Journal of Information Systems Engineering and Management
2024, 9(4)
e-ISSN: 2468-4376
https://www.jisem-journal.com/

Research Article

Stochastic Analysis of Two Non-Identical Units System Model Subject to Inspection Policy

Dr. Urvashi Gupta[1*], Prof. Pawan Kumar[2], Dr. Rakesh Chib[3]

[1*, 2 ,3]Department of Statistics University of Jammu, Jammu-180006 Email:- urvigupta2791@gmail.com

ARTICLE INFO

Received: 01 Nov 2024

Revised: 15 Dec 2024

Accepted: 25 Dec 2024

ABSTRACT

The purpose of the present paper is to analyse a two non-identical units system model in which unit A is in operative mode and unit B is in standby mode. Two repairmen expert and ordinary are considered to repair the failed units. Unit A is repaired by expert repairman who is not always available with the system while unit B by ordinary repairman which is always with the system. Expert repairman takes some significant time to repair the failed unit and after the repair, inspection is carried out to ascertain whether the repair is perfect or not. If repair is perfect then the repaired units becomes operative otherwise the repair is referred to the expert repairman. The failures of the unit are independent and the failure time distribution of both the units are taken as general. All random variables are statistically independent. Using regenerative point technique we find the following characteristics of the system such as Transition Probabilities and Mean Sojourn times, Mean time to system failure (MTSF), Availability of the system, Busy Period for Expert and ordinary repairman, Expected number of Repairs of the unit, Expected number of Replacement of the unit, Expected number of Inspection of the unit, Net Expected Profit earned by the system during the interval (0,t) and in steady state.

Keywords: Repair, Replacement, Inspection

1 INTRODUCTION

The technique of redundancy has been used extensively in many industrial systems in order to improve their performance in terms of reliability and mean life. Repair maintenance is one of the important measures for increasing the effectiveness of a system. Many authors including [3,8,44,38]have analyzed two unit system models considering different repair policies viz. two types of repair, inspection, post repair, preparation time for repair, expert and regular repairman with patience time of regular repairman etc. Therefore, several research papers have been investigated and analyzed by various authors in this direction under different repair and operation policies. Vibha Goyal and K. Murari [86] analyzed two unit standby system with two types of repairman. Keeping in view the fact that the expert repairman takes some time to repair the failed unit, Gupta et.al. [48] developed a system model with administrative delay in repair and correlated lifetimes. Pawan Kumar and Neha Kumari [71] analyzed stochastic analysis of a two non-identical unit parallel system model with preparation time for repair and proviso of rest. Further, Bhatti et.al. [14] have analyzed a system with inspection and two types of failure. Gopalan and Naidu[39] studied stochastic behavior of a two unit repairman system subject to inspection. Also, Bashir et.al. [8] developed a two unit system model with repair and inspection policies. Chander et.al. [17] worked on different types of inspection subject to degradation.
The purpose of the present paper is to analyze a two non identical unit system model consisting of two units A and B. Unit A is in operative mode and unit B is in standby mode. Two repairmen expert and ordinary are considered to repair the failed units. Unit A is repaired by expert repairman who is not always available with the system while unit B is repaired by ordinary repairman which is always available with the system. Expert repairman takes some significant time to repair the failed unit and after the repair, inspection is carried out to ascertain whether the repair is perfect or not. If repair is perfect then the repaired units become operative

Journal of Information Systems Engineering and Management
2024, 9(4)
e-ISSN: 2468-4376
https://www.jisem-journal.com/ **Research Article**

otherwise the repair is referred to the expert repairman. The failures of the units are independent and the failure and repair time distributions of both the units are taken as exponential. All random variables are statistically independent.

Using semi- Markov process and regenerative point technique, the following measures are obtained-

1. Transition Probabilities and Mean Sojourn times.
2. Reliability and Mean time to system failure (MTSF).
3. Expected uptime and downtime of the system.
4. Busy period for expert repairman and ordinary repairman.
5. Expected number of repairs by expert and ordinary repairman.
6. Expected number of replacement of the unit.
7. Net expected profit earned by the system during the interval (0,t) and in steady state.

1.2 MODEL DESCRIPTION AND ASSUMPTIONS

1. The system comprises of two units- A and B. Initially unit-A is in operative mode and unit-B is in standby mode.

2. Two repairmen are available to repair the failed unit i.e. Expert and ordinary repairman. The expert repairman repairs the unit-A who takes some significant time (a random variable) to repair the failed unit while ordinary repairman is always available with the system which repairs the unit-B.

3. Upon the failure of unit A, repairman takes some time (delay time) to reach the system. After repair, inspection is carried out to ascertain whether the repair is perfect or not. If repair is perfect it goes back to the operating mode otherwise it is replaced by the expert repairman.

4. However, if unit B fails it is repaired by ordinary repairman who is always available in the system and after repair the unit becomes operative.

5. During the repair of the unit A by the expert repairman, if the unit B also fails then the repair of the unit-B is also done by the expert repairman, so the unit B has to wait for the repair until the repair of the unit A is completed.

6. The failures and repairs of the units are independent and the failure and repair time distributions of the units are taken as Exponential.

1.3 NOTATIONS AND STATES OF THE SYSTEM

We define the following symbols for generating the various states of the system.

A_o/B_o : Unit A /unit B is in operative mode.
B_s : Unit B is in standby mode.
A_d : Unit A under administrative delay for repair.
A_{re}/B_r : Unit A/unit B under repair.
B_{wr} : Unit B is waiting for repair.
A_I/A_R : Unit A under inspection/ replacement of the unit.

Thus considering the above symbols in view of the assumptions stated, the possible states of the system are as follows-

Up states:

$S_0 = (A_o, B_S)$ $S_1 = (A_d, B_o)$ $S_3 = (A_{re}, B_o)$
$S_4 = (A_I, B_o)$ $S_6 = (A_R, B_o)$ $S_9 = (A_o, B_r)$

Failed states:

$S_2 = (A_d, B_r)$ $S_5 = (A_{re}, B_{wr})$ $S_7 = (A_I, B_{wr})$
$S_8 = (A_R, B_{wr})$

Journal of Information Systems Engineering and Management
2024, 9(4)
e-ISSN: 2468-4376
https://www.jisem-journal.com/ **Research Article**

b) NOTATIONS:

E : Set of regenerative states $= \{S_0, S_1, S_3, S_4, S_6, S_9\}$

$\overline{E}$: Set of non − regenerative states $= \{S_2, S_5, S_8, S_7\}$

α_1 : Failure rate of unit A.

α_2 : Inspection Rate of unit A.

α_3 : Repair Rate of unit A.

γ_1 : Completion rate of Inspection of unit A .

p/q : Probability with which unit will get repaired/replaced.

γ_2 : Replacement Rate of unit A.

β_1 : Failure Rate of unit B.

β_2 : Repair Rate of unit B.

TRANSITION DIAGRAM

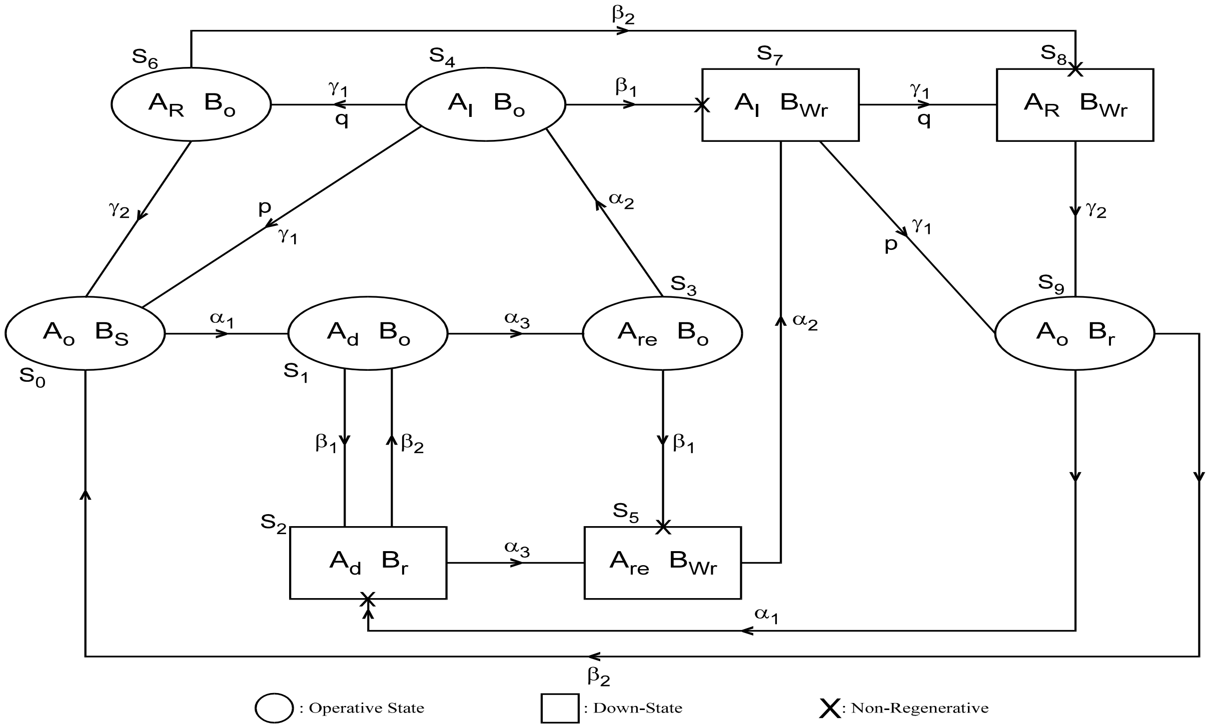

Fig 1.1

1.4 TRANSITION PROBABLITIES

Let X_n denotes the state visited at epoch T_{n^+} just after the transition at T_n, where T_1, T_2 .... represents the regenerative epochs, then$\{X_n, T_n\}$ constitute a Markov-Renewal process with state space E and

$Q_{ij}(t) = P[X_{n+1} = j, T_{n+1} - T_n \le t | X_n = i]$

is the semi Markov kernel over E.

Then the transition probability matrix of the embedded Markov chain is

$P = p_{ij} = Q_{ij}(\infty) = Q(\infty)$

(a) The various transition probabilities may be obtained as follows:

$Q_{01}(t) = \alpha_1 \int_0^t e^{-\alpha_1 u}\, du = 1 - e^{-\alpha_1 t}$

Similarly,

Journal of Information Systems Engineering and Management
2024, 9(4)
e-ISSN: 2468-4376
https://www.jisem-journal.com/ **Research Article**

$Q_{12}(t) = \beta_1 \int_0^t e^{-(\beta_1+\alpha_3)u}\, du = \frac{\beta_1}{(\beta_1+\alpha_3)}\left[1 - e^{-(\beta_1+\alpha_3)t}\right]$

$Q_{13}(t) = \alpha_3 \int_0^t e^{-(\alpha_3+\beta_1)u}\, du = \frac{\alpha_3}{(\beta_1+\alpha_3)}\left[1 - e^{-(\beta_1+\alpha_3)t}\right]$

$Q_{21}(t) = \beta_2 \int_0^t e^{-(\alpha_3+\beta_2)u}\, du = \frac{\beta_2}{(\beta_2+\alpha_3)}\left[1 - e^{-(\beta_2+\alpha_3)t}\right]$

$Q_{25}(t) = \alpha_3 \int_0^t e^{-(\alpha_3+\beta_2)u}\, du = \frac{\alpha_3}{(\beta_2+\alpha_3)}\left[1 - e^{-(\beta_2+\alpha_3)t}\right]$

$Q_{34}(t) = \alpha_2 \int_0^t e^{-(\alpha_2+\beta_1)u}\, du = \frac{\alpha_2}{(\beta_1+\alpha_2)}\left[1 - e^{-(\beta_1+\alpha_2)t}\right]$

$Q_{40}(t) = \gamma_1 p \int_0^t e^{-\gamma_1 u} e^{-\beta_1 u}\, du = \frac{\gamma_1 p}{(\gamma_1+\beta_1)}\left[1 - e^{-(\gamma_1+\beta_1)t}\right]$

$Q_{46}(t) = \gamma_1 q \int_0^t e^{-\gamma_1 u} e^{-\beta_1 u}\, du = \frac{\gamma_1 q}{(\gamma_1+\beta_1)}\left[1 - e^{-(\gamma_1+\beta_1)t}\right]$

$Q_{57}(t) = \alpha_2 \int_0^t e^{-\alpha_2 u}\, du = 1 - e^{-\alpha_2 t}$

$Q_{60}(t) = \gamma_2 \int_0^t e^{-\gamma_2 u} e^{-\beta_2 u}\, du = \frac{\gamma_2}{(\gamma_2+\beta_2)}\left[1 - e^{-(\gamma_2+\beta_2)t}\right]$

$Q_{78}(t) = \gamma_1 p \int_0^t e^{-\gamma_1 u}\, du = p(1 - e^{-\gamma_1 t})$

$Q_{79}(t) = \gamma_1 q \int_0^t e^{-\gamma_1 u}\, du = q(1 - e^{-\gamma_1 t})$

$Q_{89}(t) = \gamma_2 \int_0^t e^{-\gamma_2 u}\, du = 1 - e^{-\gamma_2 t}$

$Q_{90}(t) = \beta_2 \int_0^t e^{-\beta_2 u} e^{-\alpha_1 u}\, du = \frac{\beta_2}{(\beta_2+\alpha_1)}\left[1 - e^{-(\beta_2+\alpha_1)t}\right]$

The indirect transition probabilities are as follows:

$Q_{37}^{(5)}(t) = \beta_1\alpha_2 \int_0^t e^{-\beta_1 u} e^{-\alpha_2 u}\, du \int_u^t e^{-\alpha_2(v-u)} dv = \alpha_2 \left[\int_0^t e^{-\alpha_2 v}\, dv - \int_0^t e^{-(\alpha_2+\beta_1)v}\, dv\right]$

$Q_{48}^{(7)}(t) = \beta_1\gamma_1 q \int_0^t e^{-\beta_1 u} e^{-\gamma_1 u} du \int_u^t e^{-\gamma_1(v-u)} dv = \gamma_1 q \left[\int_0^t e^{-\gamma_1 v} dv - \int_0^t e^{-(\beta_1+\gamma_1)v} dv\right]$

$Q_{49}^{(7)}(t) = \gamma_1 p \left[\int_0^t e^{-\gamma_1 v} dv - \int_0^t e^{-(\beta_1+\gamma_1)v} dv\right]$

$Q_{69}^{(8)}(t) = \gamma_2\beta_2 \int_0^t e^{-\beta_2 u} e^{-\gamma_2 u}\, du - \int_u^t e^{-\gamma_2(v-u)} dv = \gamma_2 \int_0^t e^{-\gamma_2 v}\, dv - \int_0^t e^{-(\gamma_2+\beta_2)v} dv$

$Q_{95}^{(2)}(t) = \alpha_1\alpha_3 \int_0^t e^{-\alpha_1 u} e^{-\beta_2 u} \int_u^t e^{-\alpha_3(v-u)} e^{-\beta_2(v-u)}$

$= \frac{\alpha_1\alpha_3}{(\alpha_1-\alpha_3)} \int_0^t e^{-(\alpha_3+\beta_2)v} dv - \int_0^t e^{-(\alpha_1+\beta_2)v}\, dv$

$Q_{91}^{(2)}(t) = \alpha_1\beta_2 \int_0^t e^{-\alpha_1 u} e^{-\beta_2 u} \int_u^t e^{-\beta_2(v-u)} e^{-\alpha_3(v-u)}$

$= \frac{\alpha_1\beta_2}{(\alpha_1-\alpha_3)} \int_0^t e^{-(\alpha_3+\beta_2)v} dv - \int_0^t e^{-(\alpha_1+\beta_2)v}\, dv$

(b) STEADY STATE PROBABILITIES

The steady state transition probabilities are given by:

$p_{ij} = \lim_{t\to\infty} Q_{ij}(t) = \tilde{Q}_{ij}(s)|s = 0$ and $p_{ij}^{(k)} = \lim_{t\to\infty} Q_{ij}^{(k)}(t) = \tilde{Q}_{ij}^{(k)}(s)|s = 0$

Thus,

$p_{01} = \alpha_1 \int e^{-\alpha_1 u} du = 1$

$p_{12} = \beta_1 \int e^{-(\beta_1+\alpha_3)u} du = \frac{\beta_1}{(\beta_1+\alpha_3)}$

$p_{13} = \frac{\alpha_3}{(\beta_1+\alpha_3)}$

$p_{21} = \frac{\beta_2}{(\beta_2+\alpha_3)}$

$p_{25} = \frac{\alpha_3}{(\beta_2+\alpha_3)}$

$p_{34} = \frac{\alpha_2}{(\alpha_2+\beta_1)}$

$p_{40} = \gamma_1 p \int e^{-(\gamma_1+\beta_1)u}\, du = \frac{\gamma_1 p}{(\gamma_1+\beta_1)}$

$p_{46} = \frac{\gamma_1 q}{(\gamma_1+\beta_1)}$

Journal of Information Systems Engineering and Management
2024, 9(4)
e-ISSN: 2468-4376
https://www.jisem-journal.com/ **Research Article**

$p_{57} = \alpha_2 \int e^{-\alpha_2 u} du = 1$
$p_{60} = \frac{\gamma_2}{(\gamma_2+\beta_2)}$
$p_{78} = \gamma_1 p \int e^{-\gamma_1 u} du = p$
$p_{79} = \gamma_1 q \int e^{-\gamma_1 u} du = q$
$p_{89} = \gamma_2 \int e^{-\gamma_2 u} du = 1$
$p_{90} = \frac{\beta_2}{(\alpha_1+\beta_2)}$

The indirect transition probability may be obtained as follows:

$p_{37}^{(5)} = 1 - \frac{\alpha_2}{(\alpha_2+\beta_1)} = \frac{\beta_1}{(\alpha_2+\beta_1)}$

Similarly,

$p_{48}^{(7)} = q - \frac{\gamma_1 q}{(\gamma_1+\beta_1)} = \frac{\beta_1 q}{(\gamma_1+\beta_1)}$
$p_{49}^{(7)} = p - \frac{\gamma_1 p}{(\gamma_1+\beta_1)} = \frac{\beta_1 p}{(\gamma_1+\beta_1)}$
$p_{69}^{(8)} = 1 - \frac{\gamma_2}{(\gamma_2+\beta_2)} = \frac{\beta_2}{(\gamma_2+\beta_2)}$
$p_{91}^{(2)} = \frac{\alpha_1\beta_2}{(\alpha_1-\alpha_3)}\left[\frac{1}{(\beta_2+\alpha_3)} - \frac{1}{(\beta_2+\alpha_1)}\right] = \frac{\alpha_1\beta_2}{(\beta_2+\alpha_3)(\beta_2+\alpha_1)}$
$p_{95}^{(2)} = \frac{\alpha_1\alpha_3}{(\alpha_1-\alpha_3)}\left[\frac{1}{(\beta_2+\alpha_3)} - \frac{1}{(\beta_2+\alpha_1)}\right] = \frac{\alpha_1\alpha_3}{(\beta_2+\alpha_3)(\beta_2+\alpha_1)}$

It can be easily verified that

$p_{12} + p_{13} = 1$ $\quad p_{21} + p_{25} = 1$ $\quad p_{34} + p_{37}^{(5)} = 1$
$p_{40} + p_{46} + p_{48}^{(7)} + p_{49}^{(7)} = 1$ $\quad p_{01} = p_{57} = p_{89} = 1$
$p_{60} + p_{68} = 1$ $\quad p_{78} + p_{79} = 1$ $\quad p_{90} + p_{91}^{(2)} + p_{95}^{(2)} = 1$

A) Mean sojourn times:

The mean sojourn time in state S_i denoted by μ_i is defined as the expected time taken by the system in state S_i before transiting to any other state. To obtain mean sojourn time μ_i, in state S_i, we observe that as long as the system is in state S_i, there is no transition from S_i to any other state. If T_i denotes the sojourn time in state S_i then mean sojourn time μ_i in state S_i is:

$\mu_i = E[T_i] = \int P(T_i > t)dt$

Therefore,

$\mu_0 = \int e^{-\alpha_1 t} dt = \frac{1}{\alpha_1}$ $\quad \mu_1 = \frac{1}{(\alpha_3+\beta_1)}$ $\quad \mu_2 = \frac{1}{(\alpha_3+\beta_2)}$
$\mu_3 = \frac{1}{(\beta_1+\alpha_2)}$ $\quad \mu_4 = \frac{1}{(\gamma_1+\beta_1)}$ $\quad \mu_5 = \frac{1}{\alpha_2}$
$\mu_6 = \frac{1}{(\gamma_2+\beta_2)}$ $\quad \mu_7 = \frac{1}{\gamma_1}$ $\quad \mu_8 = \frac{1}{\gamma_2}$
$\mu_9 = \frac{1}{(\beta_2+\alpha_1)}$

. 1.5 ANALYSIS OF RELIABILITY AND MTSF

Let the random variable T_i denotes the time to system failure when system starts up from state $S_i \in E$. Then the reliability of the system is given by

$R_i(t) = P[T_i > t]$

As an illustrations, $R_0(t)$ is the sum of the following contingencies-

1) The system remains up in states S_0 without making transition to any other state up to time t, the probability of this contingency is:

$Z_0(t) = e^{-\alpha_1 t}$

2) The system transits from state S_0 to state S_1 during $(u, u+du), u < t$ and then starting from S_1 at epoch u, it remains up continuously during the remaining time $(t-u)$,the probability of this contingency is:

Journal of Information Systems Engineering and Management
2024, 9(4)
e-ISSN: 2468-4376
https://www.jisem-journal.com/ Research Article

$$\int_0^t q_{01}(u)duR_1(t-u) = q_{01}(t)©R_1(t)$$

Therefore, $R_0(t)$ becomes

$R_0(t) = Z_0(t) + q_{01}(t)©R_1(t)$

Similarly,

$R_1(t) = Z_1(t) + q_{13}(t)©R_3(t)$

$R_3(t) = Z_3(t) + q_{34}(t)©R_4(t)$

$R_4(t) = Z_4(t) + q_{40}(t)©R_0(t) + q_{46}(t)©R_6(t)$

$R_6(t) = Z_6(t) + q_{60}(t)©R_0(t)$ **(1.5.1-1.5.5)**

Taking Laplace Transforms of relations **(1.5.1-1.5.5)**, we get

$R_0^*(s) = Z_0^*(s) + q_{01}^*(s)R_1^*(s)$

$R_1^*(s) = Z_1^*(s) + q_{13}^*(s)R_3^*(s)$

$R_3^*(s) = Z_3^*(s) + q_{34}^*(s)R_4^*(s)$

$R_4^*(s) = Z_4^*(s) + q_{40}^*(s)R_0^*(s) + q_{46}^*(s)R_6^*(s)$

$R_6^*(s) = Z_6^*(s) + q_{60}^*(s)R_0^*(s)$

where,

$Z_0(t) = e^{-\alpha_1 t}$ $Z_1(t) = e^{-(\alpha_3+\beta_1)t}$ $Z_2(t) = e^{-(\alpha_3+\beta_2)t}$

$Z_3(t) = e^{-(\beta_1+\alpha_2)t}$ $Z_6(t) = e^{-(\gamma_2+\beta_2)t}$

The solution for $R_i^*(s)$ can be written in the matrix form as below:

$$\begin{bmatrix} 1 & -q_{01}^* & 0 & 0 & 0 \\ 0 & 1 & -q_{13}^* & 0 & 0 \\ 0 & 0 & 1 & -q_{34}^* & 0 \\ -q_{40}^* & 0 & 0 & 1 & -q_{46}^* \\ -q_{60}^* & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} R_0^* \\ R_1^* \\ R_3^* \\ R_4^* \\ R_6^* \end{bmatrix} = \begin{bmatrix} M_0^* \\ M_1^* \\ M_3^* \\ M_4^* \\ M_6^* \end{bmatrix}$$

For brevity the argument's' is omitted from $q_{ij}^*(s)$, $Z_i^*(s)$ and $R_i^*(s)$.

Solving for $R_0^*(s)$, we have

$R_0^*(s) = \frac{N_1(s)}{D_1(s)}$ **(1.5.6)**

Where

$N_1(s) = Z_0^* + q_{01}^*[Z_1^* + q_{13}^*(Z_3^* + q_{34}^*(Z_4^* + q_{46}^* Z_6^*))]$

$D_1(s) = (1 - q_{01}^* q_{13}^* q_{34}^*(q_{40}^* + q_{46}^* q_{60}^*))$

Taking the inverse Laplace Transform of **(1.5.6)**, one can get the reliability of the system when it starts from state S_0.

To get MTSF, we use the well known formula

$E(T_0) = \int R_0(t)dt = \lim_{s\to 0} R_0^*(s) = N_1(0)/D_1(0)$

where,

$N_1(0) = \mu_0 + p_{01}[\mu_1 + p_{13}(\mu_3 + p_{34}(\mu_4 + p_{46}\mu_6))]$

$D_1(0) = 1 - p_{01}p_{13}p_{34}(p_{40} + p_{46}p_{60})$

Since, we have $q_{ij}^*(0) = p_{ij}$ and $\lim_{s\to 0} Z_i^*(s) = \int Z_i(t)dt = \mu_i$

1.6 AVAILABILITY ANALYSIS

Let $A_i(t)$ be the probability that the system is in operative mode at epoch t, when it initially starts from $S_i \in E$. To obtain recurrence relations among different pointwise availabilities we use the simple probabilistic arguments.

As an illustrations, $A_0(t)$ is the sum of the following mutual exclusive contingencies-

Journal of Information Systems Engineering and Management
2024, 9(4)
e-ISSN: 2468-4376
https://www.jisem-journal.com/ **Research Article**

1) The system remains up in states S_0 without making transition to any other state up to time t, the probability of this contingency is:

$$Z_0(t) = e^{-\alpha_1 t}$$

2) The system transits from state S_0 to state S_1 during $(u, u + du), u < t$ and then starting from S_1 at epoch u, it remains up continuously during the remaining time $(t - u)$, the probability of this contingency is:

$$\int_0^t q_{01}(u)du R_1(t-u) = q_{01}(t) © R_1(t)$$

Therefore, $A_0(t)$ becomes

$$A_0(t) = Z_0(t) + q_{01}(t) © A_1(t)$$

By similar arguments, we have

$$A_1(t) = Z_1(t) + q_{12}(t) © A_2(t) + q_{13}(t) © A_3(t)$$
$$A_2(t) = q_{21}(t) © A_1(t) + q_{25}(t) © A_5(t)$$
$$A_3(t) = Z_3(t) + q_{34}(t) © A_4(t) + q_{37}^{(5)}(t) © A_7(t)$$
$$A_4(t) = Z_4(t) + q_{40}(t) © A_0(t) + q_{46}(t) © A_6(t) + q_{48}^{(7)}(t) © A_8(t) + q_{49}^{(7)}(t) © A_9(t)$$
$$A_5(t) = q_{57}(t) © A_7(t)$$
$$A_6(t) = Z_6(t) + q_{60}(t) © A_0(t) + q_{69}^{(8)}(t) © A_9(t)$$
$$A_7(t) = q_{78}(t) © A_8(t) + q_{79}(t) © A_9(t)$$
$$A_8(t) = q_{89}(t) © A_9(t)$$
$$A_9(t) = Z_9(t) + q_{90}(t) © A_0(t) + q_{91}^{(2)}(t) © A_1(t) + q_{95}^{(2)}(t) © A_5(t) \qquad \textbf{(1.6.1-1.6.10)}$$

where,

$Z_0(t) = e^{-\alpha_1 t}$ $\quad Z_1(t) = e^{-(\alpha_3+\beta_1)t}$ $\quad Z_3(t) = e^{-(\beta_1+\alpha_2)t}$

$Z_4(t) = e^{-(\gamma_1+\beta_1)t}$ $\quad Z_6(t) = e^{-(\gamma_2+\beta_2)t}$ $\quad Z_9(t) = e^{-(\alpha_1+\beta_2)t}$

Taking Laplace Transforms of relations **(1.6.1-1.6.10)**, we get

$$A_0^*(s) = Z_0^*(s) + q_{01}^*(s)A_1^*(s)$$
$$A_1^*(s) = Z_1^*(s) + q_{12}^*(s)A_2^*(s) + q_{13}^*(s)A_3^*(s)$$
$$A_2^*(s) = q_{21}^*(s)A_1^*(s) + q_{25}^*(s)A_5^*(s)$$
$$A_3^*(s) = Z_3^*(s) + q_{34}^*(s)A_4^*(s) + q_{37}^{(5)*}(s)A_5^*(s)$$
$$A_4^*(s) = Z_4^*(s) + q_{46}^*(s)A_6^*(s) + q_{40}^*(s)A_0^*(s) + q_{48}^{(7)*}(s)A_8^*(s) + q_{49}^{(7)*}(s)A_9^*(s)$$
$$A_5^*(s) = q_{57}^*(s)A_7^*(s)$$
$$A_6^*(s) = Z_6^*(s) + q_{60}^*(s)A_0^*(s) + q_{69}^{(8)*}(s)A_9^*(s)$$
$$A_7^*(s) = q_{78}^*(s)A_8^*(s) + q_{79}^*(s)A_9^*(s)$$
$$A_8^*(s) = q_{89}^*(s)A_9^*(s)$$
$$A_9^*(s) = Z_9^*(s) + q_{90}^*(s)A_0^*(s) + q_{91}^{(2)*}(s)A_1^*(s) + q_{95}^{(2)*}(s)A_5^*(s)$$

Solving the resultant set of equations and simplifying for $A_0^*(s)$, we have

$$A_0^*(s) = N_2(s)/D_2(s) \textbf{(1.6.11)}$$

Where

$$N_2(s) = Z_0^*\Big[\Big(1 - q_{95}^{(2)*}q_{57}^*(q_{78}^*q_{89}^* + q_{79}^*)\Big)(1 - q_{12}^*q_{21}^*) - \Big\{q_{13}^*q_{34}^*\big(q_{46}^*q_{69}^{(8)*} + q_{48}^{(7)*}q_{89}^* + q_{49}^{(7)*}\big)q_{91}^{(2)*} + q_{13}^*q_{37}^{(5)*}(q_{78}^*q_{89}^* + q_{79}^*)q_{91}^{(2)*} + q_{12}^*q_{25}^*q_{57}^*(q_{78}^*q_{89}^* + q_{79}^*)q_{91}^{(2)*}\Big\}\Big] + q_{01}^*\Big(1 - q_{95}^{(2)*}q_{57}^*(q_{78}^*q_{89}^* + q_{79}^*)\Big)[Z_1^* + q_{13}^*(Z_3^* + q_{34}^*Z_4^* + q_{34}^*q_{46}^*Z_6^*)] + q_{01}^*[q_{13}^*q_{34}^*\big(q_{46}^*q_{69}^{(8)*} + q_{48}^{(7)*}q_{89}^* + q_{49}^{(7)*}\big)q_{91}^{(2)*} + q_{13}^*q_{37}^{(5)*}(q_{78}^*q_{89}^* + q_{79}^*)q_{91}^{(2)*} + q_{12}^*q_{25}^*q_{57}^*(q_{78}^*q_{89}^* + q_{79}^*)q_{91}^{(2)*}]Z_9^*$$

and

$$D_2(s) = \Big(1 - q_{95}^{(2)*}q_{57}^*(q_{78}^*q_{89}^* + q_{79}^*)\Big)(1 - q_{12}^*q_{21}^*) - \Big\{q_{13}^*q_{34}^*\big(q_{46}^*q_{69}^{(8)*} + q_{48}^{(7)*}q_{89}^* + q_{49}^{(7)*}\big) + q_{13}^*q_{37}^{(5)*}(q_{78}^*q_{89}^* + q_{79}^*) + q_{12}^*q_{25}^*q_{57}^*(q_{78}^*q_{89}^* + q_{79}^*)\Big\}q_{91}^{(2)*} - q_{01}^*\Big(1 - q_{95}^{(2)*}q_{57}^*(q_{78}^*q_{89}^* + q_{79}^*)\Big)[q_{13}^*(q_{34}^*q_{40}^* + q_{34}^*q_{46}^*q_{60}^*)] + q_{01}^*[q_{13}^*q_{34}^*\big(q_{46}^*q_{69}^{(8)*} + q_{48}^{(7)*}q_{89}^* + q_{49}^{(7)*}\big) + q_{13}^*q_{37}^{(5)*}(q_{78}^*q_{89}^* + q_{79}^*) + q_{12}^*q_{25}^*q_{57}^*(q_{78}^*q_{89}^* + q_{79}^*)]q_{90}^*$$

Journal of Information Systems Engineering and Management
2024, 9(4)
e-ISSN: 2468-4376
https://www.jisem-journal.com/ **Research Article**

(1.6.12)
The steady state availability is given by

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} sA_0^*(s) = \frac{N_2(0)}{D_2(0)}$$

(1.6.13)
As we know that, $q_{ij}(t)$is the pdf of the time of transition from state S_i to S_j and $q_{ij}(t)dt$ is the probability of transition from state S_i to S_j during the interval $(t, t + dt)$, thus
$\lim_{s\to 0} Z_i^*(s) = \int Z_i(t)dt = \mu_i$ and $q_{ij}^*(s) = q_{ij}^*(0) = p_{ij}$, we get
Therefore,
$N_2(0) = \mu_0 \left[\left(1 - p_{95}^{(2)}p_{57}(p_{78}p_{89} + p_{79})\right)(1 - p_{12}p_{21}) - p_{12}p_{25}p_{57}(p_{78}p_{89} + p_{79})p_{91}^{(2)} + p_{13}p_{34}(p_{46}p_{69}^{(8)} + p_{48}^{(7)}p_{89} + p_{49}^{(7)})p_{91}^{(2)} + p_{13}p_{37}^{(5)}(p_{78}p_{89} + p_{79})p_{91}^{(2)}\right] + p_{01}\left(1 - p_{95}^{(2)}p_{57}(p_{78}p_{89} + p_{79})\right)[\mu_1 + p_{13}(\mu_3 + p_{34}\mu_4 + p_{34}p_{46}\mu_6)] + p_{01}[p_{12}p_{25}p_{57}(p_{78}p_{89} + p_{79}) + p_{13}p_{34}(p_{46}p_{69}^{(8)} + p_{48}^{(7)}p_{89} + p_{49}^{(7)}) + p_{13}p_{37}^{(5)}(p_{78}p_{89} + p_{79})]\mu_9$

(1.6.14)
and
$D_2(0) = \left(1 - p_{95}^{(2)}p_{57}(p_{78}p_{89} + p_{79})\right)(1 - p_{12}p_{21}) - [p_{12}p_{25}p_{57}(p_{78}p_{89} + p_{79})p_{91}^{(2)} + p_{13}p_{34}(p_{46}p_{69}^{(8)} + p_{48}^{(7)}p_{89} + p_{49}^{(7)})p_{91}^{(2)} + p_{13}p_{37}^{(5)}(p_{78}p_{89} + p_{79})p_{91}^{(2)}] - p_{01}\left(1 - p_{95}^{(2)}p_{57}(p_{78}p_{89} + p_{79})\right)[p_{13}p_{34}(p_{40} + p_{46}p_{60})] - p_{01}[p_{12}p_{25}p_{57}(p_{78}p_{89} + p_{79}) + p_{13}p_{34}(p_{46}p_{69}^{(8)} + p_{48}^{(7)}p_{89} + p_{49}^{(7)}) + p_{13}p_{37}^{(5)}(p_{78}p_{89} + p_{79})]p_{90}$
Now,
$D_2(0) = \left(1 - p_{95}^{(2)}p_{57}(p_{78}p_{89} + p_{79})\right)(1 - p_{12}p_{21}) - [p_{12}p_{25}p_{57}(p_{78}p_{89} + p_{79})p_{91}^{(2)} + p_{13}p_{34}(p_{46}p_{69}^{(8)} + p_{48}^{(7)}p_{89} + p_{49}^{(7)})p_{91}^{(2)} + p_{13}p_{37}^{(5)}(p_{78}p_{89} + p_{79})p_{91}^{(2)}] - p_{01}\left(1 - p_{95}^{(2)}p_{57}(p_{78}p_{89} + p_{79})\right)[p_{13}p_{34}(p_{40} + p_{46}p_{60})] - p_{01}[p_{12}p_{25}p_{57}(p_{78}p_{89} + p_{79}) + p_{13}p_{34}(p_{46}p_{69}^{(8)} + p_{48}^{(7)}p_{89} + p_{49}^{(7)}) + p_{13}p_{37}^{(5)}(p_{78}p_{89} + p_{79})]p_{90}$
$= (1 - p_{95}^{(2)}p_{57})(1 - p_{12}p_{21}) - [p_{12}p_{25}p_{57} + p_{13}p_{34}(p_{46}p_{69}^{(8)} + p_{48}^{(7)} + p_{49}^{(7)}) + p_{13}p_{37}^{(5)}]p_{91}^{(2)} - p_{01}\left(1 - p_{95}^{(2)}p_{57}(p_{78}p_{89} + p_{79})\right)[p_{13}p_{34}(p_{40} + p_{46}p_{60})] - p_{13}p_{34}(p_{46}p_{69}^{(8)} + p_{48}^{(7)} + p_{49}^{(7)})p_{90} + p_{12}p_{25}p_{57}p_{90} + p_{13}p_{37}^{(5)}p_{90}$
$= (1 - p_{95}^{(2)}p_{57})(1 - p_{12}p_{21}) - [p_{12}p_{25}p_{57} + p_{13}p_{34}(p_{46}p_{69}^{(8)} + p_{48}^{(7)} + p_{49}^{(7)}) + p_{13}p_{37}^{(5)}](p_{91}^{(2)} + p_{90}) - p_{01}\left(1 - p_{95}^{(2)}p_{57}(p_{78}p_{89} + p_{79})\right)[p_{13}p_{34}(p_{40} + p_{46}p_{60})]$
$= (1 - p_{95}^{(2)}p_{57})(1 - p_{12}p_{21}) - [p_{12}p_{25}p_{57} + p_{13}p_{34}(1 - p_{40} - p_{46}p_{60}) + p_{13}p_{37}^{(5)}] - (1 - p_{95}^{(2)})p_{13}p_{34}(p_{40} + p_{46}p_{60})$
$= (1 - p_{95}^{(2)})(1 - p_{12}p_{21}) - [p_{12}p_{25}p_{57} + p_{13}p_{34} - p_{13}p_{34}p_{40} - p_{13}p_{34}p_{46}p_{60} + p_{13}p_{37}^{(5)}] - (1 - p_{95}^{(2)})(p_{13}p_{34}p_{40} + p_{13}p_{34}p_{46}p_{60})$
$= (1 - p_{95}^{(2)})[1 - p_{12}p_{21} - p_{12}p_{25} - p_{13}p_{34} + p_{13}p_{34}p_{40} + p_{13}p_{34}p_{46}p_{60} - p_{13}p_{37}^{(5)} - p_{13}p_{34}p_{40} - p_{13}p_{34}p_{46}p_{60}]$
$= (1 - p_{95}^{(2)})[1 - p_{12}p_{21} - p_{12}p_{25} - p_{13}p_{34} - p_{13}p_{37}^{(5)}]$
$= (1 - p_{95}^{(2)})[1 - p_{12}(p_{21} + p_{25}) - p_{13}(p_{34} + p_{37}^{(5)})] = (1 - p_{95}^{(2)})[1 - p_{12} - p_{13}]$
$= (1 - p_{95}^{(2)})[1 - (p_{12} + p_{13})] = (1 - p_{95}^{(2)})[1 - 1] = 0$
The steady state probability that the system will be up in the long run is given by
$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} sA_0^*(s) = \lim_{s\to 0} \frac{sN_2(s)}{D_2(s)} = \lim_{s\to 0} N_2(s) \lim_{s\to 0} \frac{s}{D_2(s)}$

(1.6.15)
As s→ 0, $D_2(s)$ becomes zero. Thus **(1.6.15)** becomes indeterminate form.

Journal of Information Systems Engineering and Management
2024, 9(4)
e-ISSN: 2468-4376
https://www.jisem-journal.com/ **Research Article**

Therefore, by L' Hospital's rule, A_0 becomes
$A_0 = N_2(0)/D_2'(0)$

(1.6.16)
To obtain $D_2'(0)$, we note that
$q_{ij}^*(0) = -m_{ij}$ and $q_{ij}^{(k)*}(0) = -m_{ij}^{(k)}$
Now we collect the coefficient of $m_{ij}'s$ in $D_2'(0)$ as follows
Coefficient of $m_{01} = p_{13}p_{34}(p_{40} + p_{46}p_{60})p_{91}^{(2)} + p_{13}p_{34} + (p_{13}p_{37}^{(5)} + p_{12}p_{25})p_{90} = A$
Coefficient of $m_{12} = 1 - p_{95}^{(2)} = B$
Coefficient of $m_{13} = 1 - p_{95}^{(2)}$
Coefficient of $m_{21} = p_{12}(1 - p_{95}^{(2)}) = C$
Coefficient of $m_{25} = p_{12}(1 - p_{95}^{(2)})$
Coefficient of $m_{34} = p_{13}(1 - p_{95}^{(2)}) = D$
Coefficient of $m_{37}^{(5)} = p_{13}(1 - p_{95}^{(2)})$
Coefficient of $m_{40} = p_{13}p_{34}(1 - p_{95}^{(2)}) = E$
Coefficient of $m_{46} = p_{13}p_{34}(1 - p_{95}^{(2)})$
Coefficient of $m_{48}^{(7)} = p_{13}p_{34}(1 - p_{95}^{(2)})$
Coefficient of $m_{49}^{(7)} = p_{13}p_{34}(1 - p_{95}^{(2)})$
Coefficient of $m_{57} = p_{95}^{(2)}p_{13}(1 - p_{34}(p_{40} + p_{46}p_{60})) + p_{12}p_{25} = F$
Coefficient of $m_{60} = p_{13}p_{34}p_{46}(1 - p_{95}^{(2)}) = G$
Coefficient of $m_{69}^{(8)} = p_{13}p_{34}p_{46}(1 - p_{95}^{(2)})$
Coefficient of $m_{78} = p_{95}^{(2)}(1 - p_{12}p_{21} - p_{13}p_{34}(p_{40} + p_{46}p_{60})) + (p_{13}p_{37}^{(5)} + p_{12}p_{25})(1 - p_{95}^{(2)}) = H$
Coefficient of $m_{79} = p_{95}^{(2)}(1 - p_{12}p_{21} - p_{13}p_{34}(p_{40} + p_{46}p_{60})) + (p_{13}p_{37}^{(5)} + p_{12}p_{25})(1 - p_{95}^{(2)})$
Coefficient of $m_{89} = p_{95}^{(2)}[p_{78}(1 - p_{12}p_{21}) - p_{13}p_{34}(p_{40} + p_{46}p_{60})] + (1 - p_{95}^{(2)})$
$[p_{13}(p_{34}p_{48}^{(7)} + p_{37}^{(5)}p_{78})] + p_{12}p_{25}p_{78} = I$
Coefficient of $m_{90} = (1 - p_{12}p_{21}) - p_{13}p_{34}(p_{40} + p_{46}p_{60})$
Coefficient of $m_{91}^{(2)} = (1 - p_{12}p_{21}) - p_{13}p_{34}(p_{40} + p_{46}p_{60}) = J$
Coefficient of $m_{95}^{(2)} = (1 - p_{12}p_{21}) - p_{13}p_{34}(p_{40} + p_{46}p_{60})$
Therefore,
$D_2'(0) = m_{01}(A) + (m_{12} + m_{13})(B) + (m_{21} + m_{25})(C) + (m_{34} + m_{37}^{(5)})(D) + (m_{40} + m_{46} + m_{48}^{(7)} + m_{49}^{(7)})(E) + (m_{57})(F) + (m_{60} + m_{69}^{(8)})(G) + (m_{78} + m_{79})(H) + (m_{89})(I) + (m_{90} + m_{91}^{(2)} + m_{95}^{(2)})(J)$
Using the relation $\sum_j m_{ij} = \mu_i$
$D_2'(0) = \mu_0\{p_{13}p_{34}(p_{40} + p_{46}p_{60})p_{91}^{(2)} + p_{13}p_{34} + (p_{13}p_{37}^{(5)} + p_{12}p_{25})p_{90}\} + (1 - p_{95}^{(2)})\{\mu_1 + \mu_2p_{12} + \mu_3p_{13} + \mu_4p_{13}p_{34} + \mu_6p_{13}p_{34}p_{46}\} + \mu_5\{p_{95}^{(2)}p_{13}(1 - p_{34}(p_{40} + p_{46}p_{60})) + p_{12}p_{25}\} + \mu_7\{p_{95}^{(2)}(1 - p_{12}p_{21} - p_{13}p_{34}(p_{40} + p_{46}p_{60})) + (p_{13}p_{37}^{(5)} + p_{12}p_{25})(1 - p_{95}^{(2)})\} \mu_8\{p_{95}^{(2)}[p_{78}(1 - p_{12}p_{21}) - p_{13}p_{34}(p_{40} + p_{46}p_{60})] + (1 - p_{95}^{(2)})[p_{13}(p_{34}p_{48}^{(7)} + p_{37}^{(5)}p_{78})] + p_{12}p_{25}p_{78}\} + \mu_9\{(1 - p_{12}p_{21}) - p_{13}p_{34}(p_{40} + p_{46}p_{60})\}$

(1.6.17)
Using the results **(1.6.14)** and **(1.6.17)** in **(1.6.16),** we get the expression for A_0.
The expected up (operative) time of the system during (0,t] is given by

Journal of Information Systems Engineering and Management
2024, 9(4)
e-ISSN: 2468-4376
https://www.jisem-journal.com/ **Research Article**

$\mu_{up}(t) = \int_0^t A_0\,(u)du$
so that,
$\mu^*_{up}(s) = \frac{A^*_0(s)}{s}$
And expected down time of the system during (0,t] is given by
$\mu_{dn}(t) = t - \int_0^t A_0\,(u)du$
so that
$\mu^*_{dn}(s) = \frac{1}{s^2} - \mu^*_{up}(s)$

1.7 BUSY PERIOD ANALYSIS
a) FOR EXPERT REPAIRMAN

Let $B^e_i(t)$ be the probability that the expert repairman is busy in the repair of failed unit at epoch t, when the system initially starts operation from state $S_i \in E$. The expression for $B^e_0(t)$ can be written by the following mutually exclusive contingencies-

1) The system transits from state S_0 to S_1 during $(u, u + du), u < t$ and then repairman may be found busy at epoch $(t - u)$ starting from S_1. The probability of this event is

$$\int_0^t q_{01}(u)du\, B^e_1(t-u) = q_{01}(t) © B^e_1(t)$$

(1.7.1a)

Therefore, $B^e_0(t)$ becomes
$B^e_0(t) = q_{01}(t) © B^e_1(t)$
Similarly,
$B^e_1(t) = q_{12}(t)©B^e_2(t) + q_{13}(t)© B^e_3(t)$
$B^e_2(t) = q_{21}(t)©B^e_1(t) + q_{25}(t)©B^e_5(t)$
$B^e_3(t) = Z_3(t) + q_{34}(t)©B^e_4(t) + q^{(5)}_{37}(t)©B^e_7(t)$
$B^e_4(t) = q_{40}(t)©B^e_0(t) + q_{46}(t)©B^e_6(t) + q^{(7)}_{48}(t)©B^e_8(t) + q^{(7)}_{49}(t)©B^e_9(t)$
$B^e_5(t) = Z_5(t) + q_{57}(t)©B^e_7(t)$
$B^e_6(t) = q_{60}(t)©B^e_0(t) + q^{(8)}_{69}(t)©B^e_9(t)$
$B^e_7(t) = q_{78}(t)©B^e_8(t) + q_{79}(t)©B^e_9(t)$
$B^e_8(t) = q_{89}(t)©B^e_9(t)$
$B^e_9(t) = q_{90}(t)©B^e_0(t) + q^{(2)}_{91}(t)©B^e_1(t) + q^{(2)}_{95}(t)©B^e_5(t)$

(1.7.2a-1.7.10a)

where,
$Z_3 = e^{-(\beta_1+\alpha_2)t}$ $Z_5 = e^{-\alpha_2 t}$
Taking L.T of **(1.7.1a-1.7.10a)**, we get
$B^{e*}_0(s) = q^*_{01}(s)B^{e*}_1(s)$
$B^{e*}_1(s) = q^*_{12}(s)B^{e*}_2(s) + q^*_{12}(s)B^{e*}_2(s)$
$B^{e*}_2(s) = q^*_{21}(s)B^{e*}_1(s) + q^*_{25}(s)B^{e*}_5(s)$
$B^{e*}_3(s) = Z^*_3(s) + q^*_{34}(s)B^{e*}_4(s) + q^{(5)*}_{37}(s)B^{e*}_7(s)$
$B^{e*}_4(s) = q^*_{40}(s)B^{e*}_0(s) + q^*_{46}(s)B^{e*}_6(s) + q^{(7)*}_{48}(s)B^{e*}_8(s) + q^{(7)*}_{49}(s)B^{e*}_9(s)$
$B^{e*}_5(t) = Z^*_5(s) + q^*_{57}(s)©B^{e*}_7(s)$
$B^{e*}_6(t) = q^*_{60}(s)B^{e*}_0(s) + q^{(8)*}_{69}(s)B^{e*}_9(s)$
$B^{e*}_7(t) = q^*_{78}(s)B^{e*}_8(s) + q^*_{79}(s)B^{e*}_9(s)$
$B^{e*}_8(t) = q^*_{89}(s)B^{e*}_9(s)$
$B^{e*}_9(t) = q^*_{90}(s)B^{e*}_0(s) + q^{(2)*}_{91}(s)B^{e*}_1(s) + +q^{(2)*}_{95}(s)B^{e*}_5(s)$

Journal of Information Systems Engineering and Management
2024, 9(4)
e-ISSN: 2468-4376
https://www.jisem-journal.com/ **Research Article**

Solving the resultant set of equations and simplifying for $B_0^{e*}(s)$, we have
$B_0^{e*}(s) = N_3^e(s)/D_2(s)$

(1.7.11a)
Where,
$$N_3^e(s) = q_{01}^*\left[\left(1 - q_{95}^{(2)*}q_{57}^*(q_{78}^*q_{89}^* + q_{79}^*)\right)\left[q_{12}^*q_{25}^* + q_{13}^*q_{34}^*\left(q_{46}^*q_{69}^{(8)*} + q_{48}^{(7)*}q_{89}^* + q_{49}^{(7)*}\right) + q_{13}^*q_{37}^{(5)*}(q_{78}^*q_{89}^* + q_{79}^*) + q_{12}^*q_{25}^*q_{57}^*(q_{78}^*q_{89}^* + q_{79}^*)\right]Z_5^*\right] + q_{01}^*\left(1 - q_{95}^{(2)*}q_{57}^*(q_{78}^*q_{89}^* + q_{79}^*)\right)q_{13}^*Z_3^*$$
and $D_2(s)$ is same as given by **(1.6.12)**.

In the long run, the expected fraction of time for which the expert server is busy in the repair of failed unit is given by
$$B_0^e = \lim_{t\to\infty} B_0^e(t) = \lim_{s\to 0} s B_0^{e*}(s) = \frac{N_3^e(0)}{D_2'(0)}$$

(1.7.12a)
where
$$N_3^e(0) = p_{01}\left[\left(1 - p_{95}^{(2)}p_{57}(p_{78}p_{89} + p_{79})\right)\left(p_{12}p_{25} + p_{12}p_{25}p_{57}(p_{78}p_{89} + p_{79}) + p_{13}p_{34}\left(p_{46}p_{69}^{(8)} + p_{48}^{(7)}p_{89} + p_{49}^{(7)}\right) + p_{13}p_{37}^{(5)}(p_{78}p_{89} + p_{79})\right)\mu_5\right] + p_{01}\left(1 - p_{95}^{(2)}p_{57}(p_{78}p_{89} + p_{79})\right)p_{13}\mu_3$$

(1.7.13a)
and $D_2'(0)$ is same as given by **(1.6.17)**.
Thus using **(1.7.13a)** and **(1.6.17)** in **(1.7.12a)**, we get the expression for B_0^e.

The expected busy period of expert repairman due to repair of failed unit during the time interval (0,t] is given by
$$\mu_b^e(t) = \int_0^t B_0^e(u)du$$
So that
$$\mu_b^{e*}(s) = \frac{B_0^{e*}(s)}{s}$$

b) FOR ORDINARY REPAIRMAN

Let $B_i^{or}(t)$ be the probability that the ordinary repairman is busy in the repair of failed unit at epoch t, when the system initially starts operation from state $S_i \in E$.
Therefore, $B_0^{or}(t)$ becomes
$$B_0^{or}(t) = q_{01}(t) © B_1^{or}(t)$$

(1.7.1b)
Similarly,
$$B_1^{or}(t) = q_{12}(t)©B_2^{or}(t) + q_{13}(t)© B_3^{or}(t)$$
$$B_2^{or}(t) = Z_2(t) + q_{21}(t)©B_1^{or}(t) + q_{25}(t)©B_5^{or}(t)$$
$$B_3^{or}(t) = q_{34}(t)©B_4^{or}(t) + q_{37}^{(5)}(t)©B_5^{or}(t)$$
$$B_4^{or}(t) = q_{40}(t)©B_0^{or}(t) + q_{46}(t)©B_6^{or}(t) + q_{48}^{(7)}(t)©B_8^{or}(t) + q_{49}^{(7)}(t)©B_9^{or}(t)$$
$$B_5^{or}(t) = q_{57}(t)©B_7^{or}(t)$$
$$B_6^{or}(t) = q_{60}(t)©B_0^{or}(t) + q_{69}^{(8)}(t)©B_9^{or}(t)$$
$$B_7^{or}(t) = q_{78}(t)©B_8^{or}(t) + q_{79}(t)©B_9^{or}(t)$$
$$B_8^{or}(t) = q_{89}(t)© B_9^{or}(t)$$
$$B_9^{or}(t) = Z_9^*(s) + q_{90}(t)©B_0^{or}(t) + q_{91}^{(2)}(t)©B_1^{or}(t) + q_{95}^{(2)}(t)©B_5^{or}(t)$$

Journal of Information Systems Engineering and Management
2024, 9(4)
e-ISSN: 2468-4376
https://www.jisem-journal.com/ **Research Article**

(1.7.2b-1.7.10b)
where
$Z_2 = e^{-(\beta_2+\alpha_3)t}$ $\quad Z_9 = e^{-(\beta_2+\alpha_1)t}$
Taking L.T of **(1.7.1b-1.7.10b)**, we get
$B_0^{or*}(s) = q_{01}^*(s)B_1^{or*}(s)$
$B_1^{or*}(s) = q_{12}^*(s)B_2^{or*}(s) + q_{12}^*(s)B_2^{or*}(s)$
$B_2^{or*}(s) = Z_2^*(s) + q_{21}^*(s)B_1^{or*}(s) + q_{25}^*(s)B_5^{or*}(s)$
$B_3^{or*}(s) = q_{34}^*(s)B_4^{or*}(s) + q_{37}^{(5)*}(s)B_7^{or*}(s)$
$B_4^{or*}(s) = q_{40}^*(s)B_0^{or*}(s) + q_{46}^*(s)B_6^{or*}(s) + q_{48}^{(7)*}(s)B_8^{or*}(s) + q_{49}^{(7)*}(s)B_9^{or*}(s)$
$B_5^{or*}(t) = q_{57}^*(s)©B_7^{or*}(s)$
$B_6^{or*}(t) = q_{60}^*(s)B_0^{or*}(s) + q_{69}^{(8)*}(s)B_9^{or*}(s)$
$B_7^{or*}(t) = q_{78}^*(s)B_8^{or*}(s) + q_{79}^*(s)B_9^{or*}(s)$
$B_8^{or*}(t) = q_{89}^*(s)B_9^{or*}(s)$
$B_9^{or*}(t) = Z_9^*(s) + q_{90}^*(s)B_0^{or*}(s) + q_{91}^{(2)*}(s)B_1^{or*}(s) + q_{95}^{(2)*}(s)B_5^{or*}(s)$
Solving the resultant set of equations and simplifying for $B_0^{or*}(s)$, we have
$B_0^{or*}(s) = N_4^{or}(s)/D_2(s)$

(1.7.11b)
where,
$N_4^{or}(s) = q_{01}^*q_{12}^*\left(1 - q_{95}^{(2)*}q_{57}^*(q_{78}^*q_{89}^* + q_{79}^*)\right)Z_2^* + q_{01}^*[q_{13}^*q_{34}^*(q_{46}^*q_{69}^{(8)*} + q_{48}^{(7)*}q_{89}^* + q_{49}^{(7)*}) + q_{13}^*q_{37}^{(5)*}(q_{78}^*q_{89}^* + q_{79}^*) + q_{12}^*q_{25}^*q_{57}^*(q_{78}^*q_{89}^* + q_{79}^*)]Z_9^*$
and $D_2(s)$ is same as given by **(1.6.12)**.
In the long run, the probability that the ordinary repairman will be busy is given by
$B_0^{or} = \lim_{t\to\infty} B_0^{or}(t) = \lim_{s\to 0} sB_0^{or*}(s) = \frac{N_4^{or}(0)}{D_2'(0)}$

(1.7.12b)
where
$N_4^{or}(0) = p_{12}p_{01}\left[\left(1 - p_{95}^{(2)}p_{57}(p_{78}p_{89} + p_{79})\right)\mu_2\right] + p_{01}\left[\left(p_{12}p_{25}p_{57}(p_{78}p_{89} + p_{79}) + p_{13}p_{34}(p_{46}p_{69}^{(8)} + p_{48}^{(7)}p_{89} + p_{49}^{(7)}) + p_{13}p_{37}^{(5)}(p_{78}p_{89} + p_{79})\right)\right]\mu_9$
(1.7.13b)
and $D_2'(0)$ is same as given by **(1.6.17)**.
Then using **(1.7.13b)** and **(1.6.17)** in **(1.7.12b)**, we get the expression for B_0^{or}.
The expected busy period of ordinary repairman during the time interval (0, t] is given by
$\mu_b^{or}(t) = \int_0^t B_0^{or}(u)du$
So that
$\mu_b^{or*}(s) = \frac{B_0^{or*}(s)}{s}$

1.8) EXPECTED NUMBER OF REPLACEMENTS BY EXPERT SERVER

Let $V_i^R(t)$ be the expected number of replacements made by the expert server in (0,t] given that the system starts from the regenerative state S_i at t=0. Using the definition of $V_i^R(t)$, the recursive relations among $V_i^R(t)$ are given by
$V_0^R(t) = Q_{01}(t)® V_1^R(t)$
$V_1^R(t) = Q_{12}(t)®V_2^R(t) + Q_{13}(t) ® V_3^R(t)$
$V_2^R(t) = Q_{21}(t) ® V_1^R(t) + Q_{25}(t) ® V_5^R(t)$

Journal of Information Systems Engineering and Management
2024, 9(4)
e-ISSN: 2468-4376
https://www.jisem-journal.com/ **Research Article**

$V_3^R(t) = Q_{34}(t)Ⓡ\, V_4^R(t) + Q_{37}^{(5)}(t)Ⓡ\, V_7^R(t)$
$V_4^R(t) = Q_{40}(t)Ⓡ\, V_0^R(t) + Q_{46}(t)Ⓡ V_6^R(t) + Q_{48}^{(7)}(t)\, Ⓡ\, V_8^R(t) + Q_{49}^{(7)}(t)\, Ⓡ\, V_9^R(t)$
$V_5^R(t) = Q_{57}(t)Ⓡ V_7^R(t)$
$V_6^R(t) = Q_{60}(t)\, Ⓡ \left(1 + V_0^R(t)\right) + Q_{69}^{(8)}(t)\, Ⓡ\, V_9^R(t)$
$V_7^R(t) = Q_{78}(t)Ⓡ V_8^R + Q_{79}(t)Ⓡ V_9^R(t)$
$V_8^R(t) = Q_{89}(t)Ⓡ \left(1 + V_9^R(t)\right)$
$V_9^R(t) = Q_{90}(t)Ⓡ\, V_0^R(t) + Q_{91}^{(2)}(t)\, Ⓡ\, V_1^R(t) + Q_{95}^{(2)}(t)\, Ⓡ\, V_5^R(t)$

(1.8.1-1.8.10)
Taking the Laplace Stieltjes Transform of relations **(1.8.1-1.8.10)**, we get
$\tilde{V}_0^R(s) = \tilde{Q}_{01}(s)\tilde{V}_1^R(s)$
$\tilde{V}_1^R(s) = \tilde{Q}_{12}(s)\tilde{V}_2^R(s) + \tilde{Q}_{13}(s)\tilde{V}_3^R(s)$
$\tilde{V}_2^R(s) = \tilde{Q}_{21}(s)\ \tilde{V}_1^R(s) + \tilde{Q}_{25}(s)\tilde{V}_5^R(s)$
$\tilde{V}_3^R(s) = \tilde{Q}_{34}(s)\tilde{V}_4^R(s) + \tilde{Q}_{37}^{(5)}(s)\tilde{V}_7^R(s)$
$\tilde{V}_4^R(s) = \tilde{Q}_{40}(s)\tilde{V}_0^R(s) + \tilde{Q}_{46}(s)\tilde{V}_6^R(s) + \tilde{Q}_{48}^{(7)}(s)\tilde{V}_8^R(s) + \tilde{Q}_{49}^{(7)}(s)\tilde{V}_9^R(s)$
$\tilde{V}_5^R(s) = \tilde{Q}_{57}(s)\tilde{V}_7^R(s)$
$\tilde{V}_6^R(s) = \tilde{Q}_{60}(s)[1 + \tilde{V}_0^R(s)] + \tilde{Q}_{69}^{(8)}(s)\tilde{V}_9^R(s)$
$\tilde{V}_7^R(s) = \tilde{Q}_{78}(s)\tilde{V}_8^R(s) + \tilde{Q}_{79}(s)\tilde{V}_9^R(s)$
$\tilde{V}_8^R(s) = \tilde{Q}_{89}(s)[1 + \tilde{V}_9^R(s)]$
$\tilde{V}_9^R(s) = \tilde{Q}_{90}(s)\tilde{V}_0^R(s) + \tilde{Q}_{91}^{(2)}(s)\tilde{V}_1^R(s) + \tilde{Q}_{95}^{(2)}(s)\tilde{V}_5^R(s)$
The solution for $\tilde{V}_0^R(s)$ can be written in the following form:
$\tilde{V}_0^R(s) = \frac{N_5^R(s)}{D_2(s)}$

(1.8.11)
where,
$N_5^R(s) = \tilde{Q}_{01}\left(1 - \tilde{Q}_{95}^{(2)}\tilde{Q}_{57}(\tilde{Q}_{78}\tilde{Q}_{89} + \tilde{Q}_{79})\right)\left(\tilde{Q}_{12}\tilde{Q}_{25}\tilde{Q}_{57}\tilde{Q}_{78} + \tilde{Q}_{13}(\tilde{Q}_{34}\tilde{Q}_{46} + \tilde{Q}_{37}^{(5)}\tilde{Q}_{78})\right) + [\tilde{Q}_{01}(\tilde{Q}_{12}\tilde{Q}_{25}\tilde{Q}_{57} + \tilde{Q}_{13}\tilde{Q}_{37}^{(5)})(\tilde{Q}_{78}\tilde{Q}_{89} + \tilde{Q}_{79}) + \tilde{Q}_{13}\tilde{Q}_{34}(\tilde{Q}_{46}\tilde{Q}_{69}^{(8)} + \tilde{Q}_{48}^{(7)}\tilde{Q}_{89} + \tilde{Q}_{49}^{(7)})\tilde{Q}_{60}]$
and $D_2(s)$ is same as given by **(1.6.12).**
In steady-state per-unit of time expected number of replacements by expert server is given by
$V_0^R = \lim_{t\to\infty} \frac{V_0^R(t)}{t} = \lim_{s\to 0} s\tilde{V}_0^R(s) = \frac{N_5^R(0)}{D_2'(0)}$

(1.8.12)
$N_5^R = p_{01}\left(1 - p_{95}^{(2)}p_{57}(p_{78}p_{89} + p_{79})\right)\left(p_{12}p_{25}p_{57}p_{78} + p_{13}(p_{34}p_{46} + p_{37}^{(5)}p_{78})\right) + [p_{01}(p_{12}p_{25}p_{57} + p_{13}p_{37}^{(5)})(p_{78}p_{89} + p_{79}) + p_{13}p_{34}(p_{46}p_{69}^{(8)} + p_{48}^{(7)}p_{89} + p_{49}^{(7)})p_{90}]$

(1.8.13)
and $D_2'(0)$ is same as given by **(1.6.17)**.
Here we have used $\tilde{Q}_{ij}(0) = p_{ij}$
Thus using **(1.8.13)** and **(1.6.17)** in **(1.8.12),** we get the expression for V_0^R.

Journal of Information Systems Engineering and Management
2024, 9(4)
e-ISSN: 2468-4376
https://www.jisem-journal.com/ **Research Article**

a) EXPECTED NUMBER OF REPAIRS BY EXPERT REPAIRMAN

Let $V_i^e(t)$ be the expected number of repairs by the expert repairman in (0,t] given that the system starts from the regenerative state S_i at t=0. The recursive relation for $V_i^e(t)$ are given by

$V_0^e(t) = Q_{01}(t)® V_1^e(t)$
$V_1^e(t) = Q_{12}(t)®V_2^e(t) + Q_{13}(t) ® V_3^e(t)$
$V_2^e(t) = Q_{21}(t) ® V_1^e(t) + Q_{25}(t) ® V_5^e(t)$
$V_3^e(t) = Q_{34}(t)®\left(1 + V_4^e(t)\right) + Q_{37}^{(5)}(t)® V_7^e(t)$
$V_4^e(t) = Q_{40}(t)® V_0^e(t) + Q_{46}(t)® V_6^e(t) + Q_{48}^{(7)}(t) ® V_8^e(t) + Q_{49}^{(7)}(t) ® V_9^e(t)$
$V_5^e(t) = Q_{57}(t)®\left(1 + V_7^e(t)\right)$
$V_6^e(t) = Q_{60}(t) ®V_0^e(t) + Q_{69}^{(8)}(t) ® V_9^e(t)$
$V_7^e(t) = Q_{78}(t)®V_8^e(t) + Q_{79}(t)®V_9^e(t)$
$V_8^e(t) = Q_{89}(t)®V_9^e(t)$
$V_9^e(t) = Q_{90}(t)® V_0^e(t) + Q_{91}^{(2)}(t) ® V_1^e(t) + Q_{95}^{(2)}(t) ® V_5^e(t)$

(1.8.1a-1.8.10a)

Taking the Laplace Stieltjes Transform of relations **(1.8.1a-1.8.10a)**,we get

$\tilde{V}_0^e(s) = \tilde{Q}_{01}(s)\tilde{V}_1^e(s)$
$\tilde{V}_1^e(s) = \tilde{Q}_{12}(s)\tilde{V}_2^e(s) + \tilde{Q}_{13}(s)\tilde{V}_3^e(s)$
$\tilde{V}_2^e(s) = \tilde{Q}_{21}(s)\tilde{V}_1^e(s) + \tilde{Q}_{25}(s)\tilde{V}_5^e(s)$
$\tilde{V}_3^e(s) = \tilde{Q}_{34}(s)[1 + \tilde{V}_4^e(s)] + \tilde{Q}_{37}^{(5)}(s)\tilde{V}_7^e(s)$
$\tilde{V}_4^e(s) = \tilde{Q}_{40}(s)\tilde{V}_0^e(s) + \tilde{Q}_{46}(s)\tilde{V}_6^e(s) + \tilde{Q}_{48}^{(7)}(s)\tilde{V}_8^e(s) + \tilde{Q}_{49}^{(7)}(s)\tilde{V}_9^e(s)$
$\tilde{V}_5^e(s) = \tilde{Q}_{57}(s)\left[1 + \tilde{V}_7^e(s)\right]$
$\tilde{V}_6^e(s) = \tilde{Q}_{60}(s)\tilde{V}_0^e(s) + \tilde{Q}_{69}^{(8)}(s)\tilde{V}_9^e(s)$
$\tilde{V}_7^e(s) = \tilde{Q}_{78}(s)\tilde{V}_8^e(s) + \tilde{Q}_{79}(s)\tilde{V}_9^e(s)$
$\tilde{V}_8^e(s) = \tilde{Q}_{89}(s)\tilde{V}_9^e(s)$
$\tilde{V}_9^e(s) = \tilde{Q}_{90}(s)\tilde{V}_0^e(s) + \tilde{Q}_{91}^{(2)}(s)\tilde{V}_1^e(s) + \tilde{Q}_{95}^{(2)}(s)\tilde{V}_5^e(s)$

The solution for $\tilde{V}_0^e(s)$ can be written in the following form:

$$\tilde{V}_0^e(s) = \frac{N_6^e(s)}{D_2(s)}$$

(1.8.11a)

where,

$$N_6^e(s) = \left(\tilde{Q}_{12}\tilde{Q}_{25} + \tilde{Q}_{13}\tilde{Q}_{34}\right)\left(1 - \tilde{Q}_{95}^{(2)}\right) + \tilde{Q}_{13}\tilde{Q}_{95}^{(2)}\left[\tilde{Q}_{34}\tilde{Q}_{46}\tilde{Q}_{69}^{(8)} + \tilde{Q}_{34}\tilde{Q}_{48} + \tilde{Q}_{34}\tilde{Q}_{49}^{(7)} + \tilde{Q}_{12}\tilde{Q}_{25}\tilde{Q}_{37}^{(5)}\tilde{Q}_{78} + \tilde{Q}_{37}^{(5)}\tilde{Q}_{79}\right]$$

and $D_2(s)$ is same as given by **(1.6.12).**

In steady-state per-unit of time expected number of repairs by expert repairman is given by

$$V_0^e = \lim_{t\to\infty}\frac{V_0^e(t)}{t} = \lim_{s\to 0} s\tilde{V}_0^e(s) = \frac{N_6^e(0)}{D_2'(0)}$$

(1.8.12a)

$$N_6^e(0) = (p_{12}p_{25} + p_{13}p_{34})\left(1 - p_{95}^{(2)}\right) + p_{13}p_{95}^{(2)}\left[p_{34}p_{46}p_{69}^{(8)} + p_{34}p_{48} + p_{34}p_{49}^{(7)} + p_{12}p_{25}p_{37}^{(5)}p_{78} + p_{37}^{(5)}p_{79}\right]$$

(1.8.13a)

and $D_2'(0)$ is same as given by **(1.6.17).**

Here we have used $\tilde{Q}_{ij}(0) = p_{ij}$

Thus using **(1.8.13a)** and **(1.6.17)** in **(1.8.12a),** we get the expression for V_0^e

Journal of Information Systems Engineering and Management
2024, 9(4)
e-ISSN: 2468-4376
https://www.jisem-journal.com/ **Research Article**

b) EXPECTED NUMBER OF REPAIRS BY ORDINARY REPAIRMAN

Let $V_i^o(t)$be the expected number of repairs by the ordinary repairman in (0,t] given that the system starts from the regenerative state S_i at t=0. The recursive relation for $V_i^o(t)$ are given by

$V_0^o(t) = Q_{01}(t)® V_1^o(t)$
$V_1^o(t) = Q_{12}(t)®V_2^o(t) + Q_{13}(t) ® V_3^o(t)$
$V_2^o(t) = Q_{21}(t) ® \left(1 + V_1^o(t)\right) + Q_{25}(t) ® V_5^o(t)$
$V_3^o(t) = Q_{34}(t)® V_4^o(t) + Q_{37}^{(5)}(t)® V_7^o(t)$
$V_4^o(t) = Q_{40}(t)® V_0^o(t) + Q_{46}(t)® V_6^o(t) + Q_{48}^{(7)}(t) ® V_8^o(t) + Q_{49}^{(7)}(t) ® V_9^o(t)$
$V_5^o(t) = Q_{57}(t)®V_7^o(t)$
$V_6^o(t) = Q_{60}(t) ®V_0^o(t) + Q_{69}^{(8)}(t) ® V_9^o(t)$
$V_7^o(t) = Q_{78}(t)®V_8^o(t) + Q_{79}(t)®V_9^o(t)$
$V_8^o(t) = Q_{89}(t)®V_9^o(t)$
$V_9^o(t) = Q_{90}(t)® \left(1 + V_0^o(t)\right) + Q_{91}^{(2)}(t) ® \left(1 + V_1^o(t)\right) + Q_{95}^{(2)}(t) ® V_5^o(t)$

(1.8.1b-1.8.10b)

Taking the Laplace Stieltjes Transform of relations **(1.8.1b-1.8.10b)**,we get

$\tilde{V}_0^o(s) = \tilde{Q}_{01}(s)\tilde{V}_1^o(s)$
$\tilde{V}_1^o(s) = \tilde{Q}_{12}(s)\tilde{V}_2^o(s) + \tilde{Q}_{13}(s)\tilde{V}_3^o(s)$
$\tilde{V}_2^o(s) = \tilde{Q}_{21}(s)\left(1 + \tilde{V}_1^o(s)\right) + \tilde{Q}_{25}(s)\tilde{V}_5^o(s)$
$\tilde{V}_3^o(s) = \tilde{Q}_{34}(s)\tilde{V}_4^o(s) + \tilde{Q}_{37}^{(5)}(s)\tilde{V}_7^o(s)$
$\tilde{V}_4^o(s) = \tilde{Q}_{40}(s)\tilde{V}_0^o(s) + \tilde{Q}_{46}(s)\tilde{V}_6^o(s) + \tilde{Q}_{48}^{(7)}(s)\tilde{V}_8^o(s) + \tilde{Q}_{49}^{(7)}(s)\tilde{V}_9^o(s)$
$\tilde{V}_5^o(s) = \tilde{Q}_{57}(s)\tilde{V}_7^o(s)$
$\tilde{V}_6^o(s) = \tilde{Q}_{60}(s)\tilde{V}_0^o(s) + \tilde{Q}_{69}^{(8)}(s)\tilde{V}_9^o(s)$
$\tilde{V}_7^o(s) = \tilde{Q}_{78}(s)\tilde{V}_8^o(s) + \tilde{Q}_{79}(s)\tilde{V}_9^o(s)$
$\tilde{V}_8^o(s) = \tilde{Q}_{89}(s)\tilde{V}_9^o(s)$
$\tilde{V}_9^o(s) = \tilde{Q}_{90}(s)\left(1 + \tilde{V}_0^o(s)\right) + \tilde{Q}_{91}^{(2)}(s)\left(1 + \tilde{V}_1^o(s)\right) + \tilde{Q}_{95}^{(2)}(s)\tilde{V}_5^o(s)$

The solution for $\tilde{V}_0^o(s)$ can be written in the following form:

$\tilde{V}_0^o(s) = \frac{N_7^o(s)}{D_2(s)}$

(1.8.11b)

where,

$N_7^o(s) = \left(1 - \tilde{Q}_{95}^{(2)}\right)[\tilde{Q}_{12} + \tilde{Q}_{13}\tilde{Q}_{37}^{(5)} + \tilde{Q}_{13}\tilde{Q}_{34}\left(\tilde{Q}_{46}\tilde{Q}_{69}^{(8)} + \tilde{Q}_{48}^{(7)} + \tilde{Q}_{49}^{(7)}\right) + \tilde{Q}_{90} + \tilde{Q}_{91}^{(2)}]$

and $D_2(s)$ is same as given by **(1.6.12).**

In steady-state per-unit of time expected number of repairs by ordinary repairman is given by

$V_0^o = \lim_{t\to\infty} \frac{V_0^o(t)}{t} = \lim_{s\to 0} s\tilde{V}_0^o(s) = \frac{N_7^o(0)}{D_2'(0)}$

(1.8.12b)

$N_7^o(0) = \left(1 - p_{95}^{(2)}\right)[p_{12} + p_{13}p_{37}^{(5)} + p_{13}p_{34}\left(p_{46}p_{69}^{(8)} + p_{48}^{(7)} + p_{49}^{(7)}\right) + p_{90} + p_{91}^{(2)}]$

(1.8.13b)

and $D_2'(0)$ is same as given by **(1.6.17).**

Here we have used $\tilde{Q}_{ij}(0) = p_{ij}$

Thus using **(1.8.13b)** and **(1.6.17)** in **(1.8.12b),** we get the expression for V_0^o.

Journal of Information Systems Engineering and Management
2024, 9(4)
e-ISSN: 2468-4376
https://www.jisem-journal.com/ Research Article

1.9 PROFIT FUNCTION ANALYSIS

Two profit functions $P_1(t)$ and $P_2(t)$ can be easily obtained for the system model under study with the help of characteristics obtained earlier.

The expected total profits incurred during (0,t] are:

$P_1(t)$ = Expected total revenue in $(0, t]$ − Expected total expenditure in $(0, t]$

$$= K_0\mu_{up}(t) - K_1\mu_b^e(t) - K_2\mu_b^o(t) \quad (1)$$

Similarly,

$$P_2(t) = K_0\mu_{up}(t) - K_3V_0^{rp}(t) - K_4V_0^e(t) - K_5V_0^o(t) \quad (2)$$

where,

K_0 is revenue per unit up time of the system.

K_1 is repair cost per unit of time by expert repairman.

K_2 is repair cost per unit of time by ordinary repairman.

K_3 is per unit repalcement cost of the failed unit.

K_4 is per unit repair cost by expert repairman.

K_5 is per unit repair cost by ordinary repairman.

Now the expected total profits per unit time, in steady state, is given by

$$P_1 = \lim_{t\to\infty}\frac{P_1(t)}{t} = \lim_{s\to 0} s^2 P_1^*(s)$$

and

$$P_2 = \lim_{t\to\infty}\frac{P_2(t)}{t} = \lim_{s\to 0} s^2 P_2^*(s)$$

so that

$$P_1 = K_0A_0 - K_1B_0^e - K_2B_0^o \quad (3)$$

and

$$P_2 = K_0A_0 - K_3V_0^{re} - K_4V_0^e - K_5V_0^o \quad (4)$$

1.10 CONCLUSION

To study the behavior of MTSF, Availability and Profit function through graphs w.r.t various parameters, we plot curves for these three characteristics w.r.t failure parameter α_1 in Fig.1.2 and 1.3 respectively for three different values of repair rate $\alpha_3 = (0.60, 0.70, 0.80)$ whereas other parameters are kept fixed as $\alpha_2 = 0.75, \gamma_1 = 0.30, \gamma_2 = 0.20, \beta_1 = 0.20, \beta_2 = 0.45, p = 0.5, q = 0.5, K_0 = 900, K_1 = 750, K_2 = 600, K_3 = 500, K_4 = 350, K_5 = 200,$

Fig 1.2 indicates that the graph for MTSF decreases steeply with the increase in the failure rate α_1 and increases with the increase in repair rate α_3 .

Fig 1.3 clearly shows that the graph for availability decreases almost exponentially with the increase in the failure rate α_1 and increases with the increase in repair rate α_3 .

A similar pattern is exhibited for the profit functions shown in fig 1.4 and shows that the graph decreases with the increase in failure rate α_1 and increases with the increase in repair rate α_3. It can also be observed from fig 1.4 that Profit function P_2 is always better than Profit function P_1 for some values of failure parameter and for fixed values of repair parameter.

Journal of Information Systems Engineering and Management
2024, 9(4)
e-ISSN: 2468-4376
https://www.jisem-journal.com/

Research Article

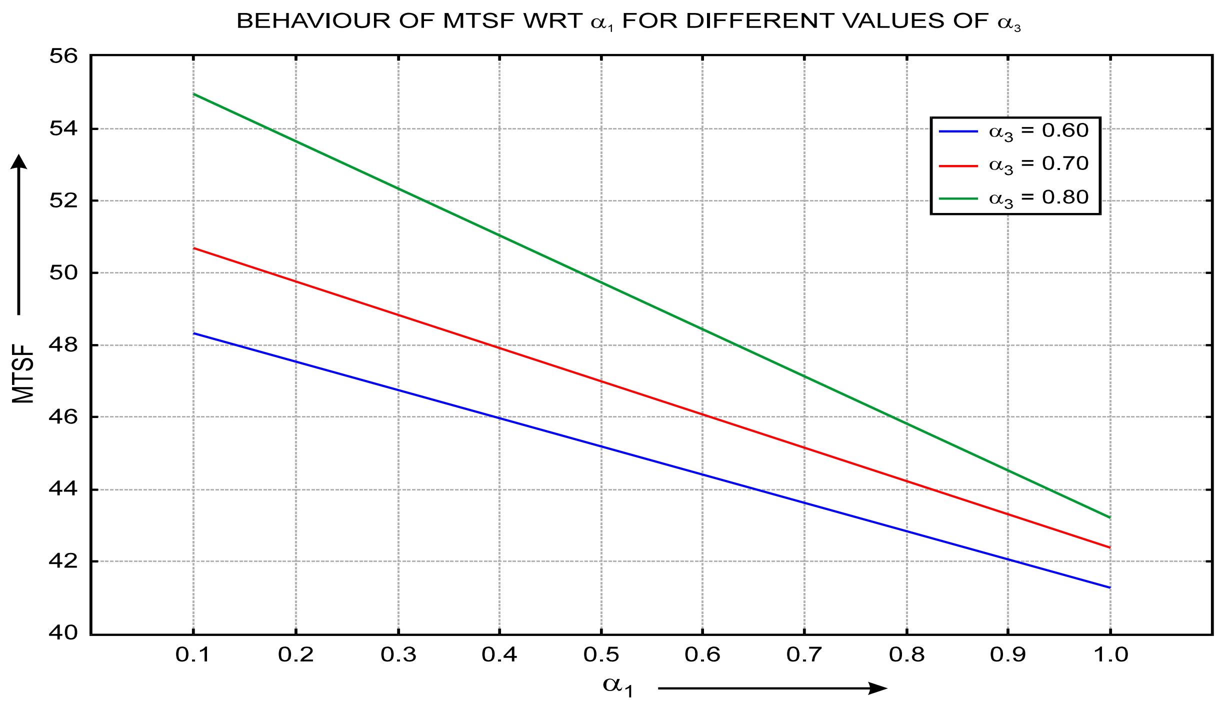

Fig 1.2

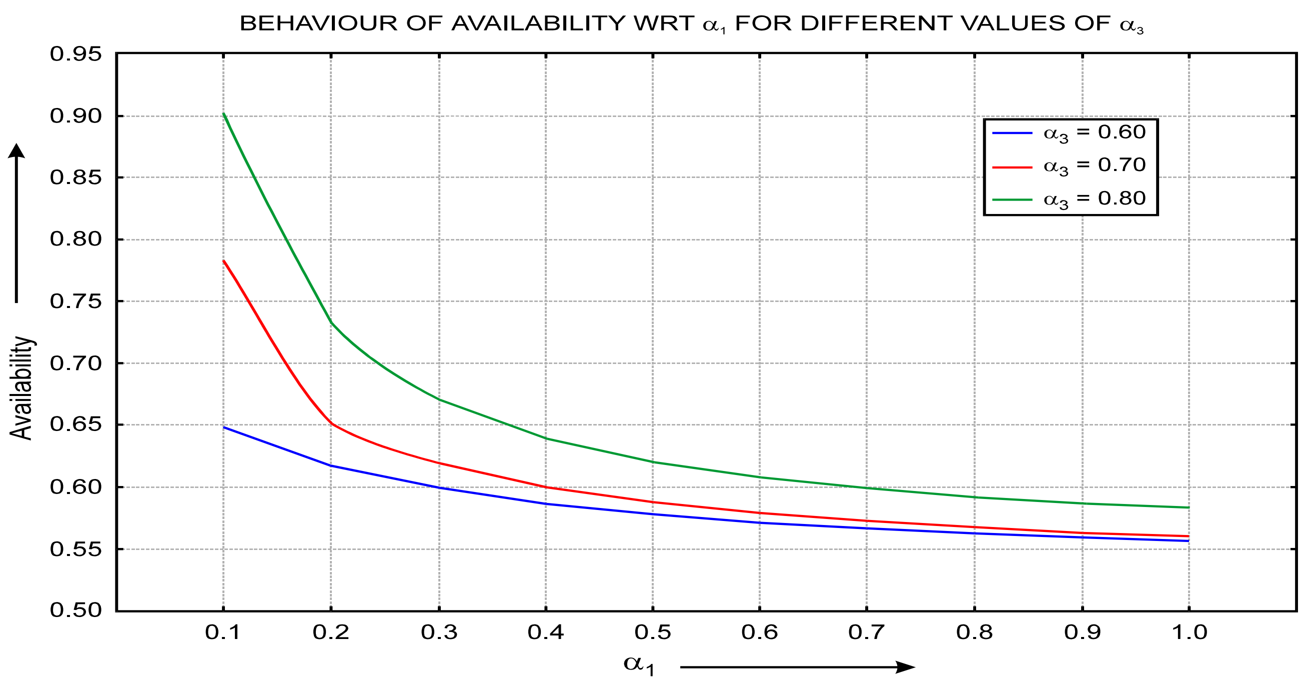

Fig 1.3

Journal of Information Systems Engineering and Management
2024, 9(4)
e-ISSN: 2468-4376
https://www.jisem-journal.com/

Research Article

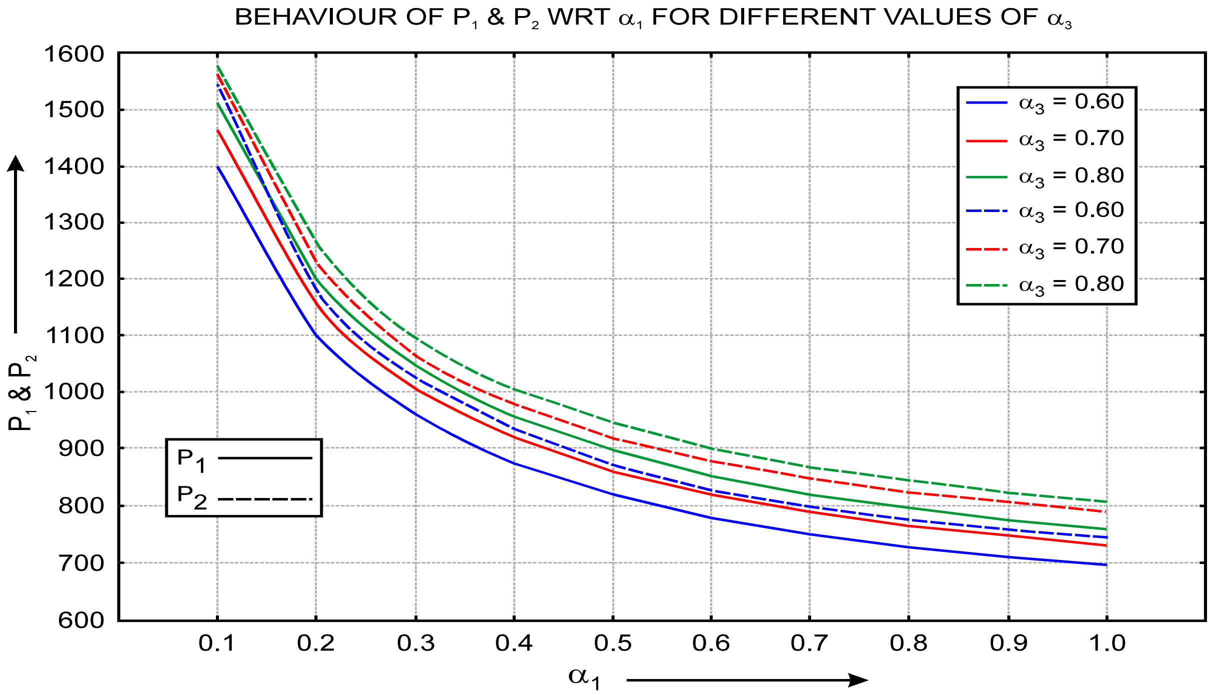

Fig 1.4