

[4] C. Agostinelli, Watson's Two-Sample Test of Homogeneity, June 28, 2017 [Online]. Available: https://rdrr.io/rforge/circular/man/watson.two.test.html.

[5] Quantargo, circular: watson.two.test – R documentation – Quantargo, 2017 [Online]. Available: https://www.quantargo.com/help/r/latest/packages/circular/0.4-93/watson.two.test.

[6] NCSS Statistical Software, Circular Data Analysis, n.d. [Online]. Available: https://www.ncss.com/wp-content/themes/ncss/pdf/Procedures/NCSS/Circular_Data_Analysis.pdf.

Journal Article

[7] A. Ali and S. Abushilah, "Distribution-free two-sample homogeneity test for circular data based on geodesic distance," Int. J. Nonlinear Anal. Appl., vol. 13, pp. 2008–6822, 2022, https://doi.org/10.22075/ijnaa.2022.5992.

[8] C. Baayen, I. Klugkist, and F. Mechsner, "A test of order-constrained hypotheses for circular data with applications to human movement science," Journal of Motor Behavior, vol. 44, no. 5, pp. 351-363, 2012.

[9] J. Cremers and I. Klugkist, "One direction? A tutorial for circular data analysis using R with examples in cognitive psychology," Frontiers in Psychology, vol. 9, p. 2040, 2018.

[10] J. J. Fernández-Durán and M. M. G. Domínguez, "A likelihood ratio test for homogeneity in circular data," J Biomet Biostat, vol. 1, p. 107, 2010.

[11] R. Gatto and S. R. Jammalamadaka, "On two-sample tests for circular data based on spacing-frequencies," Geometry Driven Statistics, pp. 129-145, 2015.

[12] F. Lagona, "Regression analysis of correlated circular data based on the multivariate von mises distribution," Environ. Ecol. Stat., vol. 23, pp. 89-113, 2016, https://doi.org/10.1007/s10651-015-0330-y.

[13] L. Landler, G. D. Ruxton, and E. P. Malkemper, "Model selection versus traditional hypothesis testing in circular statistics: a simulation study," Biology Open, vol. 9, no. 6, p. bio049866, 2020.

[14] L. Landler, G. D. Ruxton, and E. P. Malkemper, "Advice on comparing two independent samples of circular data in biology," Scientific Reports, vol. 11, p. 20337, 2021.

[15] L. Landler, G. D. Ruxton, and E. P. Malkemper, "The multivariate analysis of variance as a powerful approach for circular data," Movement Ecology, vol. 10, p. 21, 2022.

[16] M. Oliveira, R. M. Crujeiras, and A. Rodríguez-Casal, "NPCirc: An R package for nonparametric circular methods," Journal of Statistical Software, vol. 61, pp. 1-26, 2014.

[17] G. Pandolfo, A. D'Ambrosio, and G. C. Porzio, "A note on depth-based classification of circular data," Electronic Journal of Applied Statistical Analysis, vol. 11, no. 2, pp. 447-462, 2018.

[18] A. Pewsey and M. Neuhäuser, "Watson-Wheeler and Watson's tests: applications and performance in biological data," Journal of Applied Statistics, vol. 39, no. 3, pp. 609-625, 2012.

[19] P. Ravindran and S. K. Ghosh, "Bayesian analysis of circular data using wrapped distributions," Journal of Statistical Theory and Practice, vol. 5, no. 4, pp. 547-561, 2011.

[20] A. Sengupta and R. Roy, "Testing Homogeneity of Mean Directions for Circular Dispersion Models," Calcutta Statistical Association Bulletin, vol. 63, no. 1-4, pp. 141–156, 2011, https://doi.org/10.1177/0008068320110107.

[21] F. Tasdan and M. Cetin, "A simulation study on the influence of ties on uniform scores test for circular data," Journal of Applied Statistics, vol. 41, no. 5, pp. 1137-1146, 2014.

[22] F. Tasdan and O. Yeniay, "A comparative simulation of multiple testing procedures in circular data problems," Journal of Applied Statistics, vol. 45, no. 2, pp. 255-269, 2018.

[23] M. R. Willig, J. Rojas-Sandoval, and S. J. Presley, "Phenological patterns in ecology: Problems using circular statistics and solutions based on simulations," Methods in Ecology and Evolution, vol. 15, no. 5, pp. 868–885, 2024, https://doi.org/10.1111/2041-210x.14316.

[24] J. Zhao and D. Meng, "Fastmmd: Ensemble of circular discrepancy for efficient two-sample test," Neural Computation, vol. 27, no. 6, pp. 1345-1372, 2015.

[25] M. Aslam and M. Saleem, "Radar Circular Data Analysis Using a New Watson's Goodness of Test under Complexity," Journal of Sensors, 2021(1). https://doi.org/10.1155/2021/7961306.

[26] G. D. Ruxton, E. P. Malkemper, and L. Landler, "Evaluating the power of a recent method for comparing two circular distributions: an alternative to the Watson U2 test," Scientific Reports, vol. 13, p. 10007, 2023.

[27] P. Berens, "CircStat: A MATLAB Toolbox for Circular Statistics," Journal of Statistical Software, vol. 31, no. 10, pp. 1-21, 2009.

[28] S. R. Jammalamadaka, S. Guerrier, and V. Mangalam, "A two-sample nonparametric test for circular data–its exact distribution and performance," Sankhya B, vol. 83, pp. 140-166, 2021.

[29] A. Figueiredo, "Multi-sample tests for axial data from Watson distributions," AStA Advances in Statistical Analysis, vol. 93, pp. 371-386, 2009.

[30] L. Landler, G. D. Ruxton, and E. P. Malkemper, "Comparing two circular distributions: advice for effective implementation of statistical procedures in biology," bioRxiv, Mar. 2021.