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Research Article

Steady State Creep in Isotropic Composite Disc with Thickness Variation

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ABSTRACT

The present work examines the influence of creep on the plastic stress distribution and creep deformation in a rotating functionally graded material (FGM) disc having a linearly varying thickness under the influence of thermal gradients. A mathematical model is developed to evaluate the combined effects of material gradation, disc geometry, and temperature variation on the creep response. The results reveal that thermal gradients considerably modify the stress and strain-rate distributions within the rotating isotropic disc. It is observed that the temperature field has a pronounced impact on the overall creep behavior, highlighting the necessity of incorporating thermal effects in the design and analysis of rotating discs operating under high-temperature conditions.

Keywords: Mathematical modeling, Composite materials, Creep, functionally graded materials, Thermal gradients, Rotating disc.

INTRODUCTION

Rotating discs are widely employed in numerous engineering systems, including jet engines, gas turbines, automotive brake systems, flywheels, compressors, and computer storage devices. During service, these components are often subjected to high rotational speeds and elevated temperature gradients, which significantly influence their mechanical performance and structural integrity. Under such severe operating conditions, time-dependent deformation due to creep becomes an important factor affecting the reliability and service life of the disc.

Consequently, understanding the creep behavior of rotating discs has become an active area of research. Accurate prediction of stress distribution and creep deformation is essential for ensuring safe and efficient operation in high-temperature environments. Motivated by these considerations, the present study investigates the creep response of a rotating functionally graded material disc with linearly varying thickness under both thermal and non-thermal conditions, with particular emphasis on the influence of thermal gradients on stress and strain-rate distributions.

ANALYSIS

The constitutive equations for creep are described under multiaxial stress, are expressed,

$$\dot{\varepsilon}_r = \frac{d\,\dot{u}_r}{d\,r} = \frac{\left[2x(r)-1\right]\left[M(r)\left(\bar{\sigma}-\sigma_0(r)\right)\right]^8}{2\left[x^2(r)-x(r)+1\right]^{1/2}} \qquad (4.12)$$

$$\dot{\varepsilon}_\theta = \frac{\dot{u}_r}{r} = \frac{\left[2-x(r)\right]\left[M(r)\left(\bar{\sigma}-\sigma_0(r)\right)\right]^8}{2\left[x^2(r)-x(r)+1\right]^{1/2}} \qquad (4.2)$$

$$\dot{\varepsilon}_z = -\left(\dot{\varepsilon}_r + \dot{\varepsilon}_\theta\right) \qquad (4.3)$$

The effective strain rate of disc is assumed, as described by following Sherby's law [1977],

$$\dot{\bar{\varepsilon}} = \left[M\left(\bar{\sigma}-\sigma_0\right)\right]^n \qquad (4.4)$$

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where $\dot{\bar{\varepsilon}}, M, \bar{\sigma}, \sigma_0, n$ are the effective strain rate, creep parameter, effective stress, threshold stress, the stress exponent.

Dividing Eq. (4.1) by Eq. (4.2) and integrating the resulting equation by taking limit a to r on both sides,

$$\frac{\dot{u}_r}{\dot{u}_{r_i}} = \exp\int_a^r \frac{\phi(r)}{r}\,dr \tag{4.5}$$

where, $x(r) = \frac{\sigma_r}{\sigma_\theta}$, is the ratio of radial and tangential stresses at any radius and $\dot{u}_r = du/dt$, is the radial deformation rate and $\dot{u}_{r_i}$, is the radial deformation rate at the inner radius.

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The equilibrium force in the radials direction is,

$$\frac{d}{dr}(r\ h\ \sigma_r) + \rho\omega^2 r^2 h = h\sigma_\theta \tag{4.6}$$

Boundary Conditions are $\sigma_r(a) = 0 = \sigma_r(b)$ (4.7)

Dividing Eq. (4.5) by r and equated to Eq. (4.2),

$$\frac{\bar{\sigma} - \sigma_0}{\psi(r)} = \frac{(\dot{u}_{r_i})^{1/n}}{M} \tag{4.8}$$

where,

$$\psi(r) = \left\{\frac{2}{r}\cdot\frac{\left[x^2(r) - x(r) + 1\right]^{1/2}}{\left[2 - x(r)\right]}\ \exp.\int_a^r \frac{\varphi(r)dr}{r}\right\}^{1/n} \tag{4.9}$$

and

$$M(r) = e^{-35.38} P^{0.2077} T(r)^{4.98} V^{-0.622} \tag{4.10}$$

$$\sigma_0(r) = -0.03507P + 0.01057\,T(r) + 1.00536 - 2.11916 \tag{4.11}$$

Simplify Eq. (4.8) into Eq. (4.1) to get the tangential stress (σ_θ),

$$\sigma_\theta = \frac{\bar{\sigma} - \sigma_0}{\psi(r)}\ \psi_1(r) + \psi_2(r) \tag{4.12}$$

where,

$$\psi_1(r) = \frac{\psi(r)}{\left[x^2(r) - x(r) + 1\right]^{1/2}} \tag{4.17}$$

$$\psi_2(r) = \frac{\sigma_0(r)}{\left[x^2(r) - x(r) + 1\right]^{1/2}} \tag{4.14}$$

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$$\sigma_{\theta_{avg}} = \frac{1}{b-a}\int_a^b \sigma_\theta \, dr \tag{4.15}$$

where, $\sigma_{\theta_{avg}}$ is the average tangential stress.

Now radial stress can be expressed by integrating Eq. (4.6),

$$\sigma_r(r) = \frac{1}{r.h}\left[\int_a^r \sigma_\theta \, dr - \frac{\omega^2 \rho (r^3 - a^3)}{3}\right] \tag{4.16}$$

For a disc, the thermal conductivity can be calculated as,

$$K(r) = \frac{[100 - V(r)]K_m + V(r)K_d}{100} \tag{4.17}$$

Where $K_m = 247 W/m\,K$ is matrix conductivity and $K_d = 100 W/m\,K$ is diserpersoid conductivity.

$$T(r) = 619.69 + 0.6083\, r - 0.0208\, r^2 + 3.27\times10^{-4} r^3 - 1.96\times10^{-6} r^4 + 4.43\times10^{-9} r^5 \tag{4.18}$$

where $T(r)$ is the temperature taken from Gupta et al.[4].

RESULTS AND DISCUSSION

A computational analysis was performed to evaluate the influence of thermal gradients on the creep response and plastic stress distribution in isotropic functionally graded material (FGM) rotating discs. A numerical program was developed to carry out the analysis. The study considers isotropic FGM discs reinforced with silicon carbide (SiC) whiskers embedded in a pure aluminum matrix. The results obtained under thermal gradient conditions are compared with those corresponding to a uniform temperature field to assess the influence of temperature variation. The material properties and geometric parameters adopted for the analysis are listed in Table 1.

The variation of tangential stress along the radial direction is presented in Figure 1. The results indicate that the presence of thermal gradients causes a marginal increase in tangential stress near the inner radius, whereas a slight reduction is observed toward the outer radius when compared with the disc subjected to uniform temperature conditions.

The radial stress distribution is illustrated in Figure 2. It is observed that the thermal gradient has only a minor influence on the radial stress, and the overall stress pattern remains nearly unchanged for both thermal and non-thermal cases.

The distribution of tangential strain rate is shown in Figure 3. The strain rate decreases in magnitude when thermal gradients are considered, although the overall variation along the radial direction follows the same trend as that of the disc without thermal gradients. This indicates that temperature variation primarily affects the magnitude rather than the nature of the tangential creep deformation.

The radial strain-rate distribution is presented in Figure 4. A comparison between the two cases reveals that incorporating thermal gradients reduces the radial strain rate throughout the disc. This reduction demonstrates that the temperature distribution has a measurable effect on the creep deformation characteristics of isotropic whisker-reinforced rotating discs.

Density of disc material $\rho = 2812.4\, kg/m^3$

Angular velocity, $\omega = 15{,}000\, rpm$

Inner radius of disc, $a = 31.75 mm$

Outer radius of disc, $b = 152.4 mm$

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Particle size, $P = 1.7\ \mu m$

Particle content, $V = 20\%$

Operating temperature, $T = 623\ K$

Creep parameters for whisker reinforced disc: $m = 53.0 \times 10^{-4}\ s^{-1/8} / MPa$ and $\sigma_0 = 52.83$ MPa

The ratio of isotropic constants, $G/F = 1.34,\ H/F = 1.64$

Table 1

CONCLUSION

The present investigation demonstrates that thermal gradients have a significant influence on the creep response of isotropic silicon carbide whisker-reinforced rotating discs. Although the changes in radial and tangential stress distributions are relatively small, the effect on creep strain rates is more pronounced. In particular, the inclusion of thermal gradients reduces both tangential and radial strain rates, thereby affecting the overall deformation behavior of the disc. These findings emphasize that thermal effects should be incorporated into the analysis and design of rotating discs operating under elevated temperatures to ensure improved structural reliability and safe long-term performance.

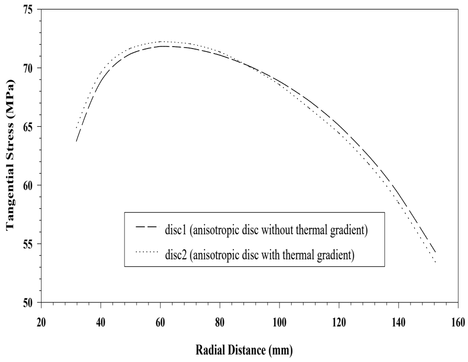

FIGURE 1. Variation of tangential stresses in isotropic disc with/without thermal gradients.

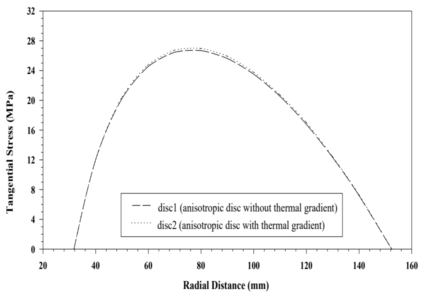

FIGURE 2. Variation of radial stresses in isotropic disc with/without thermal gradients.

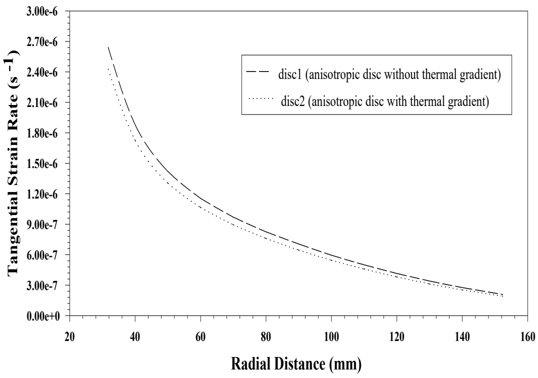

FIGURE 3. Variation of tangential strain rates in isotropic disc with/without thermal gradients.

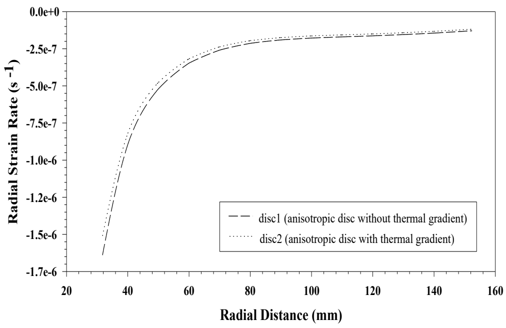

FIGURE 4. Variation of radial strain rates in isotropic disc with/without thermal gradients.

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