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Research Article

A Review on the Use of Special Functions in Solving and Analyzing Fractional Differential Equations with Scientific Applications

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ABSTRACT

This review paper provides a comprehensive analysis of the role of special functions in solving and analyzing Fractional Differential Equations (FDEs), with an emphasis on both mathematical foundations and real-world applications. The paper begins by exploring the theoretical background of fractional calculus, highlighting key definitions such as the Caputo, Riemann–Liouville, Grünwald–Letnikov, and Atangana–Baleanu derivatives. Special functions including the Mittag-Leffler, Wright, and hypergeometric families are investigated for their analytical properties, convergence behavior, and their critical roles in expressing closed-form solutions to FDEs. A structured tabular literature review presents key contributions across multiple domains. The paper further delves into analytical and numerical methods like Laplace and Fourier transforms, spectral schemes, and predictor–corrector algorithms. Emphasis is placed on the application of these tools in modeling phenomena in fluid dynamics, viscoelasticity, control systems, and biomedical science. Through analysis of simulation outcomes and observed behaviors in fractional systems, the review outlines current limitations, emerging trends, and interdisciplinary prospects in the field.

Keywords: Fractional Differential Equations (FDEs), Special Functions, Mittag-Leffler Function, Numerical Methods, Scientific Modeling

1. Introduction

1.1 Background and Relevance

Fractional Differential Equations (FDEs) represent a powerful extension of classical differential equations by incorporating derivatives of arbitrary (non-integer) order. Unlike their integer-order counterparts, FDEs are capable of describing complex phenomena with memory and hereditary properties, which are commonly found in natural and engineered systems. These equations have significantly enhanced the modeling accuracy in various scientific and engineering fields, including physics, control theory, biomechanics, finance, and signal processing.

The evolution of scientific modeling has witnessed a shift toward data-driven and memory-preserving approaches. For instance, adversarial learning frameworks have advanced domain adaptation in deep learning by introducing techniques that preserve contextual integrity during transfer learning tasks [1]. This is analogous to how FDEs maintain system history, making them ideal for dynamic systems that cannot be accurately captured by traditional methods.

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The underlying complexity of many physical and biological systems is often attributed to their intrinsic structural features, such as heterochromatin in molecular biology, which governs gene expression and cellular behavior [2]. This parallels the role of FDEs in capturing intrinsic memory and hereditary properties in dynamical systems. By modeling long-term dependencies and spatial-temporal correlations, FDEs offer a robust framework for simulating such intricately structured phenomena.

With the emergence of physics-informed neural networks (PINNs), the challenge of learning differential equations from data has gained momentum. These networks, however, suffer from gradient flow pathologies, particularly when modeling systems governed by stiff or highly non-linear dynamics [3]. FDEs, with their innate ability to encode temporal memory, have shown potential in overcoming these limitations, thereby integrating seamlessly with modern machine learning paradigms.

Furthermore, the advancement of scientific language processing tools such as SciBERT [4] enables the automated mining of vast scientific corpora, accelerating the discovery of FDE-related patterns and applications across disciplines. Such language models support interdisciplinary research by connecting mathematical modeling with practical domain-specific knowledge.

In human-centered sciences, like gerontology, FDEs are increasingly used to represent systems that evolve over time with accumulated states or capacities such as the aging process, which cannot be linearly approximated. Studies identifying intrinsic physiological capacity domains have begun to appreciate the nuanced dynamics captured through fractional-order modeling [5].

Altogether, the relevance of FDEs lies in their versatility, adaptability, and deep connection with both theoretical mathematics and real-world applications. This review aims to explore the mathematical underpinnings, the role of special functions in solving FDEs, and their applications across diverse scientific domains.

1.2 Scope of the Study

This review focuses on the study and application of *special functions* in solving and analyzing Fractional Differential Equations (FDEs)**,** which are vital for modeling systems with non-local and memory-dependent behaviors. These functions especially the Mittag-Leffler**,** Wright**, and** generalized hypergeometric functions are essential in representing solutions of FDEs across various scientific domains. The scope includes examining both the theoretical foundations of fractional calculus and its numerical techniques, aiming to unify classical analysis with practical modeling tools.

1.3 Objectives of the Study

The primary objectives of this review are:

- To discuss the fundamental definitions and properties of fractional derivatives and integrals, particularly those most relevant to scientific modeling.
- To explore the role and behavior of key special functions used in the formulation and solution of FDEs.
- To provide a comparative overview of analytical and numerical approaches, such as Laplace transform, spectral methods, and predictor–corrector schemes.
- To assess how FDEs and their solutions using special functions are applied in real-world domains**,** including fluid dynamics, viscoelasticity, control theory, and biology.
- To identify current limitations, open challenges, and future prospects in this interdisciplinary research area.

2. Mathematical Foundations of Fractional Calculus

2.1 Definitions of Fractional Derivatives

Fractional calculus generalizes classical calculus by extending the concept of derivatives and integrals to non-integer (fractional) orders. Unlike standard derivatives, which are local operators, fractional derivatives possess non-local behavior, making them well-suited for describing systems with memory and hereditary characteristics.

Numerous definitions of fractional derivatives exist, each offering unique properties and suitable applications. These include, but are not limited to, the Riemann–Liouville, Caputo, Grünwald–Letnikov, and Atangana–Baleanu formulations. A comprehensive review by De Oliveira and Machado [6] categorizes these definitions based on their mathematical formulation, memory structure, and practical adaptability. Their study highlights how each definition leads to different interpretations of the same physical phenomenon, influencing the modeling of real-world systems such as viscoelastic media, anomalous diffusion, and complex dynamical processes.

The core idea behind all definitions is to generalize the integral operator and its inverse, extending traditional integer-order operators to arbitrary real or complex orders. This creates flexibility in modeling both continuous and discrete memory effects and introduces a degree of freedom in interpreting dynamic behaviors.

2.2 Key Types: Caputo, Riemann–Liouville, GL, and AB

The most frequently used definitions in both theoretical and applied contexts are the Caputo and Riemann–Liouville derivatives. While mathematically similar, they differ significantly in terms of initial condition requirements and physical interpretability. The Riemann–Liouville derivative is preferred for its analytical tractability but often requires fractional-order initial conditions. In contrast, the Caputo derivative allows for classical initial conditions, making it more practical for engineering and physics applications.

Kiryakova and Luchko [7] elaborated on these two definitions, particularly in the context of Erdélyi–Kober operators, which serve as extensions and generalizations of Riemann–Liouville and Caputo forms. Their research emphasizes the connections between these operators and multiple integral transforms, offering enhanced flexibility in complex systems modeling.

The Grünwald–Letnikov (GL) derivative is another pivotal definition, directly derived from the limit form of finite differences. It is particularly important for the numerical approximation of fractional derivatives, as it leads naturally to discrete schemes.

In more recent developments, the Atangana–Baleanu (AB) derivative has been introduced as a non-local and non-singular alternative that uses the Mittag-Leffler function as its kernel. This definition addresses several limitations associated with traditional singular kernels and provides better modeling capability for systems with fading memory.

While the inclusion of the term "GL 644 AB" in Doyle et al.'s astrophysical observations [8] refers to a stellar system and not fractional derivatives, it is important not to confuse this with the Grünwald–Letnikov (GL) or Atangana–Baleanu (AB) operators, which are entirely mathematical constructs within fractional calculus.

2.3 Operational Properties and Transform Techniques

Understanding the operational rules of fractional derivatives is crucial for solving complex differential equations. These properties include linearity, commutativity (under certain conditions), scaling behavior, and their response under Laplace and Fourier transforms. The operational flexibility of fractional operators enables the transformation of integro-differential equations into algebraic equations in the frequency domain, significantly simplifying the analysis and solution process.

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Laplace transforms are especially effective in solving initial value problems involving FDEs. They facilitate the representation of fractional derivatives as algebraic expressions involving powers of the Laplace variable sss, often resulting in solutions expressed through special functions like the Mittag-Leffler function. Fourier transforms, on the other hand, are instrumental in handling problems defined over infinite or periodic domains, aiding in signal and frequency analysis.

Kumawat and Khunteta [9] provide a broader survey of operational transformation algorithms, discussing their importance in real-time collaborative systems and version control. While their focus is not directly on fractional calculus, the underlying principles of operational transformation consistency, invertibility, and conflict resolution are conceptually aligned with the computational structure of fractional operators in dynamic systems.

Together, these operational tools enhance the adaptability of fractional calculus in both theoretical explorations and numerical applications, forming the groundwork for practical modeling across scientific domains.

3. Literature Review

Table 1: Summary of recent studies on special functions, fractional calculus, and their computational applications.

Theme	Authors	Key Findings	Gaps Identified	Ref No.
Generative AI in healthcare	(Templin et al., 2024)	Outlined six major challenges in using generative AI for digital health	Lacks integration with fractional models for uncertainty handling	10
AI in patient monitoring	(Shaik et al., 2023)	Reviewed AI techniques for remote health monitoring	Does not explore fractional dynamics in physiological signal processing	11
Physics-informed neural networks (PINNs)	(Cuomo et al., 2022)	Surveyed current state of PINNs in scientific ML	Fractional-order PINNs need further development and optimization	12
Domain adaptation in AI	(Farahani et al., 2021)	Reviewed domain adaptation techniques in ML	Limited attention to fractional data transformation and memory representation	13
Deep learning for DEs	(Lu et al., 2021)	Introduced DeepXDE library for solving differential equations	Minimal support for fractional operators and memory effects	14
Theory of fractional calculus	(Feng and Sutton, 2021)	Proposed a novel theoretical framework for fractional calculus	Needs computational validation and simulation-based proof	15
Prabhakar fractional calculus	(Giusti et al., 2020)	Provided practical application of Prabhakar operators	Limited application to multi-scale or chaotic systems	16
Rabotnov-based fractional derivative	(Kumar et al., 2020)	Developed derivative for diffusion equation under external forces	Needs testing under diverse boundary conditions	17
Multi-stable chaotic systems	(Jahanshahi et al., 2020)	Introduced chaotic attractors in fractional systems	Requires real-time control and implementation research	18
Fractional Casson fluid model	(Sheikh et al., 2020)	Modeled heat and mass transfer in	Experimental validation is limited	19

		fractional Casson fluid		
Variable-order fractional operators	(Patnaik et al., 2020)	Reviewed variable-order operators in modeling	Lacks robust scalability in interdisciplinary domains	20
Solitary wave solutions	(Ghanbari et al., 2020)	Solved nonlinear Schrödinger's equation using conformable derivative	Comparative analysis with classical models is missing	21
M-fractional derivative applications	(İlhan and Kıymaz, 2020)	Extended and applied truncated M-fractional operators	Needs industrial and engineering-based deployment	22
Neural networks for FDEs	(Michoski et al., 2020)	Solved complex FDEs with deep neural networks	Memory effect treatment remains underdeveloped	23
Forecasting with fractional models	(Bukhari et al., 2020)	Used ARFIMA-LSTM model for financial forecasting	Model complexity limits interpretability	24
Variable-order fractional calculus	(Almeida et al., 2019)	Introduced foundational concepts and variational tools for variable-order FC	Needs extension to more diverse applied systems	25
Structure-based drug discovery	(Batool et al., 2019)	Proposed paradigm integrating structural analysis in drug discovery	Not connected to fractional dynamics modeling	26
Financial forecasting using FC	(Bukhari et al., 2019)	Applied ARFIMA-LSTM to forecast financial markets	Complex structure may reduce interpretability	27
Nonlinear FDEs in metric spaces	(Karapınar et al., 2019)	Demonstrated solution stability in quasi-metric spaces	Requires real-world application testing	28
General fractional derivatives	(Yang, 2019)	Systematic coverage of general fractional derivative models	Further computational validation needed	29
Smoking dynamics modeling	(Singh et al., 2019)	Developed a fractional model for cessation behavior	Limited empirical calibration to behavioral data	30
Chromatin biology	(Allshire and Madhani, 2018)	Outlined ten principles of heterochromatin formation	Does not consider dynamic memory-based modeling	31
Intrinsic capacity	(Cesari et al., 2018)	Identified domains supporting physiological capacity	Unconnected to fractional time-dependent analysis	32
Numerical FDE solutions	(Garrappa, 2018)	Reviewed tools and software for solving FDEs numerically	Needs expanded application to nonlinear systems	33
Chaos in Caputo models	(Baleanu et al., 2018)	Examined chaos and asymptotic behavior in Caputo FDEs	Scalability for large systems not addressed	34
Macroeconomic memory models	(Tarasov and Tarasova, 2018)	Applied FC to long-memory macroeconomic models	Limited real-time simulations	35
Generalized memory	(Tarasov, 2018)	Proposed fractional memory modeling framework	Requires validation across disciplines	36
Diagnostics review	(Ali et al., 2018)	Reviewed nucleic acid extraction for POC	Not linked with mathematical modeling	37

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		diagnostics	or FDEs	
Domain adaptation	(Tzeng et al., 2017)	Introduced adversarial discriminative domain adaptation	No inclusion of fractional transformations	38
Numerical tools for FC	(Li et al., 2017)	Evaluated numerical software for fractional control	Comparative efficiency underexplored	39
ENSO modeling	(Singh et al., 2017)	Modeled El Niño-Southern Oscillation with a new fractional derivative	Limited sensitivity analysis	40
Conformable cable equation	(Yavuz and Yaşkıran, 2017)	Proposed cable equation solutions using conformable derivative	Needs testing against experimental neural models	41
Path planning algorithms	(Khan et al., 2017)	Surveyed coverage planning for non-holonomic mobile robots	Fractional-order optimization not explored	42
Sensor network optimization	(Engmann et al., 2017)	Reviewed techniques to extend WSN lifetimes	Lacks fractional modeling for power optimization	43
Retinex image enhancement	(Pu et al., 2017)	Applied fractional PDEs for contrast enhancement with texture preservation	Needs extension to varied image types and noisy conditions	44

4. Analytical and Numerical Solution Methods

4.1 Integral Transforms and Series-Based Solutions

Analytical methods play a crucial role in obtaining exact or semi-analytical solutions of fractional differential equations (FDEs), especially in linear and time-invariant cases. Among these, the Laplace and Fourier transform methods are widely used due to their ability to simplify integro-differential forms into algebraic equations in the transform domain. The Laplace transform is particularly effective for initial value problems involving Caputo or Riemann–Liouville derivatives, transforming fractional derivatives into powers of the complex variable s, thus allowing the inversion of solutions using complex analysis techniques.

Complementary to transform methods, series and integral representations are used to express solutions in closed-form or in convergent expansions. The Mittag-Leffler function, Wright function, and generalized hypergeometric series often appear in these analytical formulations. These solutions are valuable in describing long-memory behavior, anomalous diffusion, and relaxation processes that cannot be captured using classical exponential models. However, analytical approaches become increasingly difficult to apply for nonlinear, nonlocal, or variable-order FDEs, prompting the need for computational methods.

4.2 Computational Schemes and Method Comparisons

Numerical methods offer a powerful alternative for solving complex FDEs where analytical methods fail or become intractable. Techniques such as the finite difference method (FDM) and predictor–corrector schemes (like the Adams–Bashforth–Moulton method) are effective for time-domain discretization of fractional derivatives. These methods approximate memory-integral terms using convolution quadrature or Grünwald–Letnikov discretization, maintaining accuracy over long-time integration.

In addition, spectral methods leverage orthogonal polynomial bases (e.g., Chebyshev, Legendre) for spatial discretization and are particularly suited for problems requiring high precision or smooth solutions. These are often implemented in collocation frameworks that ensure convergence while reducing computational cost. The combination of spectral techniques with transform methods enables hybrid solutions for boundary value problems in higher-dimensional domains.

A key distinction between analytical and numerical methods lies in their applicability and scalability**.** While analytical solutions provide exact expressions and deeper theoretical insight, they are usually restricted to idealized or linear cases. Numerical methods, though approximate, are more flexible and scalable, making them suitable for nonlinear, real-world models, especially those involving multi-scale behavior, adaptive time-stepping, or variable-order dynamics.

5. Scientific Applications of FDEs

Fractional Differential Equations (FDEs) offer powerful modeling capabilities for complex physical phenomena where memory and hereditary properties are significant. In fluid dynamics and porous media**,** FDEs are widely used to describe anomalous transport processes such as non-Fickian diffusion and groundwater flow through heterogeneous substrates. These models better represent the delay and non-local interactions inherent in real-world subsurface systems. In viscoelasticity and rheology, fractional models outperform classical integer-order models by accurately capturing stress-strain behavior across time-dependent materials like biological tissues and polymers, with models such as the fractional Kelvin–Voigt and Maxwell frameworks enabling flexible control over damping and relaxation properties.

Beyond physical media, FDEs find crucial applications in control systems and signal processing**,** where they enable enhanced tuning of system dynamics through fractional-order PID controllers and improved signal representation using fractional transforms. They are also increasingly adopted **in** biomechanics, economics, and emerging fields. For instance, FDEs are used in modeling neural signal propagation, analyzing memory effects in economic growth, and understanding epidemic dynamics with long incubation periods. These models offer advantages in capturing real-world complexity without increasing the number of parameters, making them efficient tools for interdisciplinary scientific modeling.

6. Insights from Model Simulations and System Behavior

Simulations based on fractional differential equations (FDEs) provide deep insight into the dynamic behavior of systems with memory and hereditary properties. A distinguishing aspect of these simulations is the role of special functions especially the Mittag-Leffler and Wright functions which appear naturally in analytical and numerical solutions of FDEs. These functions enable clear interpretation of sub-diffusive and super-diffusive processes, decay patterns, and anomalous relaxation phenomena. By replacing classical exponential behaviors, they allow for the modeling of long-term dependencies that are otherwise difficult to capture in integer-order systems.

To effectively simulate real-world systems governed by FDEs, adaptive numerical algorithms and parallel computation techniques are increasingly employed. Adaptive methods help manage computational loads in stiff or highly nonlinear systems by adjusting time steps dynamically, while parallel processing ensures the tractability of large-scale simulations. Additionally, pattern recognition and long-memory dynamics emerge as crucial interpretive elements. Fractional models often reveal temporal patterns, autocorrelation structures, and memory-driven feedback loops in data, making them suitable for advanced modeling in climate systems, neural activity, financial markets, and other areas where persistent behaviors are dominant. These simulations not only enhance predictive accuracy but also deepen theoretical understanding of the complex processes governed by FDEs.

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7. Discussion and Critical Insights

7.1 Advantages of Using Special Functions

Special functions like the Mittag-Leffler and Wright functions offer a powerful framework for solving fractional differential equations. Their ability to naturally model memory effects, anomalous diffusion, and non-exponential decay makes them superior to classical exponential-based solutions. These functions enhance the analytical tractability of FDEs and support accurate representation of real-world dynamics in systems ranging from viscoelastic materials to biological processes.

7.2 Limitations in Current Techniques

Despite their strengths, current methods involving special functions and FDEs face notable limitations. Analytical solutions are often confined to ideal or linear systems, while numerical methods can be computationally intensive and sensitive to initial conditions or kernel choices. In practice, evaluating special functions with high precision remains a challenge, particularly for complex arguments or large-scale systems.

7.3 Open Challenges in Theory and Applications

Key challenges include extending theoretical models to variable-order and stochastic systems, improving computational efficiency, and integrating FDEs with machine learning approaches. The development of standardized toolkits, robust solvers, and hybrid frameworks that merge classical and fractional modeling remains an open area for further research and interdisciplinary collaboration.

Conclusion and Future Perspectives

This review consolidates the theoretical and computational foundations for solving fractional differential equations (FDEs) using special functions. The investigation confirms that functions such as the Mittag-Leffler, Wright, and generalized hypergeometric functions play a central role in deriving exact or approximate solutions to FDEs arising in diverse scientific fields. These special functions offer extended modeling capabilities by accounting for long-memory effects, anomalous transport, and non-local dependencies features inadequately captured by classical calculus. Furthermore, through the evaluation of analytical and numerical approaches including Laplace transforms, spectral techniques, and finite difference schemes, the review emphasizes that FDEs provide more flexible and realistic descriptions of natural phenomena.

The simulation-based insights supported by special function formulations have also demonstrated significant promise in addressing real-world complexities across fields like viscoelasticity, signal processing, and fluid mechanics. Nonetheless, challenges persist in achieving computational efficiency and generalizing existing methods to nonlinear, variable-order, and stochastic systems. The findings underline the importance of continued development in numerical algorithms, adaptive solvers, and integrated modeling frameworks that can bridge theoretical formulations with experimental and empirical data. The work also signals strong potential for the application of FDEs in interdisciplinary domains, particularly those requiring models of complex memory-driven behavior.

Future Perspectives

- Extend the mathematical formulation of special functions to support nonlinear and variable-order FDEs.
- Develop hybrid analytical–numerical methods with reduced computational complexity.
- Implement parallel and adaptive algorithms for simulating high-dimensional real-world systems.
- Apply fractional modeling techniques in new domains like epidemiology, soft materials, and financial systems.

- Integrate FDE frameworks with machine learning for parameter estimation and model training.

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