

Thermal Convection in a Couple Stress Micropolar Fluid Flow Saturating in a Porous Medium

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ABSTRACT

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In this paper, problem of thermal convection in couple-stress micro-polar with porous medium is considered. We use linear stability mode and normal mode analysis to find out the result. Critical Rayleigh number for the onset of instability is determined analytically and graphically. For the case of stationary convection. It is found that couple stress, coupling, medium permeability and heat flux have both stabilizing and destabilizing effects under the conditions.

Keywords: thermal convection, micropolar fluid, couple stress, porous medium.

Introduction

We consider the thermal instability of an infinite horizontal layer of an incompressible couple-stress micropolar fluid[1] containing uniformly distributed dust particles and saturating an isotropic, homogeneous porous medium.[2] The fluid layer is confined between two infinite, perfectly horizontal,[3] stress-free boundaries located at $z = 0$ and $z = d$, where d denotes the depth of the fluid layer.[4] A uniform adverse temperature gradient is maintained across the layer by heating the lower boundary[5] and cooling the upper boundary, thereby establishing a steady conductive basic state.[6]

The fundamental theory of micropolar fluids was introduced by Eringen, who incorporated the effects of particle rotation and microinertia into the continuum description. Eringen subsequently[8] developed the theory of thermomicrofluids by including thermal effects in the micropolar framework.[9] These developments provided a more realistic mathematical model for studying thermal convection[10] in fluids possessing internal rotational degrees of freedom. Datta and Sastry investigated the thermal instability of a horizontal layer of micropolar fluid heated from below, while Ahmadi examined the stability characteristics of a micropolar fluid layer.[11] Their studies demonstrated that micropolar effects can significantly modify the critical conditions for the onset of convection.[12]

The porous medium is assumed to obey Darcy's law together with the Boussinesq approximation.[13] The fluid is considered to be electrically non-conducting, incompressible and isotropic,[14] while the suspended dust particles are assumed to be identical, spherical, uniformly distributed[15] and sufficiently dilute so that particle-particle interactions, collisions and volume[16] fraction effects may be neglected. The interaction between the fluid and the suspended particles is taken into account through the Stokes drag force.[17]

The micropolar characteristics of the fluid are incorporated through the inclusion of microrotation, spin inertia and vortex viscosity, whereas the couple-stress effects are represented[18] by the higher-order viscosity terms.[19] The porous matrix is characterized by constant permeability and porosity.[20] Initially, the entire system is in a quiescent equilibrium state with no fluid motion or particle motion.[21] When the imposed temperature gradient exceeds a certain critical value,[22] the conductive equilibrium loses its stability and convective motion sets in[23].

2.1 Governing Equations

Under the Boussinesq approximation, the governing equations describing the motion of an incompressible couple-stress micropolar fluid containing suspended dust particles and saturating a homogeneous porous medium consist of the conservation of mass, conservation of linear momentum, conservation of angular momentum, conservation of thermal energy and the equation of state. The porous medium is assumed to be isotropic and homogeneous with constant permeability and porosity. The interaction between the fluid and dust particles is represented through the drag force, whereas the micropolar characteristics are incorporated by considering the microrotation vector and spin inertia. The governing equations may therefore be written as follows.

Continuity Equation

The equation expressing the conservation of mass is

$$\nabla \cdot \mathbf{q} = 0, \quad (1)$$

where $\mathbf{q} = (u, v, w)$ denotes the fluid velocity vector.

Momentum Equation

The conservation of linear momentum for the couple-stress micropolar fluid in a porous medium is expressed as

$$\frac{d\mathbf{v}}{dt} = \varepsilon \left[-\nabla \left(\frac{p}{\rho_0} \right) + \frac{\kappa}{\rho_0} \nabla \times \boldsymbol{\omega} - \frac{1}{\varepsilon \rho_0 k_1} (\mu + \kappa) \mathbf{v} - \frac{\rho g \hat{\mathbf{e}}_z}{\rho_0} + \frac{1}{\varepsilon} \left(\nu - \frac{\mu'}{\rho_0} \nabla^2 \right) \nabla^2 \mathbf{v} \right] \quad (2)$$

where p is the hydrodynamic pressure, g denotes the acceleration due to gravity, ρ represents the density, μ is the dynamic viscosity, κ is the vortex viscosity coefficient, k_1 is the permeability of the porous medium, ε is the porosity of the porous matrix and $\mathbf{F}_d$ denotes the interaction force due to suspended dust particles.

Angular Momentum Equation

The conservation of angular momentum governing the microrotation field is given by

$$\rho_0 j \left(\frac{\partial \boldsymbol{\omega}}{\partial t} + (\mathbf{q} \cdot \nabla) \boldsymbol{\omega} \right) = (\alpha + \beta) \nabla (\nabla \cdot \boldsymbol{\omega}) + \gamma \nabla^2 \boldsymbol{\omega} + \frac{\kappa}{\varepsilon} \nabla \times \mathbf{q} - 2\kappa \boldsymbol{\omega}, \quad (3)$$

where $\boldsymbol{\omega}$ denotes the microrotation vector, j is the micro-inertia per unit mass and α , β and γ are spin-gradient viscosity coefficients.

Temperature Equation

The thermal energy equation for the fluid-porous system may be written as

$$[\rho_0 C_v \varepsilon + \rho_s C_s (1 - \varepsilon)] \frac{\partial T}{\partial t} + \rho_0 C_v (\mathbf{q} \cdot \nabla) T = k_T \nabla^2 T + \delta (\nabla \times \boldsymbol{\omega}) \cdot \nabla T, \quad (4)$$

where T is the temperature, C_v is the specific heat of the fluid at constant volume, C_s is the specific heat of the porous solid matrix, k_T denotes the effective thermal conductivity and δ is the micropolar heat-conduction coefficient.

Equation of State

Under the Boussinesq approximation, the density variation is assumed to be linearly dependent on temperature and is expressed as

$$\rho = \rho_0[1 - \alpha(T - T_0)], \quad (5)$$

where α is the coefficient of thermal expansion and T_0 is the reference temperature.

$$\mathbf{q} = \mathbf{q}_b(0,0,0), \mathbf{\omega} = \mathbf{\omega}_b(0,0,0) \quad \mathbf{p} = \mathbf{p}_b(0,0,0), \rho = \rho_b(0,0,0), \mathbf{T} = \mathbf{T}_b(0,0,0).$$

Substituting the basic state into the momentum equation yields the hydrostatic pressure distribution

$$\frac{dp_b}{dz} + \rho_b g = 0. \quad (6)$$

Similarly, the steady-state temperature distribution is obtained from the energy equation as

$$T_b(z) = T_0 - \beta z \quad (7)$$

Where

$$\beta = \frac{T_2 - T_1}{d}$$

where T_2 and T_1 denote the constant temperatures of the lower and upper boundaries, respectively. The quantity β represents the imposed uniform vertical temperature gradient responsible for initiating buoyancy-driven convection. When this temperature gradient exceeds a certain critical value, the conductive equilibrium state loses its stability and convective motion begins.

$$\rho = \rho_0(1 + \alpha\beta z) \quad (8)$$

Where b use for initial state

LINEARIZE PERTURBATION EQUATIONS

We consider small perturbation for stability around basic state. The basic state is

$$\mathbf{v} = \mathbf{v}_b + \mathbf{u}, \vartheta = \vartheta_b + \omega, \mathbf{p} = p_b(z) + \delta p, \varrho = \varrho_b(z) + \delta \varrho, \mathbf{T} = T_b(z) + \theta$$

where $u(u_x, u_y, u_z)$, $\omega(u_x^l, u_y^l, u_z^l)$, $\omega(\omega_x, \omega_y, \omega_z)$, $\delta \varrho$, δp , θ are used for perturbation in velocity $\mathbf{v}$, particle velocity $\mathbf{v}_d$ spin ϑ , pressure p , density ϱ , temperature T respectively. Equation (1) – (5) after perturbation are,

$$\nabla \cdot \mathbf{u} = 0, \quad (9)$$

$$\frac{d\mathbf{u}}{dt} = \varepsilon \left(-\nabla \frac{\delta p}{\rho_0} + \frac{\kappa}{\rho_0} \nabla \times \mathbf{\omega} - \frac{1}{\varepsilon \rho_0 k_1} (\mu + \kappa) \mathbf{u} + g \alpha \theta \hat{\mathbf{e}}_z + \frac{1}{\varepsilon} \left(\nu - \frac{\mu'}{\rho_0} \nabla^2 \right) \nabla^2 \mathbf{u} \right). \quad (10)$$

$$\rho_0 j \frac{d\mathbf{\omega}}{dt} = (\varepsilon' + \beta') \nabla (\nabla \cdot \mathbf{\omega}) + \frac{\kappa}{\varepsilon} \nabla \times \mathbf{u} + \gamma \nabla^2 \mathbf{\omega} - 2\kappa \mathbf{\omega}. \quad (11)$$

$$(E + b^l \varepsilon) \frac{d\theta}{dt} = (k_T \nabla^2 \theta - \delta (\nabla \times \mathbf{\omega})_z \beta + \rho_0 C_v \beta u_z) + \frac{b^l \beta u_z}{L_1}. \quad (12)$$

$$\text{Where } b^l = \frac{m N_o C_{pt}}{\varrho_o C_v}, E = \varepsilon + \frac{\varrho_s C_s}{\varrho_o C_s} (1 - \varepsilon) \text{ and } L_1 = \left(\frac{m}{K}, \frac{\partial}{\partial t} + 1 \right)$$

On putting, $t = \frac{\rho_0 d^2}{\mu} t^*$, $z = z^* d$, $\theta = \theta^* \beta d$, $u = \frac{\kappa_T}{d} u^*$, $\omega = \frac{\kappa_T}{d^2} \omega^*$, $p = \frac{\mu \kappa_T}{d^2} p^*$, $u^l = \frac{\kappa_l}{d} (u^l)^*$, $L_1 = \pi \frac{\partial}{\partial t^*} + 1$ where $\pi = \frac{m\mu}{K \rho_0 d^2}$ in equation (9) - (12) and removing stars for convenience. We get non-dimensional equations as,

$$\nabla \cdot \mathbf{u} = 0, \quad (13)$$

$$\frac{d\mathbf{u}}{dt} = \varepsilon \left[-\nabla \delta p + \kappa (\nabla \times \boldsymbol{\omega}) - \frac{1}{\kappa_1} (1 + K) \mathbf{u} + R \theta \hat{\mathbf{e}}_z + F \left(1 - \frac{\mu'}{\rho_0} \nabla^2 \right) \mathbf{u} \right] \quad (14)$$

$$j \frac{d\boldsymbol{\omega}}{dt} = C_1 \nabla (\nabla \cdot \boldsymbol{\omega}) + K \left(\frac{1}{\varepsilon} \nabla \times \mathbf{u} - 2\boldsymbol{\omega} \right) - C_0 \nabla \times \nabla \times \boldsymbol{\omega}. \quad (15)$$

$$L_1 E_1 P_1 \frac{d\theta}{dt} = L_1 [\nabla^2 \theta - \bar{\delta} (\nabla \times \boldsymbol{\omega})_z + u_z] \quad (16)$$

where new non-dimensional coefficient are,

$$\bar{\delta} = \frac{\delta}{\rho_0 C_v d^2}, \quad \bar{j} = \frac{j}{d^2}, \quad K = \frac{\kappa}{\mu}, \quad \bar{k}_1 = \frac{k_1}{d^2}, \quad C_1 = \frac{\varepsilon^l + \beta^l}{\mu d^2}$$

$$C_0 = \frac{\gamma}{\mu d^2}, \quad F = \frac{\rho_0 d^2}{\mu \varepsilon}, \quad E_1 = E + b l_E, \quad f = \frac{m N_o}{\rho_o}$$

$$P_1 = \frac{\mu}{\rho_0 \kappa_T}, \quad R = \frac{g \alpha \beta \rho_0 d^4}{\mu \kappa_T}$$

Here R = Rayleigh number and P_1 = Prandtl number.

LINEAR THEORY AND DISPERSION RELATION

In linear stability we only consider the linear term and due to very small disturbance the non-linear term in namely $(u \cdot \nabla) \omega$, $\nabla \theta \cdot \nabla \times \omega$, $(u \cdot \nabla) u$, $(u \cdot \nabla) \theta$ may be neglected.

Apply curl operator twice on (14) and take only z-component, then linearized form of (14) is

$$\left[\frac{1}{\varepsilon} \frac{\partial}{\partial t} + \frac{1}{\kappa_1} (1 + K) \right] \nabla^2 u_z = \left[R \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) + K \nabla^2 \xi + F \left(L - \frac{\mu'}{\rho_0} \nabla^2 \right) \nabla^2 u_z \right]. \quad (17)$$

Where $\xi = \left(\frac{\partial \omega_y}{\partial x} - \frac{\partial \omega_x}{\partial y} \right) = (\nabla \times \boldsymbol{\omega})_z$ consider only z-components

Apply curl operator once on (15) and take only z-component, then linearized form of (15) is

$$j \frac{\partial \xi}{\partial t} = C_0 \nabla^2 \xi - K \left(\frac{1}{\varepsilon} \nabla^2 u_z + 2\xi \right). \quad (18)$$

Where K use for coupling between vorticity and spin effects and C_0 use for spin diffusion.

Now, linearized form of (16) is

$$EP \frac{\partial \theta}{\partial t} = \nabla^2 \theta + u_z - \bar{\delta} \xi. \quad (19)$$

NORMAL MODE ANALYSIS

By normal mode method we assume solution of equation (17) - (19) are

$$[u_z, \xi, \theta] = [U(z), G(z), \Theta(z)] \exp[i(k_x x + k_y y) + \sigma t]. \quad (20)$$

Now we consider $k = (k_x^2 + k_y^2)^{\frac{1}{2}}$, k is wave number and σ (complex number in general) is stability parameter

After use of (20) equation (17) – (19) become,

$$(D^2 - k^2) \left[\varepsilon^{-1} \sigma - M + B(D^2 - k^2) + \frac{1}{k_1} (1 + K) \right] U = K(D^2 - k^2)G - R\Theta k^2. \quad (21)$$

$$[J\sigma + 2A - (D^2 - k^2)]G = -A \varepsilon^{-1} (D^2 - k^2)U. \quad (22)$$

$$[E_1 P_1 \sigma - (D^2 - k^2)]\Theta = -\delta G + U b l. \quad (23)$$

Where $A = \frac{K}{C_0}$, $l = j \frac{A}{K}$, $D = \frac{d}{dz}$, $M = F_v$, $B = F \frac{\mu l}{q_0}$

Now on eliminating G and Θ between (22) – (24), we get,

$$(D^2 - k^2) \left[\varepsilon^{-1} \sigma - M + \frac{1}{k_1} (1 + K) + B(D^2 - k^2) \right] [l\sigma - (D^2 - k^2) + 2A][EP\sigma - (D^2 - k^2)]U \\ = -Rk^2 [l\sigma + (D^2 - k^2)(\delta \varepsilon^{-1} A - 1) + 2A]U - KA\sigma \varepsilon^{-1} PE(D^2 - k^2)U + KA\varepsilon^{-1} (D^2 - k^2)U \quad (24)$$

Now boundary condition transforms to

$$U = D^2 U = 0, G = 0 = \Theta \text{ at } z = 0 \text{ and } 1 \quad (25)$$

For proper solution of U characterizing lowest mode is

$$U = U_0 \sin \pi z \quad (26)$$

where U_0 is constant.

Put value of U by (26) in (24) and also put $\pi^2 + k^2 = b$ then we get

$$b[\varepsilon^{-1} \sigma - M - Bb + \frac{1}{k_1} (1 + K) + \frac{f}{\varepsilon} \sigma][l\sigma + b + 2A][E_1 P_1 \sigma + b] \\ = Rk^2 [(l\sigma + b + 2A)b - \delta \varepsilon^{-1} bA] + KA\sigma \varepsilon^{-1} P_1 E_1 b^2 + KA\varepsilon^{-1} b^3. \quad (27)$$

ANALYTICAL DISCUSSION

(i) Stationary convection

At stationary convection, when stability sets, the marginal state will be characterized by $\sigma = 0$. So, put $\sigma = 0$ in (27), we get

$$b^2 \left[\frac{(1+K)}{k_1} - M - Bb \right] (2A + b) = Rk^2 [(2A + b)b - \delta \varepsilon^{-1} bA] + KA\varepsilon^{-1} b^3. \\ R = \frac{\left[\left(\frac{1+K}{k_1} - M - 2BA - KA\varepsilon^{-1} \right) b^3 + \left[\frac{2A(1+K)}{k_1} - 2AM \right] b^2 - Bb^4}{k^2 [(2A + b)(1 - \delta \varepsilon^{-1} A)]}. \quad (28)$$

This relation is in Rayleigh Number R as a function of parameters K(coupling), B(couple-stress), k_1 (permeability) and δ (heat flux).

For behavior of couple stress(B), we find $\frac{dR}{dB}$

$$\frac{dR}{dB} = \frac{-2Ab^3 - b^4}{k^2 [2A + b(1 - \delta \varepsilon^{-1} bA)]} \quad \text{So, Couple-stress has a destabilizing effect on the system under the condition} \\ 1 - \delta \varepsilon^{-1} bA > 0 \quad (29)$$

For behavior of coupling(K), we find $\frac{dR}{dK}$

$$\frac{dR}{dK} = \frac{\left(\frac{b^3}{k_1} - A\varepsilon^{-1} b^3 + \frac{2A}{k_1} b^2 \right)}{k^2 [(2A + b)b - \delta \varepsilon^{-1} bA]}. \quad \text{So, Coupling has a stabilizing effect on the system under the condition } \frac{1}{k_1} - A\varepsilon^{-1} > 0 \text{ and } \delta \varepsilon^{-1} bA < 0$$

On the other hand, coupling has a destabilizing effect on system under condition $\frac{b^3}{k_1} - A\varepsilon^{-1}b^3 + \frac{2A}{k_1}b^2 < 0$ and $\delta\varepsilon^{-1}bA < 0$ (30)

For behavior of permeability ($\bar{k}_1$) we find $\frac{dR}{dk_1}$

$$\frac{dR}{dk_1} = \frac{-\frac{(1+K)b^3}{k_1^2} - \frac{2A(1+K)}{k_1^2}b^2}{k^2[(2A+b)bl - \delta\varepsilon^{-1}bA]}.$$

So, Permeability has a stabilizing effect on the system under the condition

$$A=0 \text{ and } 1+K < 0$$

On the other hand, permeability has a destabilizing effect under the condition

$$0 < \delta\varepsilon^{-1}A \quad (31)$$

For behavior of heat flux ($\bar{\delta}$) we find $\frac{dR}{d\delta}$

$$\frac{dR}{d\delta} = \frac{\left[\left(\frac{1+K}{k_1}\right) - M - 2BA - KA\varepsilon^{-1}\right]b^3 + \left[\frac{2A(1+K)}{k_1} - 2AM\right]b^2 - Bb^4 - k^2b\varepsilon^{-1}A}{k^4[(2A+b)b\delta - \delta\varepsilon^{-1}A]^2}.$$

So, heat flux has a stabilizing effect on the system under the condition

$$\frac{1+K}{k_1} < 0 \text{ and } 0 < b\varepsilon^{-1}A$$

So, heat flux has destabilizing effect on the system under the condition

$$\frac{1+K}{k_1} > 0 \text{ and } 0 > b\varepsilon^{-1}A \quad (32)$$

(ii) Oscillatory mode

Equation (22) $\times U^*$ (conjugate of U) and integrate with the range of z after that use equation (23) $\times G^*$ and (24) $\times \odot^*$ with boundary conditions (26), we get

$$\left(Ab\left[(M+Rk^2) - \frac{\sigma}{\varepsilon} + \frac{1+K}{k_1}\right] + \varepsilon K\right)I_1 - ABbI_2 + \varepsilon Kb(l\sigma^* + 2A) - RAk^2E_1P_1\sigma^*I_3 = 0.$$

Where

$$I_1 = \int [|D^2U|^2 + 2k^2|D^2U| + k^4|U|^2] dz.$$

$$I_2 = \int (|D^2U|^2 + 3k^2|D^2U|^2 + 3k^4|DU|^2 + k^6|U|^2) dz.$$

$$I_3 = \int (|D^2U| + k^2|U|) dz.$$

All these integral from I_1 to I_3 are positive definite.

Now put $\sigma = \sigma_r + i\sigma_i$ in equation (35) and on taking imaginary part, we get

$$\begin{aligned} \frac{(1+f)Ab\sigma}{\varepsilon}(D^2-k^2)^2 - \varepsilon b l K I_\sigma + RAk^2E_1P_1\sigma(D^2-k^2) &= 0. \\ \Rightarrow \sigma \left(\frac{(1+f)b l A}{\varepsilon} I_1 - Kb l \varepsilon + RAk^2E_1P I_2 \right) &= 0. \end{aligned}$$

We have taken (36) positive definite in the absence of coupling between spin and heat flux because $|I_1|$ and $|I_2|$ are all positive definite. So, $\sigma = 0$. Therefore oscillatory modes are not allowed and principle of exchange of stabilities is satisfied for the problem.

NUMERICAL COMPUTATIONS

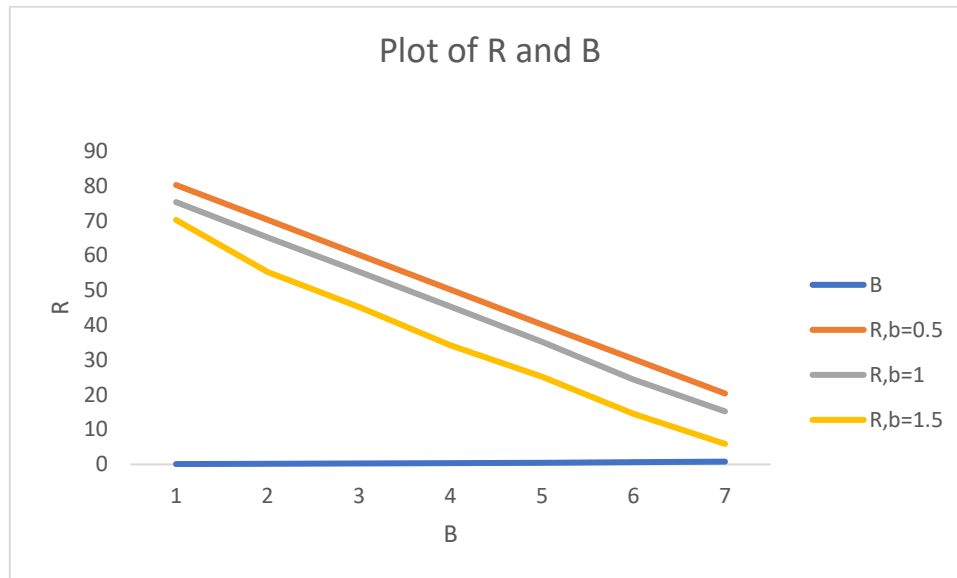
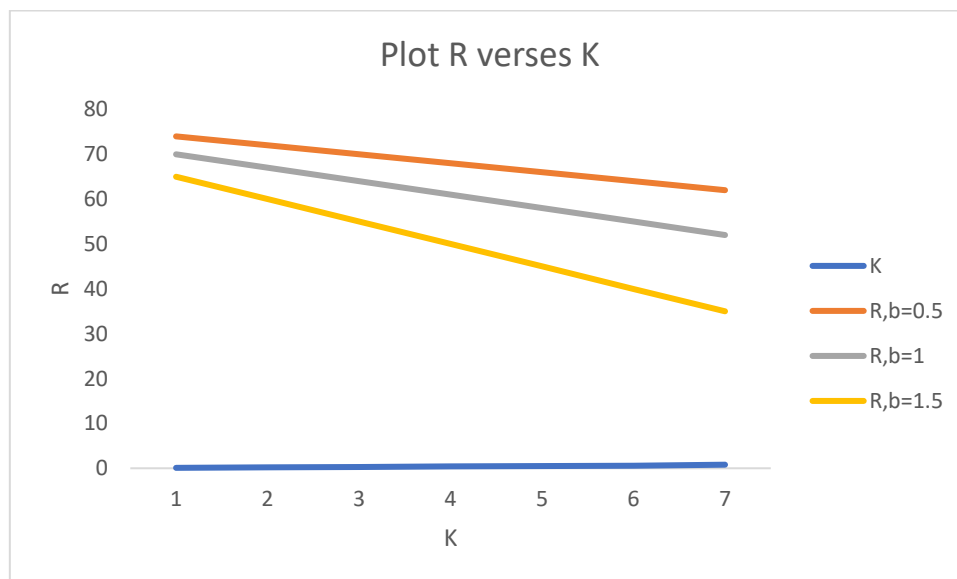


Figure 1.1

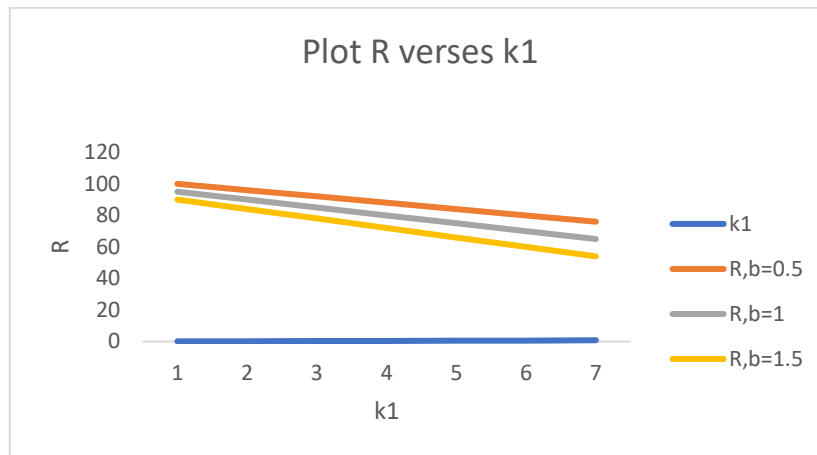
$$K=0.4, k_1=3, M=0.2, \varepsilon = 0.3, \delta=0.7, k=0.3, b=0.5, 1, 1.5$$

Plot of Rayleigh Number R versus B. R decreases when B increases.



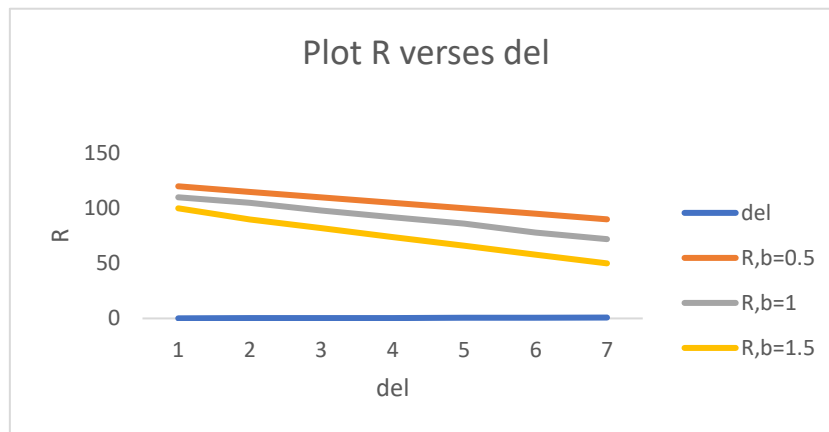
$$B=10, k_1=3, M=0.2, \varepsilon = 0.3, \delta=0.7, k=0.3, b=0.5, 1, 1.5$$

Plot of Rayleigh Number R versus K. R decreases when K increases.

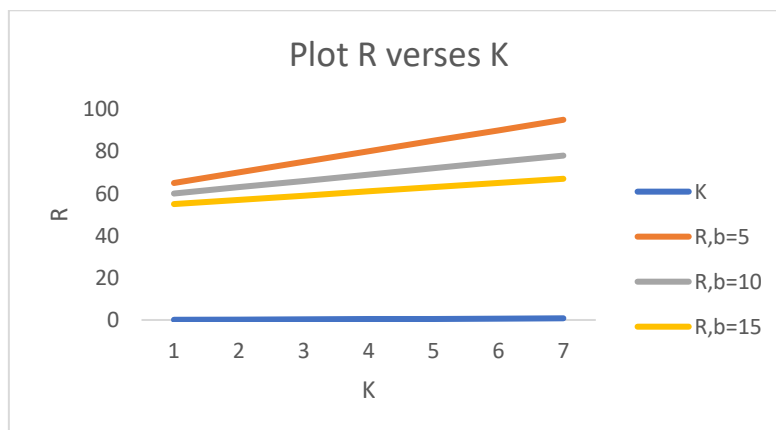


$B=10, K=0.2, M=0.2, \varepsilon = 0.3, \bar{\delta}=0.7, k=0.3, b=0.5, 1, 1.5$

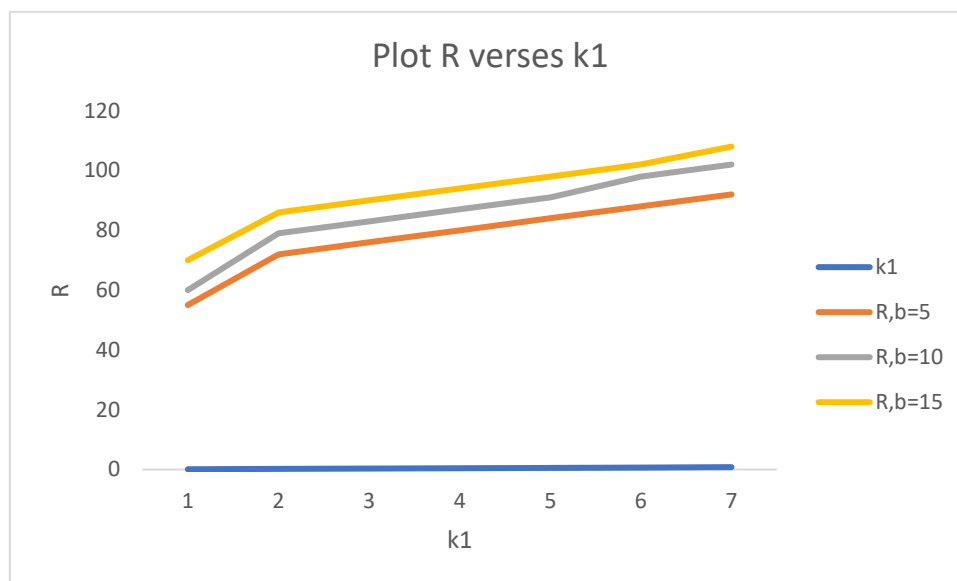
Plot of Rayleigh Number R versus K. R decreases when k1 increases.



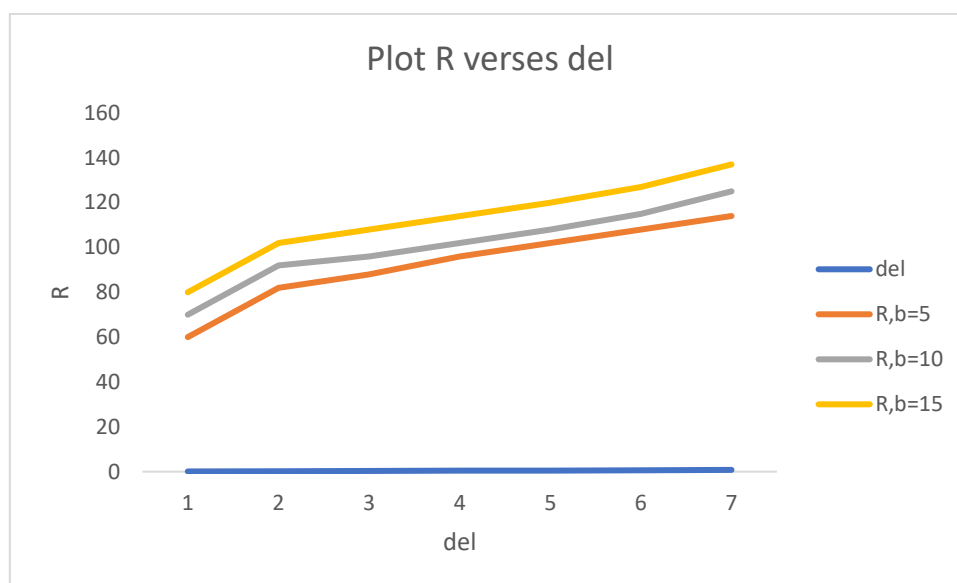
$K=0.4, k_1=3, M=0.2, \varepsilon = 0.3, B=10, k=0.3, b=0.5, 1, 1.5$



$B=10, k_1=0.6, M=0.7, A=5, \varepsilon = 0.5, \bar{\delta}=0.8, k=0.2, b=5, 10, 15$



$B=10, K=0.9, M=0.7, A=5, \varepsilon=0.5, \delta=0.8, k=0.2, b=5,10,15$



$B=10, K=0.9, M=0.7, A=5, \varepsilon=0.5, k_1=0.6, k=0.2, b=5,10,15$

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