Journal of Information Systems Engineering and Management
2024, 9(4)
e-ISSN: 2468-4376
https://www.jisem-journal.com/

Research Article

Soft Bipartite Graph: A New Bipartite Graph in the Horizon

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ARTICLE INFO

Received: 18 Oct 2024

Revised: 10 Nov 2024

Accepted: 27 Dec 2024

ABSTRACT

In order to discuss uncertainty in soft graphs, a new graph valued function called the Soft Bipartite graph is introduced in this paper. Let $V = \{x_1, x_2, x_3, \ldots, x_k\}$ be a non-empty set, $V_1 = \{X_1, X_2, X_3, \ldots, X_n\}$ and $V_2 = \{Y_1, Y_2, Y_3, \ldots, Y_m\}$ be two partitions of V, where $X_i \subseteq V(i = 1,2, \ldots, n)$ and $Y_j \subseteq V(j = 1,2, \ldots, m)$. The Soft bipartite graph, G_{sb} is a bipartite graph with $V(G_{sb})) = V_1 \cup V_2$ and any two vertices $X_i \in V_1$ and $Y_j \in V_2$ are adjacent if and only if $X_i \cap Y_j \neq \emptyset$. Several basic properties of this newly defined graph are investigated. We also present an application of this concept in the decision-making problem.

Keyword: Soft set, Soft graph, Soft Bipartite graph .

1. INTRODUCTION

Many problems in engineering, medical science, economics and so forth, have various uncertainties. Molodtsov [5] introduced the concept of soft set theory as a mathematical tool for dealing with uncertainties. It has been shown that soft sets have potential applications in various fields. Graph theory was first introduced by the Swiss mathematician Leonhard Euler [2]. Since then, graph theory has become a most important part of combinatorial mathematics. A graph is used to create a relationship between a given set of elements. Each element can be represented by a vertex and the relationship between them can be represented by an edge. Graphs have been widely applied across various fields to model and solve real-world problems. The concept of soft graphs and their different operations can be seen in [7]. A number of generalizations of soft graphs are available in [1]. In this paper, we define a new graph valued function named as soft bipartite graph and some basic properties of this graph are investigated. Also, an application of this concept in the decision- making problem is presented.

2. Preliminaries

In this section the basic ideas regarding graphs, soft sets and soft graphs are given.

Definition 2.1 [6]: A graph $G = (V, E)$ is a pair of sets, where V is a finite nonempty set called the vertex set and E is a set of unordered pairs of distinct vertices called the edge set. An edge of a graph that joins a vertex to itself is called a self-loop. More than one edge between a pair of vertices is called multigraph and these edges are called parallel edges. A graph is called a simple graph if it has no self-loops and multiple edges.

Definition 2.2 [6]: A bipartite graph is a graph whose vertex set V can be partitioned into two sets V_1 and V_2 such that every edge connects a vertex in V_1 to one in V_2.

Definition 2.3 [4]: Let T be the set of parameters. A pair (K, T) is called a soft set over the set U of the universe, where $K : T \to P(U)$ is a set valued mapping and $P(U)$ is the power set of U.

Definition 2.4 [7]: A quadruple (G, λ, μ, T) is called a soft graph, where

i) $G = (V, E)$ is a simple graph

ii) (λ, T) is a soft set over V

iii) (μ, T) is a soft set over E

Journal of Information Systems Engineering and Management
2024, 9(4)
e-ISSN: 2468-4376
https://www.jisem-journal.com/

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$(\lambda(a), \mu(a))$ is a subgraph of G for all $a \in T$.

Any undefined definition can be found in [3].

3. Soft bipartite graph

In this section we introduce the concept of a Soft bipartite graph and its properties.

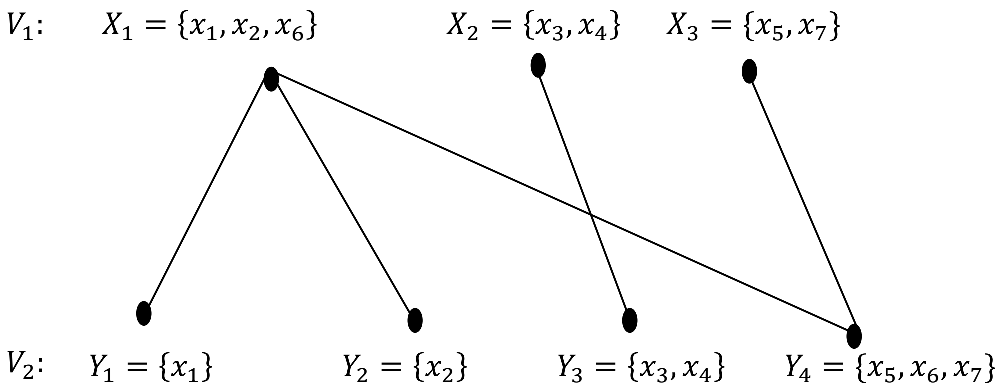
Definition: Let $V = \{x_1, x_2, x_3, \ldots, x_k\}$ be a non-empty set (sample set). $V_1 = \{X_1, X_2, X_3, \ldots, X_n\}$ and $V_2 = \{Y_1, Y_2, Y_3, \ldots, Y_m\}$ be two partitions of V, which represents parameters, where $X_i \subseteq V$and $Y_j \subseteq V, 1 \leq i \leq n, 1 \leq j \leq m$. The **soft bipartite graph** $\boldsymbol{G_{sb}}$, is a bipartite graph with $V_1 \cup V_2$ as vertices and any two vertices $X_i \in V_1$and $Y_j \in V_2$ are adjacent if and only if $X_i \cap Y_j \neq \emptyset$.

Note: $X_i \cap Y_j$ gives the elements of V satisfying the parameters X_i and Y_j.

Example:

Let $V = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$ such that $V_1 = \{\{x_1, x_2, x_6\}, \{x_3, x_4\}, \{x_5, x_7\}\} = \{X_1, X_2, X_3\}$ and $V_2 = \{\{x_1\}, \{x_2\}, \{x_3, x_4\}, \{x_5, x_6, x_7\}\} = \{Y_1, Y_2, Y_3, Y_4\}$.

Then G_{sb} is:



Theorem 3.1: G_{sb} has no isolated vertex.

Proof:

Suppose G_{sb} has an isolated vertex $X_k \in V_1, 1 \leq k \leq n$ or $Y_k \in V_2, 1 \leq k \leq m$. Then $X_k \cap Y_j = \emptyset, \forall j$ or $X_i \cap Y_k = \emptyset$, $\forall i$ which is a contradiction to the fact that $V = X_1 \cup X_2 \cup \ldots \cup X_n = Y_1 \cup Y_2 \cup \ldots \cup Y_m$.

Theorem 3.2: For any $v \in V(G_{sb}), deg(v) = k$ if and only if $v \subseteq X_1 \cup X_2 \cup \ldots \cup X_k$ and $v \cap \{X_i\}_{i=k+1}^{n} = \emptyset$ Or $v \subseteq Y_1 \cup Y_2 \cup \ldots \cup Y_k$ and $v \cap \{Y_j\}_{j=k+1}^{m} = \emptyset$.

Proof:

By definition of G_{sb}, $deg(v) = k$ if and only if the elements of v are distributed to $'k' X_i's$, say, $X_1, X_2, X_3, \ldots, X_k$ or $'k' Y_j's$, say, $Y_1, Y_2, Y_3, \ldots, Y_k$.Then $v \cap \{X_i\}_{i=1}^{k} \neq \emptyset$ and $v \cap \{X_i\}_{i=k+1}^{n} = \emptyset$. Therefore, $v \subseteq X_1 \cup X_2 \cup \ldots \cup X_k$ and $v \cap \{X_i\}_{i=k+1}^{n} = \emptyset$. If the elements of v are distributed to $'k' Y_j's$, then $v \cap \{Y_j\}_{j=1}^{k} \neq \emptyset$ and $v \cap \{Y_j\}_{j=k+1}^{m} = \emptyset$, that is, $v \subseteq Y_1 \cup Y_2 \cup \ldots \cup Y_k$ and $v \cap \{Y_j\}_{j=k+1}^{m} = \emptyset$.

Theorem 3.3: For any $v \in V(G_{sb}), deg(v) = 1$ if and only if $X_i \cap Y_j = X_i (or\ Y_j)$ for some $X_i \in V_1, Y_j \in V_2$.

Proof:

Journal of Information Systems Engineering and Management
2024, 9(4)
e-ISSN: 2468-4376
https://www.jisem-journal.com/

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Suppose $deg(v) = 1$, then $N\left(v = X_i(or\ Y_j)\right) = Y_j(or\ X_i)$ for some $X_i \in V_1$ and $Y_j \in V_2$, that is, $X_i \subseteq Y_j$ or $Y_j \subseteq X_i$. Hence $X_i \cap Y_j = X_i(or\ Y_j)$.

Conversely, for any $v \in V(G_{sb})$, then $v = X_i(or\ Y_j)$. Suppose, $X_i \cap Y_j = X_i$ then $X_i \subseteq Y_j$ which implies $N(X_i) = Y_j$. So $deg(X_i) = 1$. Suppose, $X_i \cap Y_j = Y_j$ then $Y_j \subseteq X_i$ which implies $N(Y_j) = X_i$. So $deg(Y_j) = 1$. Hence for any $v \in V(G_{sb}), deg(v) = 1$.

Theorem 3.4: G_{sb} is a complete bipartite graph if and only if $X_i \cap Y_j \neq \emptyset, \forall i, j$.

Proof:

$G_{sb}$is a complete bipartite graph if and only if $deg(X_i) = m$ for each $X_i \in V_1$ and $deg(Y_j) = n$ for $Y_j \in V_2$. Then by Theorem 3.2, $X_i \subseteq Y_1 \cup Y_2 \cup \ldots \cup Y_m$ and $Y_j \subseteq X_1 \cup X_2 \cup \ldots \cup X_n$, that is, $X_i \cap \{Y_j\}_{j=1}^{m} \neq \emptyset$ for each $1 \leq i \leq n$ and $Y_j \cap \{X_i\}_{i=1}^{n} \neq \emptyset$ for each $1 \leq j \leq m$.

Hence $X_i \cap Y_j \neq \emptyset, \forall i, j$.

Corollary 3.4.1: G_{sb} is a regular graph if and only if $|V_1| = |V_2|$ |and $X_i \cap Y_j \neq \emptyset, \forall i, j$.

Corollary 3.4.2: G_{sb} is a biregular graph if and only if $|V_1| \neq |V_2|$ and $X_i \cap Y_j \neq \emptyset, \forall i, j$.

Theorem 3.5: G_{sb} is a cycle if and only if $|V_1| = |V_2|$ and for any $v \in V(G_{sb})$, $v \subseteq (Y_1 \cup Y_2)$ or $(X_1 \cup X_2)$ and $v \cap [\{Y_j\}_{j=3}^{m} or \{X_i\}_{i=3}^{n}] = \emptyset$, where $X_i \in V_1$ and $Y_j \in V_2$, $1 \leq i \leq n, 1 \leq j \leq m$.

Proof:

Any graph is a cycle graph if and only if $deg(v) = 2$. Using Theorem 3.2, $v \subseteq \{X_1 \cup X_2\}$ or $\{Y_1 \cup Y_2\}$ and $v \cap [\{Y_j\}_{j=3}^{m} or \{X_i\}_{i=3}^{n}] = \emptyset$, for every $v \in V(G_{sb})$. Using Corollary 3.4.1, we get $|V_1| = |V_2|$.

Hence the proof.

Theorem 3.6: A graph G_{sb} is planar if and only if G_{sb} has no sub graph homeomorphic to $K_{3,3}$.

Proof:

Suppose G_{sb} is a planar graph. By Kuratowski's theorem, G_{sb} has no sub graph homeomorphic to either K_5 or $K_{3,3}$. Since G_{sb} is bipartite, it cannot have sub graph homeomorphic to K_5. Hence the proof.

Theorem 3.7: G_{sb} is disconnected if and only if $\{X_1 \cup X_2 \cup \ldots \cup X_i\} = \{Y_1 \cup Y_2 \cup \ldots \cup Y_j\}$ for $i < n$ and $j < m$ and $\{X_1 \cup X_2 \cup \ldots \cup X_i\} \cap \{Y_{j+1} \cup Y_{j+2} \cup \ldots \cup Y_m\} = \emptyset$ and/or $\{Y_1 \cup Y_2 \cup \ldots \cup Y_j\} \cap \{X_{i+1} \cup X_{i+2} \cup \ldots \cup X_n\} = \emptyset$.

Proof:

Suppose $\{X_1 \cup X_2 \cup \ldots \cup X_i\}$= $\{Y_1 \cup Y_2 \cup \ldots \cup Y_j\}$ for $i < n$ and $j < m$ and $\{X_1 \cup X_2 \cup \ldots \cup X_i\} \cap \{Y_{j+1} \cup Y_{j+2} \cup \ldots \cup Y_m\} = \emptyset$ and/or $\{Y_1 \cup Y_2 \cup \ldots \cup Y_j\} \cap \{X_{i+1} \cup X_{i+2} \cup \ldots \cup X_n\} = \emptyset$. Then, for any $i < n$, $X_i \subset \{Y_1 \cup Y_2 \cup \ldots \cup Y_j\}$ or for any $j < m$, $Y_j \subset \{X_1 \cup X_2 \cup \ldots \cup X_i\}$. So, each X_i is adjacent with Y_1 and/or $Y_2$and/or Y_3...and/or Y_j. Similarly, Y_j is adjacent with X_1 and/or X_2 and/or X_3... and/or X_i . Therefore, $\{X_1, X_2, \ldots, X_i, Y_1, Y_2, \ldots, Y_j\}$ lies in one component of G_{sb}. Since$\{X_1 \cup X_2 \cup \ldots \cup X_i\} \cap \{Y_{j+1} \cup Y_{j+2} \cup \ldots \cup Y_m\} = \emptyset$, any of $X_1, X_2, \ldots, X_i's$ are not adjacent with any of $Y_{j+1}, Y_{j+2}, \ldots, Y_m$. Therefore G_{sb} is disconnected.

Conversely, suppose G_{sb} is disconnected, then there exists $X_{i+1} \in V_1$ for $i + 1 \leq n$ such that $X_{i+1} \cap \{Y_1 \cup Y_2 \cup \ldots \cup Y_j\} = \emptyset$, $j < m$. Since G_{sb} has no isolates, there exists some Y_{j+1} such that $X_{i+1} \cap Y_{j+1} \neq \emptyset$ which implies that $\{X_{i+1}, Y_{j+1}\}$ lies in one component of G_{sb}. If there exists any other $X_{i+2} \in V_1$ such that $X_{i+2} \cap \{Y_1 \cup Y_2 \cup \ldots \cup Y_j\} = \emptyset$, then $X_{i+2} \cap Y_{j+1} \neq \emptyset$ and/or$X_{i+2} \cap Y_{j+2} \neq \emptyset$. So $\{X_{i+1}, X_{i+2}, Y_{j+1}, Y_{j+2}\}$ lies in same component of G_{sb}.

Journal of Information Systems Engineering and Management
2024, 9(4)
e-ISSN: 2468-4376
https://www.jisem-journal.com/

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Therefore$\{X_{i+1} \cup X_{i+2}\} \cap \{Y_1 \cup Y_2 \cup \ldots \cup Y_j\} = \emptyset$. If there exists some more vertices in V_1 which are not subsets of $\{Y_1 \cup Y_2 \cup \ldots \cup Y_j\}$, then, $\{Y_1 \cup Y_2 \cup \ldots \cup Y_j\} \cap \{X_{i+1} \cup X_{i+2} \cup \ldots \cup X_n\} = \emptyset$.. Since the above intersection is empty, and G_{sb} has no isolates, $\{Y_1, Y_2, \ldots, Y_j\} \in V_2$ must be adjacent with some $\{X_1, X_2, \ldots, X_i\} \in V_1$ so that $\{X_1 \cup X_2 \cup \ldots \cup X_i\} = \{Y_1 \cup Y_2 \cup \ldots \cup Y_j\}$

Similarly, we can prove that $\{X_1 \cup X_2 \cup \ldots \cup X_i\} \cap \{Y_{j+1} \cup Y_{j+2} \cup \ldots \cup Y_m\} = \emptyset$.

Theorem 3.8: G_{sb} is a tree if and only if there is no $X_1 = \{a_1, a_2, a_3, \ldots, a_p\}$ and $X_2 = \{b_1, b_2, b_3, \ldots, b_q\}$, $p, q < n$ such that $\{a_1, a_2, a_3, \ldots, a_i, b_1, b_2, b_3, \ldots, b_j\} \in Y_1$ and $\{a_{i+1}, a_{i+2}, \ldots, a_l, b_{j+1}, b_{j+2}, b_{j+3}, \ldots, b_k\} \in Y_2$where $X_1, X_2 \in V_1$ and $Y_1, Y_2 \in V_2$, $l, i < p$ and $j, k < q$.

Proof:

Suppose there exists $X_1 = \{a_1, a_2, a_3, \ldots, a_p\} \in V_1$, $X_2 = \{b_1, b_2, b_3, \ldots, b_q\} \in V_1$, $X_3 = \{c_1, c_2, c_3, \ldots, c_r\} \in V_1$,... such that $\{a_1, a_2, \ldots, a_i, b_1, b_2, \ldots, b_j, c_1, c_2, \ldots, c_h\} \in Y_1$, $\{a_{i+1}, a_{i+2}, \ldots, a_l, b_{j+1}, b_{j+2}, \ldots, b_k, c_{h+1}, c_{h+2}, \ldots, c_g\} \in Y_2$, $\{a_{l+1}, a_{l+2}, \ldots, b_{k+1}, b_{k+2}, \ldots, c_{g+1}, c_{g+2}, \ldots,\} \in Y_3$ and so on. Then $X_1, Y_1, X_2, Y_2, \ldots, X_1$ forms a cycle. Hence G_{sb} is not a tree.

Conversely, suppose G_{sb} is not a tree which implies that G_{sb} has atleast one cycle say $\{X_1, Y_1, X_2, Y_2, \ldots, X_1\}$, where $X_i \in V_1$ and $Y_j \in V_2$ Then $a_1 \in X_1 \cap Y_1, a_2 \in X_1 \cap Y_2, b_1 \in X_2 \cap Y_1, b_2 \in X_2 \cap Y_2, \ldots$ That is, $a_1, a_2 \in X_1, b_1, b_2 \in X_2, a_1, b_1 \in Y_1, a_2, b_2 \in Y_2$ and so on.

In general, there exists $X_1 = \{a_1, a_2, a_3, \ldots, a_p\}$ and $X_2 = \{b_1, b_2, b_3, \ldots, b_q\}, p, q < n, \{a_1, a_2, a_3, \ldots, a_i, b_1, b_2, b_3, \ldots, b_j\} \in Y_1$ and $\{a_{i+1}, a_{i+2}, \ldots, a_l, b_{j+1}, b_{j+2}, b_{j+3}, \ldots, b_k\} \in Y_2$

Hence the proof.

Theorem 3.9: Any connected graph G_{sb} is Eulerian if and only if for each $X_i \in V_1$ and $Y_j \in V_2$, $1 \leq i \leq n, 1 \leq j \leq m$, the elements of X_i and Y_j belong to an even number of Y_j's and X_i's respectively.

Proof:

Suppose G_{sb} is a Eulerian graph. Then $deg(v_i) = 2k$ for each $v_i \in V(G_{sb})$. For any $v_i \in V_1$, by Theorem 3.2,$v_i \cap \{Y_i\}_{i=1}^{2k} \neq \emptyset$ and $v_i \cap \{Y_i\}_{i=2k+1}^{m} = \emptyset$, which implies that, for any $v_i \in V_1$, the elements of v_i belong to an even number of Y_j's and similar is the situation for any $v_i \in V_2, v_i \cap \{X_i\}_{i=1}^{2k} \neq \emptyset$ and $v_i \cap \{X_i\}_{i=2k+1}^{m} = \emptyset$.Therefore for any $v_i \in V_2$, the elements of v_i belong to an even number of X_i's.

Hence the proof.

Theorem 3.10: G_{sb} is a ladder graph if and only if G_{sb} is connected with $|V_1| = |V_2| = n$ and for any $v_i \in V(G_{sb})$, $v_i \subset \{Y_{i-1} \cup Y_i \cup Y_{i+1}\}$ or $\{X_{i-1} \cup X_i \cup X_{i+1}\}$ for $i = 1,2, \ldots, n$.

Proof:

Let G_{sb} be a ladder graph. Then $|V_1| = |V_2| = n$ and for any $v_i \in V(G_{sb})$,

$$deg(v_i) = \begin{cases} 2 & \text{for i = 1 and i = n} \\ 3 & \text{for } 1 < i < n. \end{cases}$$

Therefore, $N(v_i) \in \{Y_i \cup Y_{i+1}\}$ *or* $\{X_i \cup X_{i+1}\}$ for $i = 1$, $N(v_i) \in\subset \{Y_{i-1} \cup Y_i \cup Y_{i+1}\}$ *or* $\{X_{i-1} \cup X_i \cup X_{i+1}\}$ for $1 < i < n$, $N(v_i) \in \{Y_{i-1} \cup Y_i\}$ *or* $\{X_{i-1} \cup X_i\}$ for $i = n$.

Hence $v_i \subset \{Y_{i-1} \cup Y_i \cup Y_{i+1}\}$ *or* $\{X_{i-1} \cup X_i \cup X_{i+1}\}$.

Conversely, suppose G_{sb} is a connected graph with $|V_1| = |V_2| = n$ and for any $v_i \in V(G_{sb})$, $v_i \subset \{Y_{i-1} \cup Y_i \cup Y_{i+1}\}$ *or* $\{X_{i-1} \cup X_i \cup X_{i+1}\}$. Then $deg(v_i) \leq 3$ for $i = 1,2, \ldots, n$.

Journal of Information Systems Engineering and Management
2024, 9(4)
e-ISSN: 2468-4376
https://www.jisem-journal.com/

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For $i = 1, v_1 \subset \{Y_0 \cup Y_1 \cup Y_2\}$ or $\{X_0 \cup X_1 \cup X_2\}$. Since X_0, Y_0 do not exist in G_{sb}, $v_1 \subset \{Y_1 \cup Y_2\}$ or $\{X_1 \cup X_2\}$. Then $deg(v_1) = 2$ which implies that $deg(X_1) = deg(Y_1) = 2$.

For $1 < i < n$, $v_i \subset \{Y_{i-1} \cup Y_i \cup Y_{i+1}\}$or $\{X_{i-1} \cup X_i \cup X_{i+1}\}$. Then $deg(v_i) = 3$ which implies that $deg(X_i) = 3 = deg(Y_i)$ for $1 < i < n$.

For $i = n$, $v_n \subset \{Y_{n-1} \cup Y_n \cup Y_{n+1}\}$ or $\{X_{n-1} \cup X_n \cup X_{n+1}\}$. Since X_{n+1} and Y_{n+1} do not exist in G_{sb}, $v_n \subset \{Y_{n-1} \cup Y_n\}$ or $\{X_{n-1} \cup X_n\}$.. So $deg(v_n) = 2$. That is, $deg(X_n) = deg(Y_n) = 2$. Hence G_{sb} is a ladder graph.

Theorem 3.11: If $G_{sb}$is a connected n-regular graph ($n > 2$) with $X_i \cap Y_j = \emptyset$ for $i = j$ and $X_i \cap Y_j \neq \emptyset$ for $i \neq j$, then G_{sb} is a crown graph.

Proof:

Let G_{sb} be a connected n-regular graph with the given conditions. By corollary 3.4.1, $|V_1| = |V_2|$ and $X_i \in N(Y_j)$ or $Y_j \in N(X_i)$ for all $i \neq j$, which implies that any $X_i \in V_1$ is not adjacent with $Y_i \in V_2$ and X_i is adjacent with all other Y_j's $(i \neq j)$ and vice versa. Therefore G_{sb} is a crown graph.

Theorem 3.12: For any G_{sb}, $\gamma(G_{sb}) \leq Min\{|V_1|, |V_2|\}$.

Proof:

In G_{sb}, $V_1 = \{X_1 \cup X_2 \cup \ldots \cup X_i\}$ and $V_2 = \{Y_1 \cup Y_2 \cup \ldots \cup Y_j\}$ are dominating sets of G_{sb}. Thus, $\gamma(G_{sb}) \leq Min\{|V_1|, |V_2|\}$.

4. Applications

In this section, we present an application of the soft bipartite graph in a decision-making problem. The problem we consider is as given below.

4.1. Suppose we are analyzing a dataset related to people undergoing tests for diabetes.

Let there be eight people who have undergone investigation for diabetes, forming the universe:

$V = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8\}$.

Medical experts primarily consider parameters for identifying diabetes in people. These parameters include:

$E = \{$ Blood Sugar Levels, BMI (Body Mass Index)$\} = \{V_1, V_2\}$.

Let C be the set of opinions regarding diabetes diagnosis, where $C = \{1 = Yes, 0 = No\}$.

Here is the information collected from the investigation, which involves two primary medical parameters:

Patient	Blood Sugar Level	BMI	Diabetes Diagnosis
p_1	Normal	Normal	No
p_2	Elevated	Overweight	Yes
p_3	Very High	Overweight	Yes
p_4	Normal	Normal	No
p_5	Elevated	Overweight	No
p_6	Very High	Overweight	Yes
p_7	Elevated	Overweight	Yes
p_8	Very High	Normal	No

By the above data,

1. Blood Sugar Levels: $V_1 = \{normal, elevated, very\ high\} = \{\{p_1, p_4\}, \{p_2, p_5, p_7\}, \{p_3, p_6, p_8\}\} = \{X_1, X_2, X_3\}$.

2. BMI: $V_2 = \{normal, overweight\} = \{\{p_1, p_4, p_8\}, \{p_2, p_3, p_5, p_6, p_7\}\} = \{Y_1, Y_2\}$.

Journal of Information Systems Engineering and Management
2024, 9(4)
e-ISSN: 2468-4376
https://www.jisem-journal.com/
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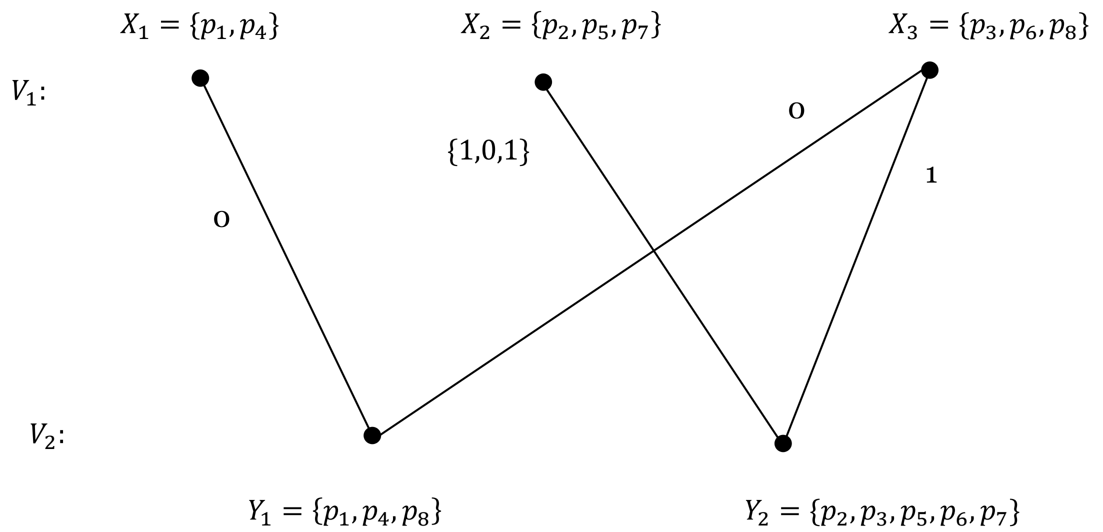
3. Diabetes Diagnosis: $C = \{1 = Yes, 0 = No\} = \{\{p_2, p_3, p_6, p_7\}, \{p_1, p_4, p_5, p_8\}\}$.

Analysis

The set of patients forms the universe V, where the patients have been categorized based on their blood sugar levels and BMI. These parameters help in identifying whether a person has been diagnosed with diabetes or not.



In the above graph, the edges are labeled '0' and/or '1', which represent the opinion with respect to diabetes diagnosis based on the parameters.

For instance, the edge between $X_1$and Y_1 which represents people who have normal blood sugar level and normal BMI is labeled '0', since $X_1 \cap Y_1 = \{p_1, p_4\}$, has no diabetes as per the report/data. Therefore, the edge (X_1, Y_1) is labeled '0'. Similarly, $X_3 \cap Y_2 = \{p_3, p_6\}$ and $\{p_3, p_6\}$ has diabetes as per the report/data. Therefore, the edge (X_3, Y_2) is labeled as '1'.

We have, $X_2 \cap Y_2 = \{p_2, p_5, p_7\} = \{1,0,1\}$. From the above graph, we observe that patients $\{p_2, p_5, p_7\}$ have same blood sugar level and BMI, report says p_2 and p_7 have diabetes and p_5 is has no diabetes which may not be correct. Therefore, the people p_2, p_5, p_7 have to undergo re-examination with respect to Diabetes.

4.2. Let us consider the Loan approval for a bank's customers based on certain financial parameters. Assume there are eight customers who have applied for a loan at a bank. The universe of customers is: $V = \{c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8\}$. Bank experts primarily consider two parameters to assess loan eligibility:

$E = \{\text{Credit Score, Annual Income}\} = \{V_1, V_2\}$.

Let C be the set of opinions regarding loan approval, where $C = \{1 = Approved, 0 = Rejected\}$.

Here is the information collected from the loan application investigation, including the two key financial parameters:

Customer	Credit Score	Annual Income	Loan Approval
c_1	High	High	Approved
c_2	Medium	High	Approved
c_3	Low	Low	Rejected
c_4	High	Medium	Approved
c_5	Low	High	Rejected
c_6	Medium	Medium	Rejected
c_7	High	Low	Approved
c_8	Low	High	Approved

Journal of Information Systems Engineering and Management
2024, 9(4)
e-ISSN: 2468-4376
https://www.jisem-journal.com/
Research Article

By the above data,

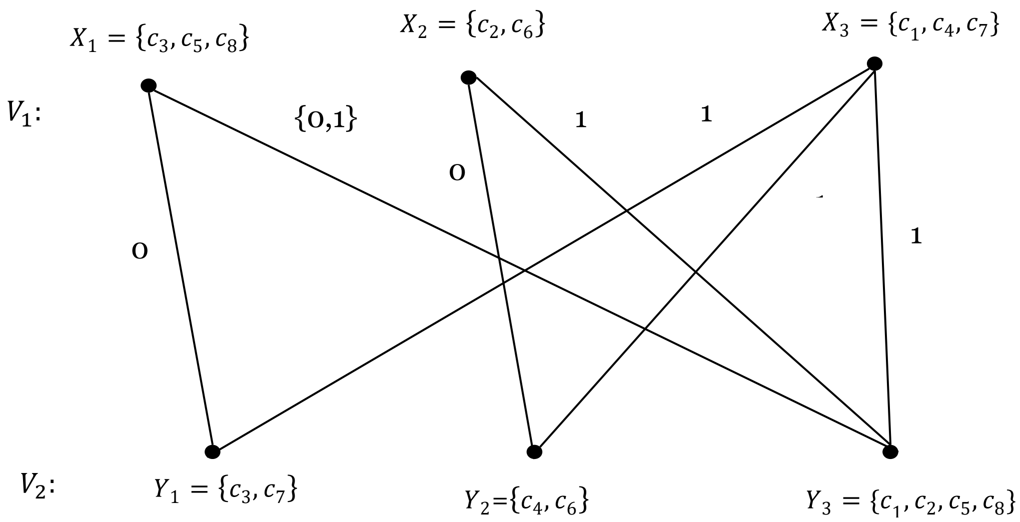
1. Credit Score: $V_1 = \{low, medium, high\} = \{\{c_3, c_5, c_8\}, \{c_2, c_6\}, \{c_1, c_4, c_7\}\} = \{X_1, X_2, X_3\}$.

2. Annual Income: $V_2 = \{low, medium, high\} = \{\{c_3, c_7\}, \{c_4, c_6\}, \{c_1, c_2, c_5, c_8\}\} = \{Y_1, Y_2, Y_3\}$.

3. Loan Approval: $C = \{1 = Approved, 0 = Rejected\} = \{\{c_1, c_2, c_4, c_7, c_8\}, \{c_3, c_5, c_6\}\}$.

Analysis

The set of customers forms the universe V, where the customers have been categorized based on their credit scores and annual incomes. These financial parameters are used to assess whether a customer is eligible for loan approval.



By the above data,

1. Credit Score: $V_1 = \{low, medium, high\} = \{\{c_3, c_5, c_8\}, \{c_2, c_6\}, \{c_1, c_4, c_7\}\} = \{X_1, X_2, X_3\}$.

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3. Loan Approval: $C = \{1 = Approved, 0 = Rejected\} = \{\{c_1, c_2, c_4, c_7, c_8\}, \{c_3, c_5, c_6\}\}$.

In the above graph, the edges are labeled '0' and/or '1', which represent the opinion with respect to loan approval based on the parameters.

For instance, the edge between $X_1$and Y_1 which represents customers who have low credit score and low annual income is labeled '0', since $X_1 \cap Y_1 = \{c_3\}$, application for loan is rejected as per the report/data. Therefore, the edge (X_1, Y_1) is labeled '0'. Similarly, $X_2 \cap Y_3 = \{c_2\}$ and c_2 application for loan is approved. Therefore, the edge (X_2, Y_3) is labeled as '1'.

We have, $X_1 \cap Y_3 = \{c_5, c_8\} = \{0,1\}$. From the above graph, we observe that patients c_5, c_8 have same financial parameters of low credit score with high income, report says loan application of c_5 is rejected and loan application of c_8 is approved, which may not be correct. Therefore, the bank customers c_5, c_8 loan applications have to undergo re-examination with respect to loan approval.

Hence, in a graph whenever an edge has more than one label one can easily conclude that those elements must be re-examined in order to avoid making wrong decision.

5. Conclusion

Graph theory is an extremely useful mathematical tool to solve complicated problems in different fields. In the decision-making problem, whenever huge collection of data is available, graph representation makes it easy to take a decision. We have illustrated through this example the application of a soft bipartite graph in the decision-making problem. Also, a case study has been taken to exhibit the technique.

REFRENCES

[1] Akram M, Nawaz S., 2015. Operations on soft graphs. Fuzzy Information and Engineering 7, 423 - 449. doi:10.1016/j.fiae.2015.11.003.
[2] Euler L., 1741. The solution of a problem relating to the geometry of position. Commentarii academiae scientiarum Petropolitanae 8, 128-140.
[3] Harary F., 1969. Graph Theory, Addison-Wesley Publishing Company.
[4] Maji P.K, Roy A.R, Biswas R., 2002. An application of soft sets in a decision-making problem. Computers & Mathematics with Applications 44, 1077 - 1083. doi:10.1016/S0898-1221(02)00216-X.
[5] Molodtsov D., 1999. Soft Set Theory-First results. Computers & Mathematics with Applications 37, 19-31. doi:10.1016/S0898-1221(99)00056-5.
[6] West D.B., 1996. Introduction to graph theory. Prentice Hall Upper Saddle River NJ USA.
[7] Zhang H, Zhan J., 2017. Rough soft lattice implication algebras and corresponding decision making methods. International Journal of Machine Learning and Cybernetics 8, 13011308. doi:10.1007/s13042-016-0502-6.
[8] P.K. Maji, R. Biswas, A.R. Roy, Soft set theory, Computers & Mathematics with Applications, Volume 45, Issues 4-5, 2003, Pages 555-562, ISSN 0898-1221
[9] Degang Chen, E.C.C. Tsang, Daniel S. Yeung, Xizhao Wang, The parameterization reduction of soft sets and its applications, Computers & Mathematics with Applications, Volume 49, Issues 5–6, 2005, Pages 757-763.
[10] Jinta Jose, Bobin George, and Rajesh K Thumbakara, Soft Graphs: A Comprehensive Survey, New Mathematics and Natural Computation, 2024, doi: 10.1142/S1793005725500474.
[11] Jyoti Dharmendra Thenge-Mashale, B.Surendranath Reddy and Rupali Shikharchand Jain, Comparative study of two Soft Graph Concepts, Global Journal of Pure and Applied Mathematics. Volume 19, Number 2 (2023), pp. 241-251.